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# A factor-augmented new Keynesian Phillips curve for the European Union countries*

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## ABSTRACT

In this paper, a factor-augmented version of the hybrid New Keynesian Phillips curve (NKPC) is assessed using a data set comprised of a large panel of European Union (EU) member countries. The factor-augmentation is natural given that country-level inflation rates are highly co-moving. The presence of unattended common factors is important because it raises the issue of omitted variables bias, as the real marginal cost, which is a regressor of the NKPC, is likely to load on the same factors as inflation. One possibility here is to employ the regular instrumental variables approach. However, if the external instruments are subject to the same factors as those in the error term of the NKPC, the instruments would be invalid and the approach would therefore be inappropriate. We propose a novel econometric approach to estimate the hybrid NKPC, which allows for very general forms of factor dependencies and endogeneity, and should as a result lead to improved identification. Our main findings provide support for the hybrid NKPC when the presence of unknown common factors as well as external instruments are accounted for, although the results differ depending on the countries included in the estimation. More specifically, the evidence is stronger when the full sample of EU or Euro Area countries is used, rather than solely the new EU member countries which joined the EU in 2004 or later.

JEL Classification: E31; E52; C13; C23.

Keywords: New Keynesian Phillips curve, Inflation, Dynamic panel data model, Cross-sectional dependence, Common factors.

## 1 Introduction

It is well known that high and volatile inflation may lead to welfare losses. Many central banks have therefore adopted inflation targeting, which means that a proper understanding of the dynamics of inflation is crucial for monetary policy to be successful. It has also been shown on theoretical grounds that monetary policy conducted by a central bank critically depends on the underlying price stickiness (see Yun, 1996; Erceg, Henderson and Levin, 2000; Schmitt-Grohé and Uribe, 2004). Because of this, the hybrid New Keynesian Phillips curve (NKPC) based on the sticky-price model (see Calvo, 1983), which posits that inflation should depend positively on real marginal cost, as well as expected future and lagged inflation, has attracted a great deal of attention in the monetary policy literature.

The empirical support in favor of the NKPC has, however, been mixed and hence not very convincing. In particular, while there are studies in which the results are quite favorable to the NKPC (see, for example, Galí and Gertler, 1999; Sbordone, 2002; Lindé, 2005; Galí et al., 2001; Dees et al., 2009), most recent evidence goes in the other direction (see, for instance, Kurmann, 2005; Juselius, 2008; Boug et al., 2010; Norkutė, 2015). One explanation for this inconclusive evidence is that it relies to a large extent on the use of time series approaches that are not suitable when the number of time periods is relatively small (see Boug et al., 2010). This problem could be avoided by using panel data, as in Imbs et al. (2007, 2011), Lawless and Whelan (2011), and Byrne et al. (2013), to mention a few. By increasing the number of observations that can be brought to bear on the NKPC, panel data are expected to lead to more accurate estimates.

An issue that arises in the panel data context is the presence of cross-sectional dependence. Such dependence is predicted by many theories and it is not difficult to find empirical evidence to support this (see, for example, Beyer et al., 2008; Boivin et al., 2009; Imbs et al., 2011). In fact, one does not have to go to the panel strand of the literature to find evidence of cross-sectional dependence. One of the problems often cited in the time series literature is the presence of omitted variables, which obviously raises the issue of bias (see Rudd and Whelan, 2005). Factor models are able to generate a wide range of cross-sectional dependencies and are widely used in the econometric literature on panel data. In these models the dependence is generated by a small number of latent factors that are common to all cross-sectional units and whose sensitivities, or loadings, are allowed to vary across those units. Common factors can be seen as omitted variables and are in this sense easily reconcilable with the time series literature (see Beyer et al., 2008). Cross-correlation is an issue in itself, but the main problem is that the factors will make the error term in the NKPC correlated with the regressors, thereby rendering the ordinary least squares (OLS) estimator biased and inconsistent.

Another, more commonly appreciated source of endogeneity is that obtained when using the lead of actual inflation in place of expected future inflation, as is commonly done in the empirical literature on the NKPC, which makes the error term correlated with the future values of inflation. In order to address this problem, econometric techniques based on instrumental variables (IV) are often employed. The problem with this approach is that the instruments are likely to be affected by the same set of factors as inflation, which means that they may be correlated with the error term. If this is the case, the regular panel IV estimator is no better than OLS.

The present paper is motivated by the above observations. The purpose is to assess the empirical validity of the NKPC while accounting for possible endogeneity when the error term, regressors and instruments may be correlated not only among themselves but also cross-sectionally through the presence of common factors. The way we do this is by using a new approach that is based on the recently developed panel IV estimator of Norkutė, Sarafidis, Yamagata and Cui (2021), "NSYC" henceforth, which removes the common factors from the regressors and use the resulting defactored regressors and their lags as instruments. We extend this approach by augmenting the instrument set with the lead of the defactored regressors, which is necessary in the present context as one of the regressors in the NKPC is the lead of inflation. We show that the panel IV approach based on the extended instrument set has the same attractive asymptotic properties as the original estimator of NSYC. We also conduct a small-scale Monte Carlo simulation experiment where the data generating process resembles that of the NKPC and show that the panel IV estimator based on the augmented instrument set works well even when the sample size is relatively small.

The new IV estimator is applied to a panel data set covering 23 European Union (EU) member countries between 1999Q1 and 2018Q. The results can be summarized as follows. If the common factors are ignored, we find little or no evidence of the NKPC. Many estimates are insignificant; others are significant but have unexpected signs. If, on the other hand, the common factors are taken into account, the evidence is much more supportive of the NKPC, although the strength depends on countries included in the estimation. In particular, the evidence is weakest when considering only the countries that joined the EU in 2004 or later.

The rest of paper is structured as follows. While in Section 2 we discuss the NKPC, in Sections 3 and 4 we describe the IV approach, and its asymptotic and finite sample properties. Sections 5 and 6 describe the data and the empirical results. Section 7 concludes. All proofs and results of secondary nature are provided in the appendix.

## 2 The factor-augmented hybrid NKPC

We consider a theoretical model based on Galí and Gertler (1999), Sbordone (2002) and Woodford (2003), where the price stickiness is generated by assuming that the firms are allowed to set their prices optimally only at certain time periods with exogenously defined fixed probability as in Calvo (1983). It also assumes that only a fraction of firms are forward-looking and behave like in Calvo's model, while the rest of them are backward-looking and set their prices based on the previous changes in aggregate price. This leads to the relationship linking current inflation to one-step-ahead inflation expectations, inflation lag and real marginal cost, commonly known as the hybrid New Keynesian Phillips Curve (NKPC). The hybrid NKPC for country $i = 1, ..., N$ and time period $t = 1, ..., T$ is given by

$$\pi_{i,t} = \gamma_B \pi_{i,t-1} + \gamma_F E_t \pi_{i,t+1} + \lambda s_{i,t} + \eta_{i,t}, \tag{1}$$

where $E_t \pi_{i,t+1}$ is the conditional expectation of $\pi_{i,t+1}$ given time-$t$ information, $s_{i,t}$ is the real marginal cost defined as the log-deviation from its long-run steady state, and $\eta_{i,t}$ is a cost-push shock that captures all variation in inflation that is not accounted for by changes in excess demand (see Woodford, 2003).

The model in (1) constitutes a reduced form specification of a structural NKPC whose parameters are given by a subjective discount factor, $\beta$, a price rigidity parameter, $\delta$, which denotes the probability of forward-looking firms not being able to adjust their prices optimally, and a fraction of backward-looking firms, $\omega$, which captures the degree of "backwardness" in price setting. The relationship between the structural parameters and the slope coefficients in (1) is given by $\gamma_B = \phi^{-1}\omega$, $\gamma_F = \phi^{-1}\delta\beta$, $\lambda = \phi^{-1}(1-\omega)(1-\delta)(1-\delta\beta)$, where $\phi = \delta + \omega[1 - \delta(1-\beta)]$. If the rigidity parameter $\delta$ decreases and prices become more flexible as the forward-looking firms are allowed to adjust their prices more frequently, the impact of the real marginal cost, as measured by $\lambda$, increases. If, on the other hand, there is a complete price rigidity and the forward-looking firms keep their prices fixed so that $\delta = 1$, then $\lambda = 0$ and so current inflation is not affected by the changes in real marginal cost. An increase in the degree of price stickiness therefore reduces the importance of real marginal cost in determining inflation dynamics and vice versa. When $\omega = 0$, so that the price setting behavior of all firms is forward-looking, then $\gamma_B = 0$ and the hybrid NKPC therefore reduces to a purely forward-looking NKPC. Given the restrictions on the structural parameters, the NKPC is consistent with the economic theory if $\gamma_B$ is non-negative while $\gamma_F$ and $\lambda$ are strictly positive (Nyomen, 2012).

