Global Impacts of US Monetary Policy Uncertainty Shocks

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Global Impacts of US Monetary Policy Uncertainty Shocks∗

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We build a new empirical model to estimate the global impact of an increase in the volatility of US monetary policy shocks. Specifically, we admit time-varying variances of local structural shocks from a stochastic volatility specification. By allowing for rich dynamic interaction between the endogenous variables and time-varying volatility in the global setting, we find that US interest rate uncertainty not only drives local output and inflation volatility, but also causes declines in output, inflation, and the interest rate. Moreover, we document strong global impacts, making the world move in a very synchronous way. Crucially, spillback effects are found to be significant even for the US economy.

**Keywords:** US Monetary Policy, Volatility Shocks, Uncertainty, Global Economy.

**JEL codes:** C32, C54, E52, E58, F44.
Non-technical summary

Globalization is a mixed blessing. The current heavy interdependence of countries has improved access to new markets and technologies, enabled knowledge sharing, and intensified flows of trade, capital, people, and ideas. However, it has also produced challenges. These include difficulties in regulating markets, tackling unintended effects spilling across economies, dealing with correlated shocks and synchronized business cycles, and making policy decisions in a highly uncertain environment. The role of monetary policy, especially after the global financial crisis, has been critical in stabilizing macroeconomic fluctuations. However, the effect on the real economy depends not only on central banks’ actions but also on what agents expect them to do.

Such uncertainty in the macroeconomic environment can be both a source and a byproduct of macroeconomic and policy developments. This effect can be further amplified by global financial and trade integration, thereby creating spillover and spillback effects. They matter not only for small open economies but also for large countries like the United States. By focusing on US monetary policy uncertainty shocks, this paper covers a global framework where unexpected variations in uncertainty about the US monetary policy impact the United States economy, which can, in turn, affect macroeconomic uncertainty, the global economy, and can even be imported back to the US.

Specifically, we propose a new econometric model that extends the global vector autoregressive framework to estimate the global impacts of an increase in US monetary policy shocks’ volatility. The model has two distinguishing features. First, we admit time-varying variances of local structural shocks from a stochastic volatility specification. Second, there is a dynamic interaction between the endogenous variables in the vector auto-regression and the time-varying volatility, allowing for the second-moment shocks’ effects to the first-moment level. Because the model takes trade and financial linkages between economies into account, the uncertainty shocks affect the country of origin and spill over to other economies, whether connected directly or indirectly.

We document how an unexpected change in the US interest rate volatility affects the US and the global economy. In line with the recent literature, we find a significant recessionary and deflationary effect, as well as increases in output and inflation volatilities. We also find strong spillovers, making the rest of the world move in a very synchronous way, especially among the advanced economies group. This contribution highlights the role of uncertainty in generating...
synchronized contraction and rationalizes the global economy’s slow recovery.

We document only marginally less pronounced spillovers (particularly for advanced economies) after the “great trade collapse”, when we use trade weights, hinting at possible, though slowly moving, structural rebalancing in the global trade network. We find proof for US dominance globally once we instead employ financial linkages. Importantly, we establish that the global dimension is critical even for the US economy via the non-trivial spillback effects (thereby providing quantitative evidence of spillbacks from the global economy to the US economy). Additionally, we find that macroeconomic uncertainties are state-dependent and necessitate a dynamic interaction between endogenous variables and time-varying volatility. Specifically, a decrease in output growth leads to an increase in output and inflation volatility, thereby supporting recent findings in the literature that macroeconomic uncertainty is often a consequence of real economy fluctuations, whereas financially-related uncertainty is likely to be a cause.
1 Introduction

Globalization is a mixed blessing. The heavy interdependence has brought about benefits but also challenges, including correlated shocks, unintended effects spilling across economies, synchronized business cycles and unprecedented levels of uncertainty. The latter has been the subject of intense research, especially after the global financial crisis, as one of the main reasons for the unusual depth and duration of the recession as well as slow and weak recoveries thereafter. The role of monetary policy has been central in stabilizing macroeconomic fluctuations. However, the effect on the real economy depends not only on the actions of central banks but also on what agents expect them to do. Uncertainty about the reaction of monetary policy affects the behavior of investors, households and firms. Moreover, such uncertainty in the macroeconomic environment can be both a source and a byproduct of macroeconomic and policy developments. This effect can be further amplified by global financial and trade integration, thereby creating spillover and spillback effects. By focusing on the United States (US) monetary policy uncertainty shocks, this paper covers a global framework where unexpected variations in uncertainty about the US monetary policy impact the United States economy, which can, in turn, affect macroeconomic uncertainty, the global economy and can even be imported back to the US.

Specifically, we propose a new econometric model that extends the global vector autoregressive (GVAR) framework to estimate the global impacts of an increase in the volatility of US monetary policy shocks. The model has two distinguishing features. First, the variance of structural shocks in the local model is allowed to be time-varying via a stochastic volatility specification, subject to shocks as well as changes in the fundamentals in the economy. Second, there is a dynamic interaction between the endogenous variables in the VAR and the time-varying volatility, allowing for the effects of the second-moment shocks on the first-moment level. Because the model takes linkages between economies into account, the uncertainty shocks affect not only the country of origin but also can spill over to other economies, whether they are connected directly or indirectly. Our paper therefore contributes to the literature in a number of respects.

First, while most of the existing papers on policy uncertainty focus on a single country, particularly the United States, we consider the US along with 32 other countries which, taken together, account for 90% of world output. Such wide coverage allows us to evaluate the possibility of the heterogeneous effects of US policy uncertainty on economic fluctuations globally.
and, at the same time, take into account the international channel of this shock, i.e. the possibility of not only spillovers but also spillbacks on the US economy. The latter point has been recently emphasized by Obstfeld [2020] (and previously put forward by BIS [2016], Agenor and da Silva [2018], Carney [2019] in the context of increasing spillovers from and thus to advanced economies after the global financial crisis (GFC)) as the crucial component to be tracked by policymakers due to the disproportionate US weight in the global economy. These spillback effects may impact the tradeoffs between price level control and low unemployment or financial stability goals, and they may exacerbate financial risks if economic and financial cycles are not sufficiently aligned. Despite this policy-relevant background, we lack quantitative evidence on the magnitude and effects of spillbacks on the US economy, especially from a global perspective rather than that of a few selected countries. Relatedly, as has been argued by Kose et al., [2003], increasing interdependencies across countries produce a common world factor that underlies the world business cycle. We document the role of US monetary policy uncertainty as one of the drivers of similar responses across the globe. Therefore, this contribution is also important given the observation of a recent synchronized contraction and the slow recovery of the global economy.

Second, while the international transmission mechanism of first-moment shocks has been well investigated, the literature about the global impact of uncertainty shocks (second-moment shocks) is limited. Concerning the latter, the common approach is to integrate a proxy of uncertainty into a VAR-type model. We instead model the economy’s first and second moments in a unified, internally consistent framework. Our model shows that much of the variation in the proxies of monetary policy uncertainty is actually not driven by monetary policy uncertainty. Third, we explore the role of different channels of transmission, in particular, international trade and financial linkages. We also admit structural changes before and after the GFC (and the effects of the “great trade collapse”, as famously coined by Alessandria et al., 2010). Last but not least, we contribute to the global macroeconomic modeling literature, extensively covered in Garratt et al. [2006], by incorporating a marginal model of stochastic volatility-in-mean into its standard framework, the rationale of which is discussed below, and show how to build, solve and estimate a global model; this addition paves the way for applications on global uncertainty well beyond the scope of this paper.

We set out by documenting how an unexpected change in the US interest rate volatility affects
the US and the global economy. In line with the literature, we find a significant recessionary and deflationary effect as well as document increases in output and inflation volatilities. We also find strong spillovers, making the rest of the world move in a very synchronous way, especially among the advanced economies group. We provide evidence of the depreciation of the renminbi against the US dollar in China, using trade and financial linkages as well as different shock identification techniques. Moreover, there is evidence of slightly less pronounced spillovers after the “great trade collapse”, hinting at possible rebalancing in the global trade network. However, if instead of trade we make use of financial linkages, then the US confirms its prominent role globally, especially so for the economies that are further away and thus are less integrated trade-wise. Nonetheless, our results about global impacts, despite different weight schemes or identification procedures, remain largely intact. Importantly, we establish that the global dimension is critical even for the US economy via the non-trivial spillback effects. Additionally, we find that macroeconomic uncertainties are state-dependent and thus necessitate a dynamic interaction between endogenous variables and time-varying volatility. Specifically, a decrease in output growth leads to an increase in output and inflation volatility, which supports the findings in Ludvigson et al. [forthcoming] that macroeconomic uncertainty is often a consequence of real economy fluctuations, whereas financially-related uncertainty is likely to be a cause.

Only recently have the effects of unexpected variations in uncertainty about economic policy (i.e. unexpected changes in the volatility of policy innovations or unexpected second-moment shocks) received due attention. For example, Baker et al. [2016] develop an index of economic policy uncertainty based on newspaper coverage frequency and find that policy uncertainty shocks foreshadow declines in investment, output, and employment. Similar findings are also obtained by Mumtaz and Zanetti [2013], Creal and Wu [2017], Husted et al. [2019] when analyzing monetary policy uncertainty, Fernández-Villaverde et al. [2015] with fiscal policy uncertainty, and Born and Pfeifer [2014] and Mumtaz and Surico [2018] with both fiscal and monetary policy uncertainty. However, these papers focus on the US economy and do not consider the global impact of US policy uncertainty shocks.

A related emerging branch of literature deals with general uncertainty shocks and their effects on the real economy. For instance, Bloom et al. [2018] emphasize that uncertainty is strongly countercyclical both at the aggregate and the industry level, suggesting its role in driving business cycles and emphasizing a need to model recessions as negative first-moment
and positive second-moment shocks. Cesa-Bianchi et al. [2019] find that the financial common factor significantly reduces country-specific GDP growth in a multi-country model with realized equity price volatility. Bonciani and Ricci [2020] document, inter alia, adverse consequences on output, but more heterogeneous effects on nominal variables, due to increased global financial uncertainty among advanced and emerging small open economies. Mumtaz and Theodoridis [2015] provide empirical evidence of the international transmission of volatility shocks, showing that a one standard deviation increase in the volatility of the shock to US real GDP leads to a decline in the UK’s GDP of 1% relative to trend. Crespo Cuaresma et al. [2020] investigate the macroeconomic consequences of international uncertainty shocks in G7 countries and show that an international uncertainty shock has negative effects across all economies and variables under consideration, leading to strong declines in output, prices, exports, interest rates and equity prices. Unlike these other investigators, however, our focus is on monetary policy risks that affect economic fluctuations, and not only domestically but also globally. Crucially, as summarized by Chinn et al. [2017], uncertainty shocks in policies can have major repercussions for the global economy, particularly for exchange rates, interest rates and capital flows, yet the channels of cross-border spillovers remain largely unexplored.