Following Imbs et al. (2011), we assume that the error term in (1) admits to the following factor representation

$$\eta_{i,t} = \boldsymbol{\lambda}_i' \mathbf{f}_t + \varepsilon_{i,t}, \tag{2}$$

where $\mathbf{f}_t$ is an $m_f \times 1$ vector of common factors with $\boldsymbol{\lambda}_i$ being a conformable vector of factor loadings, and $\varepsilon_{i,t}$ is an idiosyncratic random shock. It is natural to think of $\mathbf{f}_t$ as comprised of shocks caused by the recent financial crisis, the oil price and other transitory factors that affect inflation in all countries but with different intensities (see Jordá and Nechio, 2018). Real marginal cost is also likely to be subject to common shocks (see Imbs et al., 2011). We therefore assume that

$$s_{i,t} = \boldsymbol{\gamma}_i' \mathbf{g}_t + v_{i,t}, \tag{3}$$

where $\mathbf{g}_t$ and $\gamma_i$ are $m_g \times 1$ vectors of unobserved factors and loadings, respectively. The idiosyncratic error $v_{i,t}$ is independent of all other random elements of the model. While the factors in $\mathbf{g}_t$ may be unrelated to those in $\mathbf{f}_t$, we want to maintain the possibility that they are in fact related, as when some of the factors are the same, which is a more general consideration.

In line with the existing literature, we assume that the inflation expectations are rational (see among others Galí and Gertler, 1999; Imbs et al., 2011), so that economic agents do not make systematic mistakes when predicting future inflation. Hence, $E_t \pi_{i,t+1} = \pi_{i,t+1} + \epsilon_{i,t}$, where $\epsilon_{i,t}$ is unpredictable given the information available at time $t$. Replacing expected inflation by actual inflation, (1) becomes

$$\pi_{i,t} = \gamma_B \pi_{i,t-1} + \gamma_F \pi_{i,t+1} + \lambda s_{i,t} + u_{i,t}, \tag{4}$$

with

$$u_{i,t} = \eta_{i,t} + \gamma_F \epsilon_{i,t} = \boldsymbol{\lambda}_i' \mathbf{f}_t + \varepsilon_{i,t} + \gamma_F \epsilon_{i,t}. \tag{5}$$

Hence, defining $\boldsymbol{\theta} = [\gamma_B, \gamma_F, \lambda]'$ and $\mathbf{x}_{i,t} = [\pi_{i,t-1}, \pi_{i,t+1}, s_{i,t}]'$, (4) can be written as

$$\pi_{i,t} = \boldsymbol{\theta}' \mathbf{x}_{i,t} + u_{i,t}. \tag{6}$$

This is the model to be estimated. However, the use of actual inflation in place of expected inflation makes $u_{i,t}$ correlated with inflation lead. This means that $\mathbf{x}_{i,t}$ is endogenous and therefore (6) has to be estimated by using IV techniques (see among others Galí and Gertler 1999; Leith and Malley 2007). Let us therefore assume that there exists a $r \times 1$ vector $\mathbf{w}_{i,t}$ of "external" variables that are correlated with $\mathbf{x}_{i,t}$ and that admits the following factor model structure:

$$\mathbf{w}_{i,t} = \boldsymbol{\Psi}_i' \mathbf{g}_t + \mathbf{e}_{i,t}, \tag{7}$$

where $\boldsymbol{\Psi}_i$ is a $m_g \times r$ vector of factor loadings and $\mathbf{e}_{i,t}$ is a $r \times 1$ vector of idiosyncratic errors which are again independent of all other random elements of the model. The independence assumption here is key because it means that $\mathbf{w}_{i,t}$ can be used to construct valid instruments for $\mathbf{x}_{i,t}$. In the next section, we elaborate on this. Before we move on, however, we stack the above model. Let us therefore denote by $\boldsymbol{\pi}_i$, $\mathbf{X}_i$, $\mathbf{u}_i$, $\mathbf{F}$, $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\epsilon}_i$ the $T$-rowed matrices obtained by stacking the time series observations on $\pi_{i,t}$, $\mathbf{x}_i$, $u_{i,t}$, $\mathbf{f}_t$, $\varepsilon_{i,t}$ and $\epsilon_{i,t}$, respectively. In this notation, (6) and (5) can be written as

$$\boldsymbol{\pi}_i = \mathbf{X}_i \boldsymbol{\theta} + \mathbf{u}_i, \tag{8}$$

$$\mathbf{u}_i = \mathbf{F} \boldsymbol{\lambda}_i + \boldsymbol{\varepsilon}_i + \gamma_F \boldsymbol{\epsilon}_i. \tag{9}$$

Similarly, if we denote by $\mathbf{s}_i$, $\mathbf{G}$, $\mathbf{v}_i$, $\mathbf{W}_i$ and $\mathbf{E}_i$ the stacked matrix versions of $s_{i,t}$, $\mathbf{g}_t$, $v_{i,t}$, $\mathbf{w}_{i,t}$ and $\mathbf{e}_{i,t}$, respectively, then (3) and (7) become

$$\mathbf{s}_i = \mathbf{G} \gamma_i + \mathbf{v}_i, \tag{10}$$

$$\mathbf{W}_i = \mathbf{G} \boldsymbol{\Psi}_i + \mathbf{E}_i. \tag{11}$$

In the next section, we make use of this stacked notation to describe the IV estimation approach.

## 3 The IV approach and its asymptotic properties

The factor-augmented hybrid NKPC in (8) can be seen as a dynamic panel data model with multifactor error structure. One difference when compared to more conventional dynamic panel data models with multifactor errors is the endogeneity caused when replacing expected inflation by actual inflation. Another source of possible endogeneity is the dependence of $s_{i,t}$ and $\mathbf{w}_{i,t}$ on $\mathbf{g}_t$, where $\mathbf{g}_t$ and $\mathbf{f}_t$ may share the same elements or be distinct but correlated, which cannot be ruled out in practice as most of macroeconomic variables are subject to the same global shocks. This makes both $s_{i,t}$ and $\mathbf{w}_{i,t}$ potentially endogenous.

We therefore look for an estimation approach that is general enough to accommodate not only the dynamics and factors in the errors, but also endogeneity. The IV estimator of NSYC fits this bill. Let $\mathbf{M}_G = \mathbf{I}_T - \mathbf{G}(\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'$. The main idea behind the estimator of NSYC is to first "defactor" $\mathbf{s}_i$ and $\mathbf{W}_i$, and to use the resulting defactored variables $\mathbf{M}_G\mathbf{s}_i$ and $\mathbf{M}_G\mathbf{W}_i$, respectively, as instruments. It is not difficult to see that $\mathbf{M}_G\mathbf{s}_i$ is a valid instrument as $E(\mathbf{s}_i'\mathbf{M}_G\mathbf{u}_i) = E[(\mathbf{G}\gamma_i + \mathbf{v}_i)'\mathbf{M}_G\mathbf{u}_i] = E(\mathbf{v}_i'\mathbf{M}_G\mathbf{u}_i) = 0$ under our assumption that $\mathbf{v}_i$ is independent of all other random elements of the model. In addition, $E(\mathbf{s}_i'\mathbf{M}_G\mathbf{X}_i) \neq 0$ by construction, implying that $\mathbf{M}_G\mathbf{s}_i$ is a relevant instrument. The lags of $\mathbf{M}_G\mathbf{s}_i$ are also valid and relevant instruments, as is $\mathbf{M}_G\mathbf{W}_i$ and its lags. The leads of $\mathbf{M}_G\mathbf{s}_i$ and $\mathbf{M}_G\mathbf{W}_i$ are valid, too, because of the independence of $v_{i,t+1}$, and they are relevant since $s_{i,t+1}$ and $\pi_{i,t+1}$ are correlated by construction, provided that $\lambda \neq 0$.

Our main econometric contribution is to extend the instrument set of NSYC to also include the lead of real marginal cost and exogenous regressors, and in this way account for the fact that the NKPC has inflation lead on the right-hand side. Let us therefore define $\mathbf{z}_i = [\mathbf{s}_i, \mathbf{W}_i]$. The matrix of instruments, henceforth denoted by $\mathbf{Z}_i$, is given by

$$\mathbf{Z}_i = [\mathbf{M}_G\mathbf{z}_i, \mathbf{M}_{G_{-1}}\mathbf{z}_{i,-1}, ..., \mathbf{M}_{G_{-h}}\mathbf{z}_{i,-h}, \mathbf{M}_{G_{+1}}\mathbf{z}_{i,+1}], \tag{12}$$

where $\mathbf{G}_{-j}$ and $\mathbf{z}_{i,-j}$ are the $j$-th lags of $\mathbf{G}$ and $\mathbf{z}_i$, respectively, while $\mathbf{G}_{+1}$ and $\mathbf{z}_{i,+1}$ are the corresponding one period leads.

The factors are not observed and have to be estimated prior to the construction of the estimator of $\boldsymbol{\theta}$. We estimate $m_g$ by using the eigenvalue method of Ahn and Horenstein (2013). Given the estimate of $m_g$, the factors are extracted by using the principal components method. The estimated factors are denoted by $\hat{\mathbf{G}}$ and are defined as $\sqrt{T}$ times the eigenvectors corresponding to the $m_g$ largest eigenvalues of the $T \times T$ matrix $(NT)^{-1}\sum_{i=1}^{N}\mathbf{z}_i\mathbf{z}_i'$. The resulting feasible instrument matrix based on using $\hat{\mathbf{G}}$ in place of $\mathbf{G}$ is denoted by $\hat{\mathbf{Z}}_i$. The first-step IV estimator can now be written as

$$\hat{\boldsymbol{\theta}}_{IV1} = (\mathbf{A}'_{1NT}\mathbf{B}^{-1}_{1NT}\mathbf{A}_{1NT})^{-1}\mathbf{A}'_{1NT}\mathbf{B}^{-1}_{1NT}\mathbf{g}_{1NT}, \tag{13}$$

where $\mathbf{A}_{1NT} = (NT)^{-1}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\mathbf{X}_i$, $\mathbf{B}_{1NT} = (NT)^{-1}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\hat{\mathbf{Z}}_i$ and $\mathbf{g}_{1NT} = (NT)^{-1}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\boldsymbol{\pi}_i$.

In the appendix, we show that while $\sqrt{NT}$-consistent, the asymptotic distribution of $\sqrt{NT}(\hat{\boldsymbol{\theta}}_{IV1} - \boldsymbol{\theta})$ is biased. Essentially, since the factors are not perfectly estimated, the effect of the endogeneity is not removed completely. To address this issue, we follow NSYC and re-estimate the model using a second-step IV estimator. To obtain this estimator, we first estimate $m_f$ by using again the method by Ahn and Horenstein (2013), and extracting factors from the first-step IV residual $\hat{\mathbf{u}}_{IV1,i} = \boldsymbol{\pi}_i - \mathbf{X}_i\hat{\boldsymbol{\theta}}_{IV1}$ by using principal components. The estimated factors are denoted by $\hat{\mathbf{F}}$ and are given by $\sqrt{T}$ times the eigenvectors corresponding to the $m_f$ largest eigenvalues of the $T \times T$ matrix $(NT)^{-1}\sum_{i=1}^{N}\hat{\mathbf{u}}_{IV1,i}\hat{\mathbf{u}}'_{IV1,i}$. The second-step IV estimator is given by

$$\hat{\boldsymbol{\theta}}_{IV2} = (\mathbf{A}'_{2NT}\mathbf{B}^{-1}_{2NT}\mathbf{A}_{2NT})^{-1}\mathbf{A}'_{2NT}\mathbf{B}^{-1}_{2NT}\mathbf{g}_{2NT}, \tag{14}$$

where $\mathbf{A}_{2NT} = (NT)^{-1}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\mathbf{M}_{\hat{F}}\mathbf{X}_i$, $\mathbf{B}_{2NT} = (NT)^{-1}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\mathbf{M}_{\hat{F}}\hat{\mathbf{Z}}_i$, $\mathbf{g}_{2NT} = (NT)^{-1}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\mathbf{M}_{\hat{F}}\boldsymbol{\pi}_i$ and $\mathbf{M}_{\hat{F}} = \mathbf{I}_T - \hat{\mathbf{F}}(\hat{\mathbf{F}}'\hat{\mathbf{F}})^{-1}\hat{\mathbf{F}}'$.

**Theorem 1.** *Under assumptions analogous to Assumptions 1–5 in NSYC and the conditions laid out in the above, as $N$, $T \to \infty$ jointly such that $N/T \to c \in (0, \infty)$,*

$$\sqrt{NT}(\hat{\boldsymbol{\theta}}_{IV2} - \boldsymbol{\theta}) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Sigma})$$

*with*

$$\boldsymbol{\Sigma} = (\mathbf{A}'_2\mathbf{B}_2^{-1}\mathbf{A}_2)^{-1}\mathbf{A}'_2\mathbf{B}_2^{-1}\boldsymbol{\Omega}\mathbf{B}_2^{-1}\mathbf{A}_2(\mathbf{A}'_2\mathbf{B}_2^{-1}\mathbf{A}_2)^{-1}$$

*where the symbol $\xrightarrow{d}$ signifies convergence in distribution, $\mathbf{A}_2$ and $\mathbf{B}_2$ are the probability limits of $\mathbf{A}_{2NT}$ and $\mathbf{B}_{2NT}$, respectively, and $\boldsymbol{\Omega}$ is defined as in Assumption 5 in NSYC.*

Theorem 1, the proof of which is outlined in the appendix, shows that while the asymptotic distribution of the first-step IV estimator is biased, the asymptotic distribution of the second-step estimator is not. The second-step estimator based on the extended instrument set therefore retains the same nice asymptotic properties as the original estimator of NSYC.

The use of the extended instrument set is also expected to lead to improved identification, given that the NKPC includes inflation lead as one of the regressors, thereby creating the need for a suitable instrument.

Inference based on Theorem 1 requires a consistent estimator $\widehat{\boldsymbol{\Sigma}}$ of $\boldsymbol{\Sigma}$, which can be constructed in the following way (see Theorem 2 of NSYC):

$$\widehat{\boldsymbol{\Sigma}} = (\mathbf{A}_{2NT}'\mathbf{B}_{2NT}^{-1}\mathbf{A}_{2NT})^{-1}\mathbf{A}_{2NT}'\mathbf{B}_{2NT}^{-1}\hat{\boldsymbol{\Omega}}\mathbf{B}_{2NT}^{-1}\mathbf{A}_{2NT}(\mathbf{A}_{2NT}'\mathbf{B}_{2NT}^{-1}\mathbf{A}_{2NT})^{-1}, \tag{15}$$

where $\hat{\boldsymbol{\Omega}} = (NT)^{-1}\sum_{i=1}^{N}\hat{\sigma}_i^2\hat{\mathbf{Z}}_i'\mathbf{M}_{\hat{F}}\hat{\mathbf{Z}}_i$ with $\hat{\sigma}_i^2 = T^{-1}\hat{\mathbf{u}}_{IV1,i}'\mathbf{M}_{\hat{F}}\hat{\mathbf{u}}_{IV1,i}$.

Byrne et al. (2013) and Imbs et al. (2011) also allow for common factors in a panel data setting similar to ours, which are estimated and controlled for by using the so-called "common correlated effects" (CCE) approach of Pesaran (2006). A major difference between this last approach and the one considered here is that it is not equipped to handle instrument endogeneity. Moreover, in contrast to the pooled IV estimators considered here, which are $\sqrt{NT}$-consistent, the mean group CCE estimator employed by Byrne et al. (2013) is only $\sqrt{N}$-consistent. This is important because in our case $N$ is small when compared to $T$.

The $\sqrt{NT}$-consistency of the IV estimators relies heavily on the orthogonality of the instruments, which is violated if the instruments in $\hat{\mathbf{Z}}_i$ are correlated with the error term in (8), or if the parameters of said equation are not constant over the cross-sectional units as assumed but varying. It is therefore important to test if the orthogonality condition is in fact met. In this paper, we therefore follow NSYC and use the overidentifying restrictions $J$-test, which in our case is given by

$$J = \frac{1}{NT}\left(\sum_{i=1}^{N}\hat{\mathbf{u}}_i'\mathbf{M}_{\hat{F}}\hat{\mathbf{Z}}_i\right)\hat{\boldsymbol{\Omega}}^{-1}\left(\sum_{i=1}^{N}\hat{\mathbf{Z}}_i'\mathbf{M}_{\hat{F}}\hat{\mathbf{u}}_i\right). \tag{16}$$

Under the null hypothesis that the orthogonality condition is met, $J$ has an asymptotic chi-squared distribution with $(r+1)(h+2)-3$ degrees of freedom, which is the number of instruments minus the number of parameters (see Theorem 3 of NSYC). Large values of $J$ should therefore be interpreted as providing evidence against orthogonality.