Our paper is closely related to Bhattarai et al. [2019] who, in a panel VAR, confirm the adverse effects of the US stock market uncertainty shock on stock prices, exchange rates, output, consumer prices and capital inflows into emerging market economies. To do so, the authors first construct a measure of the US stock market uncertainty shock and then put it in a panel VAR for a set of 15 emerging economies. In contrast, we model the economy’s first and second moments in a unified, internally consistent framework, allow the interaction between uncertainty and the economy, and consider a larger set of economies. To sum up, the transmission nature of US uncertainty shocks, the interaction between uncertainty and the macroeconomy, and the global spillovers, as well as the possibility of spillbacks to the US are missing in the literature. This is indeed the gap that our paper attempts to fill.\footnote{Our paper also relates to the literature on the spillovers from US monetary policy-related shocks. For instance, Kim [2001], Canova [2005], Dees et al. [2007], Feldkircher and Huber [2016], and Crespo Cuaresma et al. [2019] show the existence of significant spillovers from unexpected changes in the US monetary policy shocks. We contribute to this literature with a study on the global impact of the shocks to the volatility of US monetary policy.}

From the methodological perspective, our work is connected to the literature on large-scale macroeconometric models. In this literature, the global vector autoregressive model (GVAR),......
originally proposed by Pesaran et al. [2004] proves to be a convenient way of reducing the dimensionality of the estimation problem. Chudik and Pesaran [2011, 2013] establish the conditions under which the key macroeconomic variables can be arbitrarily well approximated by a set of finite-dimensional small-scale models that can be consistently estimated separately and then stacked into the global model. Crespo Cuaresma et al. [2016] develop a Bayesian variant of global vector autoregressive models to forecast an international set of macroeconomic and financial variables. Huber [2016] adds the stochastic volatility into the GVAR and finds that the stochastic volatility improves the predictive accuracy and robustness. Meanwhile, Crespo Cuaresma et al. [2019] develop the GVAR with time-varying parameters and stochastic volatility to analyze whether international spillovers of US monetary policy have changed over time and document weaker effects in the aftermath of the global financial crisis. To the best of our knowledge, the literature has not addressed data-generating processes within the VAR framework that can give rise to the impact of uncertainty shocks to the first moment of the fundamentals (i.e. the volatility-in-mean property), while at the same time allowing for a common dynamic factor, for a large number of countries. We extend this approach by accommodating stochastic volatility-in-mean, at the same time allowing second-order moments to be affected by the fundamentals. We also consider different weight matrices that help dimension reduction and have an economic interpretation. Particularly, we cover international connections through trade, post-financial crisis trade, and financial flows.

Our paper is structured as follows: we cover theoretical motivation in Section 2. We proceed with explaining the econometric framework, the global model solution, and the identification of US monetary policy shocks in Section 3. Empirical findings are summarized in Section 4, while Section 5 offers more detailed discussions and placements within the literature. Finally, Section 6 provides a few concluding remarks.

The main difference between the GVAR and standard Large-(Bayesian) VARs, for instance, Bańbura et al. [2010], are the parametric restrictions imposed through the linkage matrix. If a given country \( j \) is not strongly linked to country \( i \), the corresponding parameter estimate is pushed towards zero by setting the corresponding weight close to zero (Huber, 2016).
2 Theoretical motivation

We analyze a large set of open macroeconomies in a stylized empirical framework, which is rooted
in reduced-form open-economy New-Keynesian (NK) models. The motivation for the variables and
their theoretical underpinnings are covered, among many others, in seminal contributions
in Clarida et al. [2001] and Gali and Monacelli [2005]. The canonical model, extended to the
multi-country setting with \( i = 1, \ldots, N \) economies, features the Euler equation for output:

\[
y_{it} = \alpha_{ib} y_{i,t-1} + \alpha_{if} E_{it} y_{i,t+1} - \alpha_{ir} (r_{it} - E_{it} \pi_{i,t+1}) + \alpha_{ie} \epsilon_{it} + \alpha_{iy} y^*_i + \epsilon_{iy,t},
\]

where \( r_{it} \) is the short-term interest rate, \( r_{it} \) is the real effective exchange rate, and \( y^*_i \) is the weighted average of foreign outputs. It is assumed that foreign output is weighted by
\( w_{it} = O(N^{-1}) \) and \( \sum_{i=1}^N w_{it} = 1 \). It can be motivated by the technology structure with an unobserved factor as well as net exports being part of aggregate output (and thus driven by the real effective exchange rate and outputs across all trading partners, as in Dees et al., 2014). The New Keynesian Phillips curve is given by

\[
\pi_{it} = \beta_{ib} \pi_{i,t-1} + \beta_{if} E_{it} \pi_{i,t+1} + \beta_{iy} y_{it} + \epsilon_{i\pi,t},
\]

where \( \pi_{it} \) is inflation and \( y_{it} \) is output gap; the Taylor rule is expressed as

\[
r_{it} = \gamma_{ib} r_{i,t-1} + \gamma_{i\pi} \pi_{it} + \gamma_{iy} y_{it} + \epsilon_{ir,t},
\]

and, since we cover the open economy, the real exchange rate versus the US dollar is

\[
re_{it} = \delta_{ib} r_{i,t-1} + \delta_{i\pi} \pi_{it} + \delta_{iy} y_{it} + \delta_{ir} r_{it} + \epsilon_{ie,t}.
\]

Notice that the real exchange rate does not enter the US model (since the US dollar is used
as a numeraire), though it is featured in the rest of the models. In fact, the US model will allow for the weakly exogenous trade-weighted average of bilateral real exchange rates, i.e. the

\footnotesize{As we are analyzing an identified policy shock, we merely motivate the choice of variables. There is a large literature on the properties and solution of linear multivariate rational expectations models (e.g. Broze et al., 1995, Binder and Pesaran, 1997, Klein, 2000, Sims, 2002) and switching rational expectations models (e.g. Cho, 2011, Farmer et al., 2011), which is not our direct focus. See Section A for the solution method of our empirical model.}
The specification can be motivated by the fundamentals-driven or Taylor-rule based models (Engel and West, 2005) or it can follow a stationary first-order autoregression, as in Dees et al. [2014]. Similarly, Schmitt-Grohe and Uribe [2018] assume an exogenously given terms of trade (which drive the real exchange rate) equation for emerging economies. All these different interpretations can be reproduced by constraining interactions between variables.

In fact, the parameter restrictions for the four-variable system nest many micro-founded log-linearized models in the literature. Gali and Gertler [1999] developed a hybrid variant of the New Keynesian Phillips curve that relates inflation to real marginal cost, expected future inflation and lagged inflation. The hybrid equation can also be justified by indexation (Christiano et al., 2005). The backward-lookingness of the output equation is featured if there is a habit formation (see Ravn et al. [2006], Dennis [2009], Schmitt-Grohe and Uribe [2012] for different approaches to modeling habit formation and applications). The persistence terms are empirically supported and often featured in applied macroeconomic models (e.g. DSGE models of the type of Smets and Wouters, 2007).

There is also a strand of literature on consumption, real interest rates, as well as exchange rates – all standard elements of the NK models – that incorporates volatility risk, though it does not venture into the general equilibrium setting. Our focus is on aggregate implications where the DSGE-based theoretical framework is the most prominent one, and also the one most often used to arrive at an empirical VARMA representation. It is worth noting, however, that the popular linearized version of DSGE model is certainty-equivalent, ruling out any possibility of studying the impacts of policy-type uncertainty on the real economy (see Basu and Bundick, 2017, Born and Pfeifer, 2014, Fernández-Villaverde et al., 2015, Mumtaz and Theodoridis, 2015). It is, therefore, necessary to solve the model to at least the third-order approximation around the steady state at which the volatility shocks enter as independent arguments in the policy functions with a coefficient different from zero, thus affecting directly endogenous variables.  

\footnote{One route to allow for the consumption volatility risk as the exposure to macroeconomic uncertainty is to follow the asset pricing framework by Boguth and Kuehn [2013]. Since consumption and output are intricately linked if capital is assumed away, the dynamic Euler equation with the risk volatility can be rationalized emphasizing different channels. As for the inflation, Haubrich et al. [2012] develop a model of the nominal and real terms structures wherein time-varying volatility is shown to be driving the real interest rate and expected inflation processes. When it comes to the real effective exchange rate equation, Gomez-Gonzalez and Rees [2013] find that the volatility of the key macroeconomic variables is between 20 and 30 per cent higher when volatility shocks of terms of trade are incorporated.}
this provides a guideline for our model specification.

The various channels of uncertainty shocks can be summarized by focusing on investment, precautionary savings, productivity, labor and asset pricing channels. In particular, a rise in uncertainty is mainly transmitted via the investment channel, which is known for having a significant multiplier effect on short-term growth. In the face of uncertainty, investors tend to opt for a wait-and-see approach and postpone their investment decisions (see Bloom et al. [2007], Stokey [2016], Bachmann and Bayer [2013] among many others). Another channel of transmission is precautionary savings, which tend to weigh on household spending (see Carroll and Samwick [1998], Giavazzi and McMahon [2012], among others). This naturally leads to adjustments in the current account, thereby necessitating an open-economy setup (see Ghosh and Ostry, 1997). According to some studies, uncertainty may also reduce total factor productivity, as it leads to inefficient factor allocation between companies (Bloom et al., 2018). Increased uncertainty can also have a negative impact on the labor market, as it causes firms to delay hiring decisions and discourages workers from looking for other jobs (Schaal, 2017). Risk premiums also tend to rise, which can restrict the flow of credit to households and businesses and thus exacerbate declines in macroeconomic aggregates and equity prices (Bretscher et al., 2020).

Though our data-driven approach allows us to differentiate between different models by restricting the parameter space, we will allow the data to speak about the interactions and instead make a structural identification of policy shocks. Additionally, we will add two crucial components – shock spillovers and uncertainty\(^5\) – which are missing in standard models. According to Dees et al. [2014], spillover effects are so important that it is global, not domestic, shocks to inflation (supply) and output (demand) equations that drive the macroeconomy in the long run. Therefore, we explicitly model a common factor that acts upon all variables, albeit with a country-specific intensity. In order to deal with the curse of dimensionality in a global setting, we adopt the idea to proxy the unobserved global effects using cross-sectionally weighted averages of endogenous variables; see Pesaran et al. [2004], Pesaran [2006], effectively working with

\(^{5}\)Bloom [2009], Bloom et al. [2018] argue that periods of low and high uncertainty can explain business cycle fluctuations and demonstrate that a second-moment shock is qualitatively different from the persistent impact of a first-moment shock. Similarly, Jurado et al. [2015] show that uncertainty has large and persistent impacts on the economy. Uncertainty tends to rise significantly in recessions (Bloom, 2009, Jurado et al., 2015), partly due to monetary policy shocks (Mumtaz and Theodoridis, 2019). The source of uncertainty also matters for the endogenous effects (Ludvigson et al., forthcoming), implying that the empirical model should allow for the possibility that uncertainty responds to changes in the fundamentals.
the VAR that is augmented with weakly exogenous variables, VARX.\footnote{We also experiment with direct spatial effects, subsumed with cross-sectionally weighted policy shocks, and a dominant unit setup where the impact of the US for each country does not diminish even in the limit. See \cite{Chudik2011, Chudik2013} for the conditions of this dimension-reducing technique to work.}

To sum up, our model helps to link those strands of literature by investigating shocks to monetary policy uncertainty, allowing for feedback effects between volatilities and macroeconomic variables, along with a global dimension, and different interconnections across economies that can experience spillovers and, as a result, spillbacks.