## 4 Monte Carlo study

A small-scale Monte Carlo simulation exercise is carried out to investigate the finite sample behaviour of the proposed second-step IV estimator. The data generating process is given by a simplified version of (3), (4) and (5), where the parameters of the model are set to $\gamma_B = \gamma_F = \lambda = 0.4$, while the idiosyncratic random error terms $\varepsilon_{i,t}$, $\epsilon_{i,t}$ and $v_{i,t}$ are drawn independently from $N(0,1)$. The factors in $\mathbf{f}_t$ and $\mathbf{g}_t$ are generated as autoregressive processes of order one with autoregressive coefficient equal to 0.5 and the idiosyncratic term drawn independently from $N(0,1)$. We set $m_f = 1$ and $m_g = 3$, but the number of factors is treated as unknown and is estimated using the method of Ahn and Horenstein (2013). The loadings of $\mathbf{f}_t$ are generated as $\boldsymbol{\lambda}_i \sim U[-0.5, 0.5]$, while those of $\mathbf{g}_t$ are generated as $\boldsymbol{\gamma}_i = \delta\boldsymbol{\lambda}_i + (1-\delta)\boldsymbol{\zeta}_i$ with $\delta = 0.5$ and $\boldsymbol{\zeta}_i \sim U[-0.5, 0.5]$. This means that $\boldsymbol{\gamma}_i$ is correlated with $\boldsymbol{\lambda}_i$. The generation of $\pi_{i,t}$ through (4) is made complicated by the presence of both $\pi_{i,t-1}$ and $\pi_{i,t+1}$ on the right-hand side. In the appendix, we explain how this is done. It requires a choice of initial and terminal values. We set $\pi_{i,0} = \pi_{i,T+1} = 0$, a choice that is attenuated by generating 100 extra "warm-up" and "cool-down" observations that are discarded prior to estimation of the model. The effective sample sizes considered are $N \in \{25, 50, 100, 200\}$ and $T \in \{25, 50, 100, 200\}$.

The performance of the proposed estimator is assessed in terms of the bias, root mean squared error (RMSE), and the empirical size and size-adjusted power of a two-sided nominal 5% $t$-test for testing the null hypothesis that each of $\gamma_B$, $\gamma_F$ and $\lambda$ are equal to 0.4 against the alternative that they are equal to 0.5. The number of replications is set to 2,000. All computational work is done in Matlab R2021a.[1]

The results for estimating $\gamma_B$, $\gamma_F$, and $\lambda$ are reported in Tables 1, 2 and 3, respectively. For comparison purposes, in addition to the full-blown second-step IV estimator described in Section 3, we consider one version that sets $\mathbf{M}_{\hat{F}} = \mathbf{M}_{\hat{G}} = \mathbf{I}_T$, which means that it disregards the factors, and another version that excludes the defactored lead from the set of instruments. Since in this section there are no external variables, we construct our instrument set based on $\mathbf{M}_G\mathbf{s}_i$ and its lags and lead only. Since the larger number of instruments may lead to improved efficiency at the cost of higher bias, we choose to include five lags, so that the number of instruments is large enough to ensure identification and efficiency, yet not so large as to cause a serious bias.

INSERT TABLES 1–3 ABOUT HERE

The first thing to note is that the bias is generally much smaller for the case when factors

[1]The Matlab code that generates the Monte Carlo simulation results is available from the authors upon request.

are taken into account as compared to the case when they are ignored. The instrument lead is also important in this regard and leads to lower bias if included, as expected. We also see that the bias of the full-blown estimator with both factors and instrument lead is decreasing in $N$ and $T$, which is a reflection of Theorem 2 and the consistency of this estimator. The results for the RMSE lead to similar conclusions; the RMSE is smallest for the full-blown estimator and it tends to come down as $N$ and $T$ increase.

As for the empirical size of the IV-based $t$-test, we see noticeable distortions when the factors are disregarded and/or the instrument lead is excluded. There are some distortions also for the full-blown estimator, but these go away as $N$ and $T$ increase. Indeed, when $N = T = 200$, size is very close to its nominal 5% level. Similarly, while power can sometimes be low, even for the full-blown IV estimator, it quickly approaches 100% as $T$ and $N$ increase.

All in all, the Monte Carlo results reported here show that the proposed second-step IV estimator works very well, and that it does so even when the sample size is relatively small. The results also reveal that the performance is improved when the instrument set of NSYC is augmented with a lead of the defactored regressors, which means that augmentation is important when estimating the NKPC.

# 5 Data

Our data set has been collected from the Eurostat database. It includes all European Union countries except for Bulgaria, Romania, Malta, Cyprus and Croatia, which are excluded due to data limitations. The data are quarterly and cover the period 1999Q1–2018Q1, where the starting point is motivated by the launch of the European Monetary Union. Hence, in this paper, $N = 23$ and $T = 76$. Inflation is defined as the quarter-on-quarter percentage growth rate of the harmonized index of consumer prices (HICP). Figure 1 plots these data for each country in the sample. We see that the series exhibit strong co-movement, suggesting that the data are cross-sectionally dependent, which may be explained by the presence of common factors.

INSERT FIGURE 1 ABOUT HERE

The real marginal cost is defined as the log of the labor income share, computed as the ratio of compensation of employees to nominal gross domestic product. Following Galí and Gertler (1999), we use external variables as instruments in addition to the instrument constructed based on the real marginal cost. These variables are: wage inflation, which is calculated as the quarterly growth rate of the labour cost index; and the output gap measured as a cyclical component obtained by filtering the logarithm of the real GDP using the Hodrick-Prescott filter.

Since the principal components method is sensitive to irregularities in the data, outliers should be removed prior to estimation of the factors. In the current paper, outliers are handled by using the wavelet-based method of Grané and Veiga (2010). All the data are demeaned and seasonally adjusted by using dummy variable regressions.

# 6 Empirical Results

The second-step IV estimator was implemented exactly as in the Monte Carlo study of Section 4, except that now the instrument set also contains five lags and one lead of two external variables, namely, wage inflation and the output gap. The total number of instruments is therefore equal to 21.For the ske of comparison, in addition to the full-blown IV estimator, we report the results obtained when using non-defactored instruments, as in Secion 4. Four samples are considered; the full sample, Euro area countries, the "new" EU member countries that joined 2004 or later, and all countries but the UK.

INSERT TABLE 4 ABOUT HERE

Table 4 reports the results for the factor-augmented NKPC, the parameters of which are given by $\lambda$ (the coefficient of real marginal cost), $\gamma_B$ (the coefficient of inflation lag) and $\gamma_F$ (the coefficient of inflation lead). The table reports point estimates with standard errors in parentheses, the estimated number of factors obtained using the approach of Ahn and Horenstein (2013), and the $p$-values of the $J$-test.

The $J$-test does not lead to any rejections, which, as already pointed out, provides evidence in support of instrument orthogonality. We also see that the number of factors is estimated to be positive, suggesting that it is important to use approaches that allow for such factors.

Looking next at the actual estimation results, we see that the estimates of $\gamma_B$ and $\gamma_F$ are positive and statistically significant, which is in agreement with our a priori expectations. In particular, the estimates of $\gamma_F$ are larger than those of $\gamma_B$, except for the case when defactored instruments are used for the sample of the new EU member countries, which implies that the price-setting behaviour is mainly forward-looking. The results also reveal that when the instruments are defactored, the estimates of $\lambda$ are positive and statistically significant for all samples considered, except for that of the new EU member countries. However, when the factors are ignored and instruments are not defactored, the results for $\lambda$ are statistically insignificant for all samples, which is likely to be due to the unattended factors when using the non-defactored instruments. Hence, we find evidence in favour of the NKPC, but only when the presence of common factors is appropriately accounted for. The only exception is for the new EU member countries, for which there is little or no evidence regardless of whether the instruments are defactored or not.

INSERT TABLE 5 ABOUT HERE

The results reported in Table 4 are based on the reduced form of the hybrid NKPC. By contrast, in Table 5 we report estimates of the structural parameters of the same model. These are the subjective discount factor, $\beta$, the fraction of backward-looking price setters, $\omega$, and the rigidity parameter, $\delta$. Indirect estimates of these parameters can be obtained by using their relationship with the estimated reduced form coefficients $\lambda$, $\gamma_F$ and $\gamma_B$ (as in, for example, Imbs et al., 2011). The standard errors are obtained by using the so-called "Delta method".