3 Econometric Framework

An infinite-dimensional VAR can be approximated arbitrarily well by country-specific models, which are estimated separately and later stacked into the global system, even if there are dominant units or unobserved common factors \citep{Chudik2011}. The so-called VARX model, where weakly exogenous variables include cross-sectional averages, proxying for the common unobserved factors, augmented with stochastic volatility for country $i$, is presented as follows:

$$x_{it} = a_i + \sum_{\ell=1}^{p_i} \Phi_i x_{i,t-\ell} + \sum_{\ell=0}^{q_i} \Lambda_i x^*_{i,t-\ell} + \sum_{\ell=0}^{s_i} \Psi_i h_{i,t-\ell} + u_{it}, \quad (3.1)$$

$$u_{it} = \Omega_{it}^{1/2} e_{it}, \quad e_{it} \sim N(0, I) \quad (3.2)$$

$$\Omega_{it} = A_i^{-1} H_i A_i^{-1'}, \quad (3.3)$$

where $a_i$ denotes vector of intercepts, $x_{it} = [x_{i1t}, ..., x_{ikt}]'$ denotes the $k_i \times 1$ vector of domestic variables, $\Phi_i$ is the coefficient matrix associated with the lags of $x_{it}$, $x^*_{it}$ denotes the $k^*_i \times 1$ vector of foreign variables in country $i$ associated with the coefficient matrix $\Lambda_i$. Based on the theoretical motivation, $x_{it}$ includes annual output growth, inflation, short-term interest rate, and the real exchange rate growth, whereas $x^*_{it}$ comprises respective cross-sectional averages. $h_{it} = [h_{i1t}, h_{i2t}, ..., h_{ikt}]$ is a $k_i \times 1$ vector of log volatility of structural shocks. $H_i$ is a diagonal matrix whose values are the volatility of structural shocks $\exp(h_{it})$. The function of the $A_i$ matrix is to identify structural shocks $e_{it}$ from the reduced-form ones, which will be discussed below.
The transition equation for the stochastic volatility is given by

\[ h_{it} = c_i + \sum_{\ell=1}^{m_i} \Upsilon_i \ell h_{i,t-\ell} + \sum_{\ell=1}^{q_i} \Xi_i \ell x_{i,t-\ell} + \eta_{it} \]

\[ \eta_{it} \sim N(0, \mathbf{Q}_i) \text{ and } \mathbb{E}(e_{it}, \eta_{it}) = 0, \tag{3.4} \]

where \( c_i = [c_{i1}, \ldots, c_{ik_i}]' \) is a vector of intercepts, \( \Upsilon_i \) is a matrix of coefficients on the lagged log volatility of structural shocks, and \( \Xi_i \) captures the effects of lagged macroeconomic variables on log volatility of structural shocks in country \( i \). \( \mathbf{Q}_i \) is the covariance matrix of the \( \eta_{it} \); while most relevant studies assume that \( \mathbf{Q}_i \) is a diagonal matrix, we relax this assumption and allow the off-diagonal entries to take non-zero values.

It is worth discussing several noticeable features of the model in relation to the theoretical motivations presented in Section 2. First, the idea of the global model is to account for cross-country linkages in a coherent and computationally feasible manner, where dimension reduction is achieved by weighting endogenous variables into the so-called their foreign counterparts \( x_{it}^* \) in (3.1):

\[ x_{it}^* = \bar{W}_i x_t, \tag{3.5} \]

where elements of matrix \( \bar{W}_i \) are given by \( w_{ii} = 0 \) and \( \sum_{j=0}^{N} w_{ij} = 1 \) for all \( i, j = 0, 1, \ldots, N \) and \( x_t = [x_{1t}', \ldots, x_{Nt}']' \) is the set of \( k = \sum_{i=0}^{N} k_i \) endogenous variables of the global economy. We cover international connections through trade, trade after the financial crisis and financial linkages. These connections allow us to investigate the spillover effects of shocks. Specifically, an unexpected shock in country \( i \) first affects \( x_{it} \) via (3.1), then spillovers to countries \( j \neq i \) via their foreign specific variables \( x_{jt}^* \), and spillbacks to country \( i \) via \( x_{it}^* \).

Second, the contemporaneous and lagged log volatility of structural shocks \( h_{it} \) is included directly in (3.1), thus allowing for the impacts of the second-moment shocks to the first moment of endogenous variables.\(^7\) This specification is similar to the reduced form of a DSGE model approximated to the third order around the steady state. Although the model specification does not capture all non-linear terms in DSGE models, Mumtaz and Theodoridis [2015] argue that such a time series specification is rich enough to capture the responses of macroeconomic

\(^7\)As discussed in Mumtaz and Zanetti [2013], we included log volatility instead of its level because the former is substantially more computationally stable than the latter. In addition, the level specification is sensitive to the scaling of the variables.
aggregates to uncertainty, i.e. when estimated on data generated from a nonlinear DSGE model, it closely matches the underlying DSGE responses. Third, the transition equation for stochastic volatility (3.4) indicates that the volatility of shocks are allowed to respond to changes in the fundamentals, following the discussion above.