The first thing to note about the results is that the estimated structural parameters are all positive and statistically significant. According to economic theory, the subjective discount factor should be close to but slightly smaller than one, and the reference value often mentioned in the literature is 0.99 (Galí and Gertler, 1999). The estimates reported in Table 5 are lower than 0.99, but after accounting for their standard errors they are all reasonably close to one. The estimated discount factor is equal to 0.987 for the full sample, while for the Euro area and new EU member countries the same estimates are 0.982 and 0.936, respectively. The estimates of the fraction of backward-looking price setters, $\omega$, are smaller than 0.5 for all samples except for the new EU countries, which again means that price-setting behaviour is mainly forward-looking. The estimated rigidity parameter is 0.597 for the full sample, which is smaller than the Euro area estimate of 0.560. The degree of price rigidity in the new EU countries is 0.676 and it is slightly higher when compared to the EU as a whole. These differences are, however, not significant, which means that they are not necessarily due to differences in price stickiness but could also be due to estimation uncertainty.

## 7 Conclusion

In this paper, we assess the empirical performance of a factor-augmented version of the hybrid NKPC when applied to a sample of EU member countries covering the period 1999Q1–2018Q1. The factor-augmentation makes for a very general model when it comes to the types of cross-sectional dependencies and endogeneity that can be accounted for. However, while certainly appealing, this generality invalidates all existing estimators. The only exception known to us is the recently proposed panel IV estimator of NSYC. We therefore take this estimator as our starting point; however, we augment the originally proposed instrument set by including the lead of the defactored regressors. This is necessary because the NKPC includes inflation lead as a regressor. The resulting estimator based on this augmented instrument set is shown to have excellent asymptotic and small-sample properties.

Our empirical findings can be summarized as follows. On the one hand, if the presence of common factors is ignored, we find little or no evidence in support of the NKPC. If, on the other hand, said presence is taken into account, the results are quite favorable to the NKPC, although the support depends on the countries included in the estimation. In particular, the evidence is weaker when only the new EU member countries are considered, as compared to when the Euro area countries or the full sample of EU countries are taken into account.

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# Appendix

## Asymptotic results

By using (8) and (13), we obtain

$$\sqrt{NT}(\hat{\boldsymbol{\theta}}_{IV1}-\boldsymbol{\theta}) = (\mathbf{A}'_{1NT}\mathbf{B}^{-1}_{1NT}\mathbf{A}_{1NT})^{-1}\mathbf{A}'_{1NT}\mathbf{B}^{-1}_{1NT}\frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\mathbf{u}_i, \tag{A.1}$$

which suggests that the asymptotic properties of the first-step IV estimator are determined by $(NT)^{-1/2}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\mathbf{u}_i$. Proposition 1 below provides an asymptotic expansion for this term. Before we take the proposition, however, we need to introduce some notation. Let us therefore define

$$\tilde{\mathbf{Z}}_i = [\mathbf{M}_G\tilde{\mathbf{z}}_i, \mathbf{M}_{G_{-1}}\tilde{\mathbf{z}}_{i,-1}, ..., \mathbf{M}_{G_{-h}}\tilde{\mathbf{z}}_{i,-h}, \mathbf{M}_{G_{+1}}\tilde{\mathbf{z}}_{i,+1}], \tag{A.2}$$

where $\tilde{\mathbf{z}}_i = \mathbf{z}_i - N^{-1}\sum_{n=1}^{N}\mathbf{z}_n\mathbf{\Phi}'_n\Xi^{-1}_{rN}\mathbf{\Phi}_i$, $\tilde{\mathbf{z}}_{i,-p} = \mathbf{z}_{i,-p} - N^{-1}\sum_{n=1}^{N}\mathbf{z}_{n,-p}\mathbf{\Phi}'_n\Xi^{-1}_{rN}\mathbf{\Phi}_i$ for $p = 1,...,h$, and $\tilde{\mathbf{z}}_{i,+1} = \mathbf{z}_{i,+1} - N^{-1}\sum_{n=1}^{N}\mathbf{z}_{n,+1}\mathbf{\Phi}'_n\Xi^{-1}_{rN}\mathbf{\Phi}_i$. Here, $\boldsymbol{\Phi}_i = [\gamma_i, \boldsymbol{\psi}_{1i}, ..., \boldsymbol{\psi}_{ri}]$ and $\Xi_{rN} = N^{-1}\sum_{i=1}^{N}(\gamma_i\gamma'_i + \sum_{\ell=1}^{r}\boldsymbol{\psi}_{\ell i}\boldsymbol{\psi}'_{\ell i})$ are $m_g \times (r+1)$ and $m_g \times m_g$ matrices, respectively. We also define

$$\mathbf{B}_{jNT} = [\mathbf{b}'_{jNT}, \mathbf{b}'_{jNT,-1}, ..., \mathbf{b}'_{jNT,-h}, \mathbf{b}'_{jNT,+1}]' \tag{A.3}$$

for $j \in \{1, 2\}$, where

$$\mathbf{b}_{1NT} = N^{-1}T^{-2}\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\mathbf{Q}}'_i\mathbf{Q}_j\mathbf{\Psi}'_j\Xi^{-1}_{rN}(T^{-1}\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{u}_i, \tag{A.4}$$

$$\mathbf{b}_{2NT} = -(NT)^{-1}\sum_{i=1}^{N}\mathbf{\Phi}'_i\Xi^{-1}_{rN}(T^{-1}\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\bar{\mathbf{\Sigma}}_{rNT}\mathbf{M}_G\mathbf{u}_i, \tag{A.5}$$

where $\mathbf{Q}_i = (\mathbf{v}_i, \mathbf{E}_i)$, $\tilde{\mathbf{Q}}_i = \mathbf{Q}_i - N^{-1}\sum_{n=1}^{N}\mathbf{Q}_n\mathbf{\Phi}'_n\Xi^{-1}_{rN}\mathbf{\Phi}_i$ and $\bar{\mathbf{\Sigma}}_{rNT} = N^{-1}\sum_{j=1}^{N}E(\mathbf{v}_i\mathbf{v}'_i + \sum_{\ell=1}^{r}\mathbf{e}_{\ell,i}\mathbf{e}'_{\ell,i})$ is a $T \times T$ matrix. $\mathbf{b}_{1NT,-p}$ and $\mathbf{b}_{2NT,-p}$ are defined analogously by replacing $\mathbf{Q}_i$, $\mathbf{G}$ and $\bar{\mathbf{\Sigma}}_{rNT}$ with $\mathbf{Q}_{i,-p}$, $\mathbf{G}_{-p}$ and $\bar{\mathbf{\Sigma}}_{rNT,-p}$, respectively, for $p = 1,...,h$, which are based on the $p$-th lags of $\mathbf{v}_i$, $\mathbf{E}_i$ and $\mathbf{G}$. In the same way, $\mathbf{b}_{1NT,+1}$ and $\mathbf{b}_{2NT,+1}$ are obtained by replacing $\mathbf{Q}_i$, $\mathbf{G}$ and $\bar{\mathbf{\Sigma}}_{rNT}$ with $\mathbf{Q}_{i,+1}$, $\mathbf{G}_{+1}$ and $\bar{\mathbf{\Sigma}}_{rNT,+1}$, respectively, where the latter three matrices are based on the leads of $\mathbf{v}_i$, $\mathbf{E}_i$ and $\mathbf{G}$, respectively. We now have all the notation we need in order to state the proposition.

**Proposition 1.** *Under assumptions analogous to the Assumptions 1-5 as defined in NSYC and the conditions laid out in the main text, as $N$, $T \to \infty$ jointly such that $N/T \to c \in (0, \infty)$,*

$$\frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\hat{\mathbf{Z}}'_i\mathbf{u}_i = \frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\tilde{\mathbf{Z}}'_i\mathbf{u}_i + \sqrt{\frac{T}{N}}\mathbf{B}_{1NT} + \sqrt{\frac{N}{T}}\mathbf{B}_{2NT} + o_p(1).$$

**Proof.** The result follows by using the same steps as in the proof of Proposition 1 in NSYC and using results that are analogous to their Lemmas 3, 4 and 6–8. The proof is therefore omitted. ■

Proposition 1 shows that there are two bias terms, $\mathbf{B}_{1NT}$ and $\mathbf{B}_{2NT}$, that have similar structure to those derived in NSYC but they differ in the sense that they also involve the lead of the factors, $\mathbf{G}_{+1}$, and the lead of the idiosyncratic errors of real marginal cost and external instruments, here denoted $\mathbf{v}_{i,+1}$ and $\mathbf{E}_{i,+1}$, respectively. This difference arises because of the augmentation of the instrument set with the lead of the defactored $\mathbf{s}_i$ and $\mathbf{W}_i$.

As in NSYC, it can be shown that $(NT)^{-1/2}\sum_{i=1}^{N}\tilde{\mathbf{Z}}_i'\mathbf{u}_i$ is $O_p(1)$ and that it tends to a multivariate distribution, while $\sqrt{T/N}\mathbf{B}_{1NT}$ and $\sqrt{N/T}\mathbf{B}_{2NT}$ are $O_p(1)$ as $N$, $T \to \infty$ jointly such that $N/T \to c \in (0,\infty)$. This implies that $\hat{\boldsymbol{\theta}}_{IV1}$ is $\sqrt{NT}$-consistent, but the asymptotic distribution of this estimator is biased. Essentially, since the factors are not perfectly estimated, the effect of the endogeneity is not removed completely. To address this issue, we follow NSYC and re-estimate the model using the second-step version of the IV estimator, which we can write as

$$\sqrt{NT}(\hat{\boldsymbol{\theta}}_{IV2} - \boldsymbol{\theta}) = (\mathbf{A}_{2NT}'\mathbf{B}_{2NT}^{-1}\mathbf{A}_{2NT})^{-1}\mathbf{A}_{2NT}'\mathbf{B}_{2NT}^{-1}\frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\hat{\mathbf{Z}}_i'\mathbf{M}_{\hat{F}}\mathbf{u}_i. \tag{A.6}$$

The next proposition, Proposition 2, does for $(NT)^{-1/2}\sum_{i=1}^{N}\hat{\mathbf{Z}}_i'\mathbf{M}_{\hat{F}}\mathbf{u}_i$ what Proposition 1 does for $(NT)^{-1/2}\sum_{i=1}^{N}\hat{\mathbf{Z}}_i'\mathbf{u}_i$.

**Proposition 2.** *Under Assumptions 1-5, as $N$, $T \to \infty$ jointly such that $N/T \to c \in (0,\infty)$,*

$$\frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\hat{\mathbf{Z}}_i'\mathbf{M}_{\hat{F}}\mathbf{u}_i = \frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\mathbf{Z}_i'\mathbf{M}_F\boldsymbol{\varepsilon}_i + \frac{\gamma_F}{\sqrt{NT}}\sum_{i=1}^{N}\mathbf{Z}_i'\mathbf{M}_F\boldsymbol{\epsilon}_i + o_p(1).$$

**Proof.** This proof follows by using results analogous to those reported in Lemmas 5 and 9 in NSYC, and similarly analogous manipulations of the proof of their Proposition 2. ■

Given that $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\epsilon}_i$ are independent of $\mathbf{Z}_i$ and $\mathbf{F}$ with zero means, we can deduce that the limiting expression for $(NT)^{-1/2}\sum_{i=1}^{N}\hat{\mathbf{Z}}_i'\mathbf{M}_{\hat{F}}\mathbf{u}_i$ given in Proposition 2 is $O_p(1)$ and that it is centred at zero, implying that $\hat{\boldsymbol{\theta}}_{IV2}$ is $\sqrt{NT}$-consistent and asymptotically bias-free. We can now use the steps as in NSYC to show that the asymptotic distribution of $\sqrt{NT}(\hat{\boldsymbol{\theta}}_{IV2} - \boldsymbol{\theta})$ is the one given in Theorem 1 of the main text.

### Additional details for the Monte Carlo study

As mentioned in the Monte Carlo study of Section 4, the generation of inflation using (8) is problematic in the sense that present inflation, $\boldsymbol{\pi}_i$, depends both on its lag, $\boldsymbol{\pi}_{i,-1}$, and its lead, $\boldsymbol{\pi}_{i,+1}$. To address this issue, we first introduce the following $T \times T$ matrix:

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}, \tag{A.7}$$

which can be viewed as a "lag matrix", as $\mathbf{J}\boldsymbol{\pi}_i = \boldsymbol{\pi}_{i,-1}$. The transpose of this matrix, $\mathbf{J}'$, can similarly be viewed as a "lead matrix", since $\mathbf{J}'\boldsymbol{\pi}_i = \boldsymbol{\pi}_{i,+1}$. By using these results, we can rewrite the model in (8) in the following way:

$$\boldsymbol{\pi}_i = \gamma_B \mathbf{J}\boldsymbol{\pi}_i + \gamma_F \mathbf{J}'\boldsymbol{\pi}_i + \lambda \boldsymbol{s}_i + \mathbf{u}_i. \tag{A.8}$$

Solving for $\boldsymbol{\pi}_i$ yields

$$\boldsymbol{\pi}_i = [\mathbf{I}_T - \gamma_B \mathbf{J} - \gamma_F \mathbf{J}']^{-1}(\lambda \boldsymbol{s}_i + \mathbf{u}_i). \tag{A.9}$$

This is the expression we use to generate $\boldsymbol{\pi}_i$ given the simulated series for $\boldsymbol{s}_i$ and $\mathbf{u}_i$.

Figure 1: Inflation for each country in the sample.

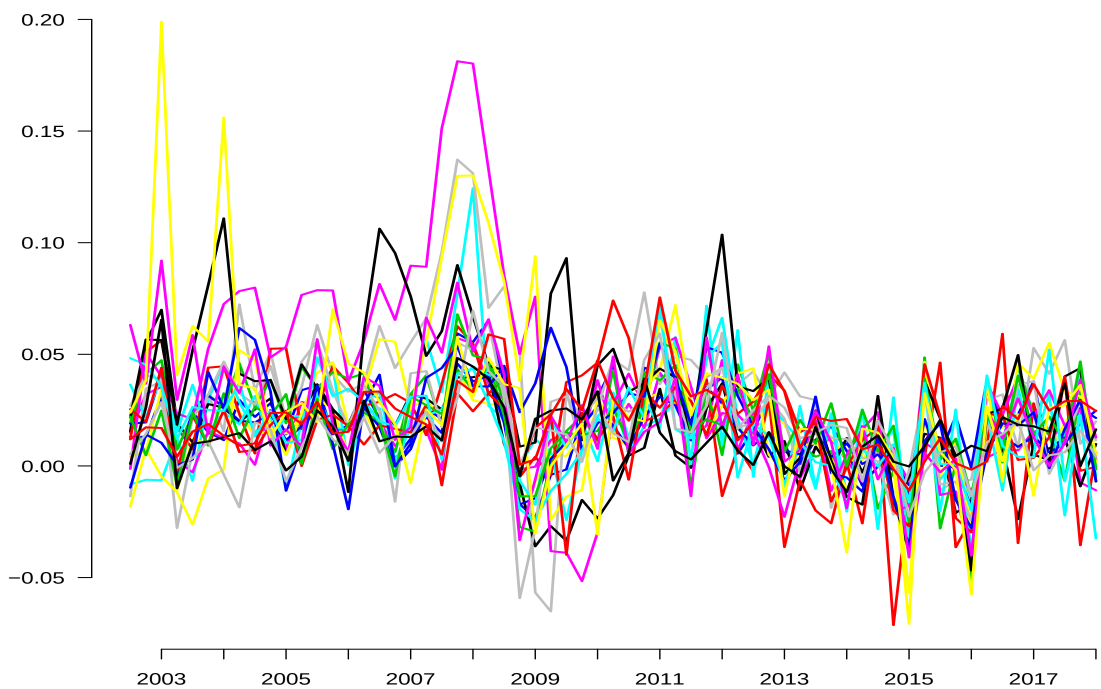

Table 1: Bias, RMSE, 5% size and power results for the IV estimator of $\gamma_B$.