After estimating the individual country models, we stack all the \( k = \sum_{i=0}^{N} k_i \) endogenous variables of country models into the global economy. We then solve the global model and obtain the dynamics of \( x_t \), as follows:

\[
x_t = x_0 + \sum_{\ell=1}^{p_z} K_\ell x_{t-\ell} + \sum_{\ell=0}^{s} \Theta_\ell h_{t-\ell} + v_t.
\] (3.6)

where \( h_t = [h_{1t}', \ldots, h_{Nt}']' \) is the stack of volatilities of country models into the global economy, associated with coefficient matrix \( \Theta_\ell \). \( K_\ell \) is the coefficient matrix associated with the lags of \( x_t \).

The derivations of \( K_\ell \) and \( \Theta_\ell \), as well as the details of the solution, are described in Appendix A.

We obtain the stochastic volatility equation, as follows:

\[
h_t = h_0 + \sum_{\ell=1}^{m} \Upsilon_\ell h_{t-\ell} + \sum_{\ell=1}^{q} \Xi_\ell x_{t-\ell} + \eta_t.
\] (3.7)

where \( \Upsilon_\ell \) and \( \Xi_\ell \) are the coefficient matrices relating to the lags of log volatilities and endogenous variables in the system, respectively.

In principle, correlations between the residuals of the GVAR model could occur both within and across countries. Regarding the latter, as widely documented in Cesa-Bianchi [2013], Eichmeier and Ng [2015], Crespo Cuaresma et al. [2019], conditioning on global averages makes cross-country dependence of the residuals become null or of second-order importance. This enables structural identification in a multi-country context in which the shocks can be considered country-specific. Therefore, based on (3.2), (3.3), (3.6), and (3.7), we can study the global impacts of the US monetary policy shocks. The identification strategy is discussed in the next section.

### 3.1 Identification of US Monetary Volatility Shocks

The VARX for the US comprises three standard endogenous variables (interest rate, output growth and inflation) and three exogenous variables, namely, foreign output growth, foreign
inflation and foreign exchange rate changes. In order to identify the US monetary volatility shock, we need to make assumptions about $A$ and $Q$.

Regarding $A$, in our baseline, we adopt the sign restrictions on $\tilde{A} = A^{-1}$, as proposed by Mumtaz and Zanetti [2013] in the context of stochastic volatility. First, we consider the following structure for $\tilde{A}$:

$$
\tilde{A} = \begin{pmatrix}
1 & 0 & 0 \\
 a_{2,1}^{(-)} & 1 & 0 \\
 a_{3,1}^{(-)} & a_{3,2} & 1
\end{pmatrix}
$$

where the superscript $(\cdot)^{(-)}$ denotes the negative sign restrictions in the corresponding parameters. Given the ordering of the endogenous variables (interest rate, output growth and inflation), these restrictions imply that an increase in interest rates causes a contemporaneous fall in output growth and inflation, an assumption which is implied by standard DSGE models, for instance Smets and Wouters [2007]. As discussed in Mumtaz and Theodoridis [2015], this agnostic approach has a distinct computational advantage. If a full set of sign restrictions is imposed, then the structure of $\tilde{A}$ is nonrecursive, and the draw of elements of $A$ becomes rather cumbersome. Unlike a standard VAR model, $\tilde{A}$ cannot be rotated after estimation to impose sign restrictions. This is because the log volatility enters the VAR equations; hence, changes to $\tilde{A}$ have an impact on the stochastic volatility which in turn affects the VAR coefficients. For $Q$, the covariance matrix of the volatility equations, we use a Cholesky decomposition for $Q$ with volatilities ordered in the same manner as the endogenous variables in the VARX.

Our interest in this study is the US monetary policy shock, so we adopt only the sign restrictions to the VARX of the US. For other countries, we simply use the Cholesky decomposition to obtain $\tilde{A}$ to save computational time. In Appendix E, we conduct robustness checks with different identifications, i.e. Cholesky decomposition with different orderings, and obtain similar results.

### 3.2 The Global Model

The global model developed in this paper includes the US along with other 32 countries, accounting for 90% of world output: i) European countries: UK, Norway, Sweden, Switzerland and 8 countries in the euro area, namely Austria, Belgium, Finland, France, Germany, Italy,

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8We drop the country notation $i$ in $A_i$ and $Q_i$ to ease the illustration.
Netherlands, Spain; ii) other developed economies: Australia, Canada, Japan, and New Zealand; iii) emerging Asian countries: China, India, Indonesia, Malaysia, Philippines, Singapore, South Korea, and Thailand; iv) Latin American economies: Argentina, Brazil, Chile, Mexico, and Peru; and v) Middle East and African economies: Turkey, Saudi Arabia, and South Africa. In line with the GVAR literature, eight countries that originally joined the euro on 1 January 1999 are grouped together using the average Purchasing Power Parity GDP weights. The GVAR model hence contains 26 countries/regions modeled individually.

Based on the theoretical motivations, with the exception of the US model, we include four endogenous variables for each country/region: annual output growth, inflation, short-term interest rate and the real exchange rate growth. The exchange rate does not enter the US model since the US dollar is used as a numeraire. For the US, the euro area and the UK, we use the shadow rate constructed by Wu and Xia [2016], which remains a useful measure of monetary policy stance at the effective lower bound.

To construct the country-foreign specific variables, in the baseline model we use the fixed trade weights, which are the average trade flows computed over the years 1990-2016. We also consider alternative weighting schemes. In line with the GVAR literature, for instance Dees et al. [2007], excepting the US, all models include three country-specific foreign variables: foreign output growth, foreign inflation and foreign interest rate, as weakly exogenous. In the case of the US model, we include foreign output growth, foreign inflation and weighted average of bilateral real exchange rates, i.e. the real effective exchange rate, as weakly exogenous. Given the importance of the US financial variables in the global economy, the US-specific foreign interest rate is not included in the US model. In contrast, the US-specific foreign output growth and inflation variables are included in the US model in order to capture the possible second-round effects of external shocks on the US.