| $N$ | $T$ | Without lead | | | | With lead | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Bias | RMSE | Size | Power | Bias | RMSE | Size | Power |
| | | | | | Without factors | | | | |
| 25 | 25 | 1.790 | 9.190 | 3.0 | 44.1 | 1.690 | 4.990 | 9.1 | 57.6 |
| 50 | 25 | 1.020 | 8.520 | 1.5 | 45.0 | 1.640 | 3.550 | 11.1 | 78.3 |
| 100 | 25 | 0.580 | 8.940 | 0.9 | 39.5 | 1.690 | 2.450 | 14.4 | 94.8 |
| 200 | 25 | 0.530 | 9.470 | 1.7 | 33.5 | 1.730 | 1.770 | 21.8 | 99.7 |
| 25 | 50 | 1.810 | 7.200 | 2.5 | 59.6 | 1.640 | 3.730 | 11.9 | 77.0 |
| 50 | 50 | 0.920 | 8.860 | 2.0 | 53.1 | 1.560 | 2.600 | 14.3 | 93.6 |
| 100 | 50 | 0.530 | 8.240 | 1.7 | 44.9 | 1.620 | 1.760 | 20.4 | 99.9 |
| 200 | 50 | 0.130 | 9.250 | 1.5 | 33.8 | 1.670 | 1.300 | 32.8 | 100.0 |
| 25 | 100 | 1.550 | 6.410 | 3.1 | 71.8 | 1.580 | 2.700 | 15.2 | 91.9 |
| 50 | 100 | 0.890 | 8.090 | 2.4 | 62.1 | 1.670 | 1.920 | 21.2 | 99.5 |
| 100 | 100 | 0.110 | 7.400 | 2.6 | 48.1 | 1.660 | 1.330 | 31.4 | 100.0 |
| 200 | 100 | −0.220 | 8.210 | 3.1 | 37.7 | 1.670 | 0.940 | 51.0 | 100.0 |
| 25 | 200 | 1.690 | 5.790 | 6.4 | 77.8 | 1.630 | 2.130 | 23.9 | 98.0 |
| 50 | 200 | 0.890 | 6.780 | 4.3 | 65.0 | 1.660 | 1.520 | 34.0 | 100.0 |
| 100 | 200 | 0.390 | 8.500 | 5.0 | 50.8 | 1.680 | 1.030 | 51.5 | 100.0 |
| 200 | 200 | −0.500 | 7.540 | 5.9 | 34.5 | 1.670 | 0.770 | 74.3 | 100.0 |
| | | | | | With factors | | | | |
| 25 | 25 | −0.300 | 9.990 | 4.2 | 31.1 | 0.600 | 7.430 | 11.6 | 35.9 |
| 50 | 25 | −0.530 | 8.810 | 3.2 | 31.9 | 0.290 | 4.700 | 8.9 | 56.8 |
| 100 | 25 | −0.950 | 8.500 | 3.0 | 26.3 | 0.350 | 3.160 | 9.0 | 83.6 |
| 200 | 25 | −1.010 | 8.620 | 3.6 | 22.3 | 0.240 | 2.270 | 8.5 | 97.5 |
| 25 | 50 | −0.190 | 7.220 | 3.8 | 43.6 | 0.430 | 5.040 | 9.3 | 58.4 |
| 50 | 50 | −0.910 | 7.690 | 3.5 | 38.7 | 0.130 | 3.180 | 7.8 | 84.3 |
| 100 | 50 | −1.190 | 7.770 | 4.1 | 26.2 | 0.070 | 2.260 | 7.6 | 97.7 |
| 200 | 50 | −1.320 | 8.630 | 3.6 | 18.8 | 0.010 | 1.650 | 7.8 | 100.0 |
| 25 | 100 | −0.250 | 5.530 | 2.7 | 64.2 | 0.190 | 3.390 | 7.4 | 80.3 |
| 50 | 100 | −0.590 | 6.730 | 4.2 | 47.6 | 0.150 | 2.330 | 7.6 | 97.8 |
| 100 | 100 | −1.040 | 7.590 | 5.0 | 27.5 | 0.010 | 1.620 | 6.1 | 100.0 |
| 200 | 100 | −1.530 | 7.740 | 3.9 | 20.0 | 0.000 | 1.140 | 5.4 | 100.0 |
| 25 | 200 | −0.090 | 4.510 | 3.7 | 78.6 | 0.120 | 2.470 | 7.6 | 95.8 |
| 50 | 200 | −0.350 | 5.840 | 6.1 | 54.4 | 0.050 | 1.650 | 6.3 | 100.0 |
| 100 | 200 | −0.500 | 7.070 | 4.6 | 35.0 | 0.010 | 1.120 | 5.1 | 100.0 |
| 200 | 200 | −1.220 | 7.510 | 3.8 | 22.3 | 0.000 | 0.820 | 5.8 | 100.0 |

*Notes*: The bias and RMSE results are multiplied by 100. The reported powers are adjusted for size. Results are reported not only for the full IV specification with the leaded regressors (factors) included in the instruments but also when said leads (factors) are excluded.

Table 2: Bias, RMSE, 5% size and power results for the IV estimator of $\gamma_F$.

| $N$ | $T$ | Without lead | | | | With lead | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Bias | RMSE | Size | Power | Bias | RMSE | Size | Power |
| | | | | | Without factors | | | | |
| 25 | 25 | 5.100 | 30.580 | 7.7 | 13.0 | 2.350 | 5.910 | 10.3 | 43.7 |
| 50 | 25 | 5.700 | 28.740 | 5.4 | 13.3 | 1.890 | 4.090 | 10.6 | 69.5 |
| 100 | 25 | 7.200 | 29.480 | 5.0 | 13.6 | 1.930 | 2.910 | 13.8 | 91.0 |
| 200 | 25 | 6.640 | 30.510 | 4.9 | 13.3 | 1.860 | 2.080 | 18.6 | 99.4 |
| 25 | 50 | 4.200 | 26.870 | 8.0 | 14.8 | 2.080 | 4.430 | 12.4 | 66.7 |
| 50 | 50 | 6.430 | 29.370 | 6.9 | 14.4 | 2.000 | 2.840 | 13.3 | 91.2 |
| 100 | 50 | 6.780 | 27.230 | 6.3 | 15.3 | 1.970 | 2.010 | 18.9 | 99.7 |
| 200 | 50 | 7.740 | 30.140 | 6.0 | 13.2 | 1.870 | 1.450 | 27.1 | 100.0 |
| 25 | 100 | 4.520 | 24.170 | 9.9 | 16.3 | 1.960 | 3.030 | 13.3 | 89.4 |
| 50 | 100 | 6.120 | 28.430 | 9.1 | 15.9 | 1.940 | 2.150 | 19.9 | 99.2 |
| 100 | 100 | 8.140 | 24.970 | 9.2 | 16.6 | 1.910 | 1.520 | 29.0 | 100.0 |
| 200 | 100 | 8.980 | 26.570 | 8.6 | 14.2 | 1.960 | 1.050 | 50.5 | 100.0 |
| 25 | 200 | 3.620 | 23.590 | 13.7 | 18.9 | 1.890 | 2.370 | 19.7 | 97.5 |
| 50 | 200 | 5.960 | 24.020 | 14.0 | 17.0 | 1.920 | 1.650 | 31.9 | 100.0 |
| 100 | 200 | 7.320 | 28.020 | 13.7 | 17.2 | 1.950 | 1.130 | 49.6 | 100.0 |
| 200 | 200 | 9.790 | 24.440 | 15.2 | 15.0 | 1.950 | 0.790 | 74.8 | 100.0 |
| | | | | | With factors | | | | |
| 25 | 25 | 8.250 | 32.470 | 10.1 | 12.7 | 1.450 | 8.270 | 12.5 | 30.4 |
| 50 | 25 | 6.290 | 33.580 | 6.4 | 12.3 | 0.410 | 5.410 | 10.7 | 49.9 |
| 100 | 25 | 6.630 | 31.900 | 6.1 | 12.2 | 0.270 | 3.660 | 9.5 | 74.6 |
| 200 | 25 | 5.580 | 32.970 | 4.9 | 12.9 | 0.070 | 2.630 | 8.7 | 94.0 |
| 25 | 50 | 4.980 | 31.260 | 7.8 | 13.3 | 0.580 | 5.820 | 10.2 | 45.8 |
| 50 | 50 | 6.300 | 34.040 | 6.8 | 13.3 | 0.300 | 3.660 | 6.9 | 78.8 |
| 100 | 50 | 6.340 | 32.560 | 4.6 | 12.4 | 0.150 | 2.480 | 6.1 | 96.8 |
| 200 | 50 | 6.000 | 36.130 | 4.0 | 11.8 | 0.010 | 1.850 | 6.8 | 99.9 |
| 25 | 100 | 4.560 | 30.680 | 8.3 | 16.6 | 0.400 | 3.880 | 8.0 | 70.1 |
| 50 | 100 | 4.930 | 32.170 | 5.5 | 14.3 | 0.170 | 2.660 | 6.1 | 96.3 |
| 100 | 100 | 5.270 | 32.860 | 4.0 | 13.5 | 0.050 | 1.800 | 5.6 | 99.9 |
| 200 | 100 | 6.890 | 32.930 | 3.1 | 12.8 | 0.030 | 1.270 | 5.1 | 100.0 |
| 25 | 200 | 1.910 | 25.990 | 8.1 | 16.6 | 0.150 | 2.820 | 7.0 | 92.8 |
| 50 | 200 | 2.720 | 28.530 | 5.7 | 14.1 | 0.060 | 1.920 | 6.6 | 99.9 |
| 100 | 200 | 2.880 | 32.120 | 3.6 | 13.6 | 0.050 | 1.320 | 5.2 | 100.0 |
| 200 | 200 | 5.590 | 32.340 | 2.8 | 15.0 | 0.020 | 0.900 | 5.2 | 100.0 |

*Notes*: See Table 1 for an explanation.