The models are estimated over the period 1979Q2-2016Q4, in which the period 1979Q2-1989Q2 is used as a training sample for the prior construction, following Cogley and Sargent [2005] and Mumtaz and Theodoridis [2015]. We estimate the VARX system for each country, which is a non-linear state space specification including equations (3.1)-(3.4), by using the Bayesian approach. The prior distribution and initialization are presented in the Appendix B. The simulation of posterior distributions is documented in the Appendix C and summarized as

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9The data are available from the GVAR quarterly database; see Mohaddes and Raissi [2018] for details.
follows:

- Step 1: We draw the elements of \( A_i \) in a manner similar to the one proposed by Cogley and Sargent [2005]. Given a draw of VARX coefficients \( F_i = (a_i, \Phi il, A il, \Psi il) \) and \( h_{it} \), we obtain that \( A_i \hat{u}_{it} = e_{it} \) where \( \hat{u}_{it} \) are the known residuals of the VARX model. This is a system of linear equations in which the form of heteroscedasticity is known. After a simple GLS transformation, we obtain the homoscedastic errors and then the conditional posterior for \( A_i \). In the identification scheme with sign restrictions, as in the baseline model, we draw the elements of \( A_i \) until the sign restrictions are satisfied.

- Step 2: Conditional on other parameters and stochastic volatility \( h_{it} \), the distribution of the VARX coefficients \( F_i \) is linear and Gaussian: \( N(F_{iT|T}, P_{iT|T}) \). The posterior mean and variance are obtained via the Kalman filter by following the Carter and Kohn [1994] algorithm. This is equivalent to a GLS transformation of the VAR model with heteroscedasticity. Note that our VARX model’s size is small (i.e. a maximum of four endogenous variables) and the GVAR approach imposes parametric restrictions through the linkage matrix, so the usage of the Kalman filter is fast.\(^{10}\)

- Step 3: We draw the elements of \( H_{it} \) conditional on the VARX coefficients and the parameters of the transition equation. The stochastic volatilities are simulated using a date-by-date independence Metropolis step, following Carlin et al. [1992], Jacquier et al. [1994], and Mumtaz and Theodoridis [2019].

- Step 4: Conditional on a draw for volatility \( h_{it} \), the transition equation is a sequence of linear equations; thus, the conditional posterior for the coefficients can be derived easily.

The MCMC algorithm is applied using 100,000 iterations, with the first 90,000 as burn-in. Appendix D shows selected criteria to illustrate convergence of our algorithm. After estimating the country VARX models, we stack them into the global model and then solve the global model as in (3.6) and (3.7).

\(^{10}\) In the case of a very large Bayesian VAR with stochastic volatility, Carriero et al. [2019] propose an efficient algorithm, based on the triangularization of the system, that produces the same results as in a system-wide algorithm.
4 Results

4.1 Uncertainty measures

Figure 1 presents the estimated volatility of US monetary policy shocks (US-MPU) (top-left panel) along with the NBER recession dates. The estimated US-MPU in our sample exhibits high levels around the periods relating to the recessions, including the early 1990s recession, the early 2000s recession and the Great Recession. Figure 1 also shows our estimated measure in comparison with the macroeconomic uncertainty measure developed by Jurado et al. [2015] (JLN in top-right panel), and two monetary policy uncertainty measures developed by Baker et al. [2016] (denoted BBD-MPU, bottom-left panel)\(^{11}\) and Husted et al. [2019] (denoted HRS-MPU, bottom-right panel), respectively. The JLN index is model-based, exploiting a data-rich environment to provide direct econometric estimates of time-varying macroeconomic uncertainty. Meanwhile, the BBD-MPU and HRS-MPU measures are based on newspaper coverage frequency of certain key words concerning monetary policy uncertainty.

Among these measures, our US-MPU estimate has the highest correlation with the JLN measure, with the correlation coefficient between the two measures being 0.6, statistically significant at 1 percent. First, such a high correlation confirms our similar approach, i.e. a model-based approach, to measuring uncertainty. Second, given that our measure concerns the volatility of (identified) monetary policy and the JLN measures general macroeconomic uncertainty, the high correlation indicates the important role of monetary policy volatility in overall macro-uncertainty during this sample. As documented in Jurado et al. [2015], between 2007 and 2009, uncertainty is highest for the monetary base, non-borrowed reserves and total reserves, contributing to the spike of the JLN measure during this period. It is interesting to note that our index also displays reasonable similarity with uncertainty measures based on newspaper coverage frequency, especially over the early 2000s recession. The BBD-MPU shares a more significant number of turning points with our measure than the latter.\(^{12}\) Nevertheless, these news-based measures exhibit quantitatively important uncertainty in a more frequent but less persistent manner, a feature that was also raised by Jurado et al. [2015]. This indicates that much of

\(^{11}\)The BBD-MPU is a sub-category of economic policy uncertainty.

\(^{12}\)According to Husted et al. [2019], the primary difference between the BBD-MPU and the HRS-MPU is caused by three main factors (listed in order of importance): i) specific set of newspapers, i.e. the BBD-MPU is based on the Access World News database of over 2,000 newspapers, while the HRS-MPU relies on the three major US newspapers, ii) keywords, and iii) scaling.
Figure 1: US Monetary Policy Uncertainty

Notes: Figure presents the estimated volatility of US monetary policy shocks (US-MPU) in top-left panel, i.e. the median (solid line) and 68% intervals (shaded area), in comparison with the macroeconomic uncertainty measure developed by Jurado et al. [2015] (JLN in top-right panel), and two monetary policy uncertainty measures developed by Baker et al. [2016] (denoted BBD-MPU, bottom-left panel) and Husted et al. [2019] (denoted HRS-MPU, bottom-right panel).