Table 3: Bias, RMSE, 5% size and power results for the IV estimator of $\lambda$.

| | | Without lead | | | | With lead | | | |
|---|---|---|---|---|---|---|---|---|---|
| $N$ | $T$ | Bias | RMSE | Size | Power | Bias | RMSE | Size | Power |
| | | | | Without factors | | | | | |
| 25 | 25 | 1.670 | 10.780 | 2.4 | 27.0 | 2.730 | 5.140 | 11.5 | 52.5 |
| 50 | 25 | 1.410 | 10.570 | 1.4 | 31.1 | 2.900 | 3.600 | 16.6 | 82.4 |
| 100 | 25 | 0.920 | 10.540 | 1.4 | 29.6 | 2.860 | 2.610 | 24.6 | 96.4 |
| 200 | 25 | 1.060 | 11.000 | 1.2 | 27.8 | 2.870 | 1.910 | 40.5 | 99.9 |
| 25 | 50 | 1.830 | 9.170 | 1.3 | 40.3 | 2.770 | 3.590 | 14.5 | 84.1 |
| 50 | 50 | 1.180 | 9.960 | 0.9 | 40.8 | 2.840 | 2.540 | 22.9 | 98.4 |
| 100 | 50 | 1.130 | 9.600 | 0.9 | 41.1 | 2.890 | 1.820 | 39.3 | 100.0 |
| 200 | 50 | 0.730 | 10.640 | 0.8 | 33.6 | 2.910 | 1.390 | 63.7 | 100.0 |
| 25 | 100 | 1.840 | 7.980 | 2.5 | 54.0 | 2.860 | 2.560 | 22.9 | 98.9 |
| 50 | 100 | 1.260 | 9.640 | 1.5 | 50.7 | 2.850 | 1.890 | 39.3 | 100.0 |
| 100 | 100 | 0.600 | 8.620 | 1.4 | 42.8 | 2.860 | 1.370 | 63.7 | 100.0 |
| 200 | 100 | 0.290 | 9.350 | 1.5 | 37.3 | 2.840 | 0.970 | 87.7 | 100.0 |
| 25 | 200 | 2.030 | 7.510 | 2.9 | 68.6 | 2.780 | 1.940 | 37.3 | 100.0 |
| 50 | 200 | 1.350 | 8.110 | 2.7 | 56.7 | 2.860 | 1.390 | 63.5 | 100.0 |
| 100 | 200 | 0.900 | 9.600 | 2.1 | 49.1 | 2.850 | 0.990 | 87.9 | 100.0 |
| 200 | 200 | 0.030 | 8.540 | 3.6 | 38.1 | 2.850 | 0.710 | 99.6 | 100.0 |
| | | | | With factors | | | | | |
| 25 | 25 | −1.580 | 9.480 | 5.4 | 16.5 | 0.470 | 5.820 | 10.4 | 39.7 |
| 50 | 25 | −1.320 | 9.480 | 3.5 | 17.9 | 0.330 | 3.910 | 8.8 | 73.0 |
| 100 | 25 | −1.550 | 8.670 | 3.4 | 16.9 | 0.130 | 2.630 | 8.5 | 95.2 |
| 200 | 25 | −1.440 | 8.710 | 2.9 | 17.7 | 0.020 | 1.840 | 7.6 | 99.9 |
| 25 | 50 | −1.040 | 7.940 | 4.2 | 25.8 | 0.290 | 3.870 | 7.8 | 74.6 |
| 50 | 50 | −1.530 | 8.590 | 3.7 | 21.0 | 0.010 | 2.560 | 6.4 | 97.3 |
| 100 | 50 | −1.480 | 8.280 | 2.6 | 22.5 | 0.060 | 1.760 | 5.9 | 100.0 |
| 200 | 50 | −1.520 | 8.860 | 3.0 | 18.4 | 0.020 | 1.290 | 7.2 | 100.0 |
| 25 | 100 | −0.870 | 7.030 | 4.0 | 40.1 | 0.180 | 2.550 | 6.0 | 97.5 |
| 50 | 100 | −1.120 | 7.630 | 3.7 | 31.6 | 0.020 | 1.820 | 6.4 | 100.0 |
| 100 | 100 | −1.260 | 8.090 | 3.3 | 24.2 | 0.020 | 1.280 | 6.3 | 100.0 |
| 200 | 100 | −1.720 | 8.060 | 2.9 | 18.2 | −0.030 | 0.900 | 6.0 | 100.0 |
| 25 | 200 | −0.360 | 5.630 | 4.2 | 60.6 | 0.050 | 1.810 | 6.1 | 100.0 |
| 50 | 200 | −0.640 | 6.630 | 3.9 | 45.7 | 0.000 | 1.260 | 6.1 | 100.0 |
| 100 | 200 | −0.710 | 7.540 | 3.4 | 32.7 | −0.010 | 0.880 | 5.6 | 100.0 |
| 200 | 200 | −1.390 | 7.920 | 2.6 | 21.6 | −0.020 | 0.610 | 4.9 | 100.0 |

*Notes*: See Table 1 for an explanation.

Table 4: IV estimates of $\gamma_B$, $\gamma_F$ and $\lambda$.

| Parameter | All | Euro | New | Without UK |
|---|---|---|---|---|
| | | Without factors | | |
| $\gamma_B$ | 0.440*** | 0.440*** | 0.427*** | 0.441*** |
| | (0.048) | (0.044) | (0.002) | (0.051) |
| $\gamma_F$ | 0.543*** | 0.529*** | 0.575*** | 0.542*** |
| | (0.047) | (0.002) | (0.056) | (0.056) |
| $\lambda$ | −0.002 | −0.001 | −0.001 | −0.002 |
| | (0.003) | (0.049) | (0.045) | (0.002) |
| | | With factors | | |
| $\gamma_B$ | 0.307* | 0.359*** | 0.473*** | 0.333** |
| | (0.158) | (0.13) | (0.002) | (0.125) |
| $\gamma_F$ | 0.487*** | 0.446*** | 0.453*** | 0.467*** |
| | (0.106) | (0.002) | (0.102) | (0.107) |
| $\lambda$ | 0.003* | 0.004** | 0.006 | 0.003* |
| | (0.005) | (0.164) | (0.141) | (0.002) |
| $m_g$ | 3 | 3 | 3 | 3 |
| $m_f$ | 1 | 1 | 1 | 1 |
| $J$-test | 0.215 | 0.523 | 0.979 | 0.252 |

*Notes*: This table contains the estimation results for the reduced form version of the NKPC. ″*″, ″**″ and ″***″ indicate statistical significance at the 10%, 5% and 1% level, respectively. The numbers within parentheses are the standard errors. $m_g$ and $m_f$ refer to the estimated number of factors in $\mathbf{g}_t$ and $\mathbf{f}_t$, respectively, and "$J$-test" refers to the $p$-value of the overidentifying restrictions test. The ″Without Factors″ results are obtained by applying the IV estimator to the data without defactoring. ″All″, ″Euro″, ″New″ and ″Without UK″ refers to the full sample, the Euro area sample, the new EU member countries, and the sample comprised of all countries but the UK.

Table 5: Indirect IV estimates of $\beta$, $\omega$ and $\delta$.

| Parameter | All | Euro | New | Without UK |
|---|---|---|---|---|
| $\beta$ | 0.987*** | 0.982*** | 0.936*** | 0.985*** |
| | (0.051) | (0.042) | (0.045) | (0.056) |
| $\omega$ | 0.371*** | 0.442*** | 0.660*** | 0.408*** |
| | (0.132) | (0.096) | (0.065) | (0.134) |
| $\delta$ | 0.597*** | 0.560*** | 0.676*** | 0.581*** |
| | (0.099) | (0.079) | (0.064) | (0.107) |

*Notes*: The structural parameter estimates reported in the table are obtained based on the reduced form IV estimates reported in Table 4. $\beta$, $\omega$ and $\delta$ refer to the discount factor, the fraction of backward-looking price setters, and the rigidity parameter, respectively. The numbers within parentheses are the standard errors, which are obtained by using the Delta method. See Table 4 for an explanation of the rest.