The variation in the proxies of monetary policy uncertainty are not driven by monetary policy uncertainty.

Figure 2 presents the volatility of shocks to the GDP growth and inflation equation in the US VARX model in the second and third columns, respectively. Note that we do not offer a direct economic interpretation of these shocks. Both volatilities peaked during the recent financial crisis.
and have high correlations with the JLN measure, 0.65 ($p < 0.01$) for the volatility of output-related shock and 0.73 ($p < 0.01$) for the volatility of inflation-related shock. As mentioned above, the JLN index captures the general macroeconomic uncertainty. We therefore construct a similar measure with the JLN by taking the first principal component of three (standardized) measures of volatilities from our model. This model-implied-macroeconomic uncertainty measure is presented in Figure 3 together with the JLN macroeconomic uncertainty. Both series share a substantially similar pattern and have a high correlation of 0.81, statistically significant at 1 percent. This evidence indicates that our VARX model, in which a small set of endogenous variables and cross-sectional averages proxy for the common unobserved factors, provides a credible description of macroeconomic uncertainty, which Jurado et al. [2015] estimates with a larger set of variables.¹³

Table 1 summarizes results from the regressions of the US volatilities in the interest rate, output growth, and inflation. We find that interest rate volatility can be mainly explained by its own shocks. Output volatility seems to be driven by past shocks in volatilities of all key variables as well as output itself. This state-dependence, along with the no-reaction of the interest rate volatility to changes in the macroeconomy, is what was recently documented in Ludvigson et al.¹⁴

¹³In Appendix F, we report the estimates of uncertainty for other economies and highlight that our measures capture reasonably well key periods of high uncertainty in these economies.
Figure 3: US Macroeconomic Uncertainty

Notes: Figure presents the first principal component of three (standardized) measures of volatilities from our model (i.e. monetary policy volatility, output volatility and inflation volatility) in comparison with the JLN macroeconomic uncertainty measure.

[forthcoming]. Once in recession, both output and inflation volatilities seem to be significantly affected. The feedback mechanism between the first and second moments during slumps provides not only a more nuanced understanding of real fluctuations but also necessitates joint modeling.

However, we argue that a missing element in the story is spillover and spillback effects, which cannot be ignored even for an economy as large as the US. The importance of the spillback impact on the US real economy as well as changes in the Fed’s costs of attaining a given price-level path have recently been put forward by Obstfeld [2020], yet we lack quantitative evidence of global factors on the US economy, taking global interdependencies into account. We thus turn to exploring how external uncertainty, stemming from the US, affects the global economy and gets imported back to the US.
Table 1: Estimates of log volatility equations: US

<table>
<thead>
<tr>
<th></th>
<th>$h_{r,t}$</th>
<th>$h_{y,t}$</th>
<th>$h_{p,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{r,t-1}$</td>
<td>0.88</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>[0.81,0.94]</td>
<td>[0.01,0.12]</td>
<td>[-0.02,0.08]</td>
</tr>
<tr>
<td>$h_{y,t-1}$</td>
<td>0.09</td>
<td>0.49</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>[0.05, 0.25]</td>
<td>[0.16,0.75]</td>
<td>[-0.10,0.15]</td>
</tr>
<tr>
<td>$h_{p,t-1}$</td>
<td>0.05</td>
<td>0.06</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>[-0.01,0.12]</td>
<td>[0.006,0.13]</td>
<td>[0.79,0.93]</td>
</tr>
<tr>
<td>$r_{t-1}$</td>
<td>0.03</td>
<td>-0.03</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>[-0.004,0.07]</td>
<td>[-0.08,0.01]</td>
<td>[-0.03,0.03]</td>
</tr>
<tr>
<td>$y_{t-1}$</td>
<td>0.02</td>
<td>-0.08</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>[-0.04, 0.08]</td>
<td>[-0.17, -0.02]</td>
<td>[-0.10, -0.002]</td>
</tr>
<tr>
<td>$p_{t-1}$</td>
<td>-0.05</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>[-0.11, 0.01]</td>
<td>[-0.11,0.03]</td>
<td>[-0.07,0.03]</td>
</tr>
</tbody>
</table>

Notes: Table shows the median estimates of coefficients in the log volatilities equation for the US, together with 68 percent credible intervals in brackets. $r, y, p$ denote interest rate, output growth and inflation, respectively.

4.2 Impacts of US monetary policy uncertainty shocks

We begin by exploring how the US macroeconomy responds to the identified shock to the interest rate volatility by doubling it (an increase of 100%). As shown in Figure 4, this leads to a rise in output volatility by about 5% on impact, increasing up to 15% within the first two years. It takes considerably longer for the maximum inflation volatility to transpire. As expected and, in line with the literature, the US output growth is reduced up to 0.4% in the first year and a half after a shock. As with volatility, it takes more time for deflationary pressures to fully manifest. Finally, as expected, the US interest rate decreases to counteract volatile and recessionary periods. Though the results for the US model are well aligned with the current evidence, we also document international effects that have received scant attention.

Figures 5 documents global impacts of the US monetary policy uncertainty to some selected countries. First, it is clear that US policy uncertainty makes the rest of the world move in a highly synchronous way. Though magnitudes of US impacts vary by country and variable, the directions remain the same as in the US. Confirming a close link of another large economic and monetary union, the euro area (EA), we document up to a 0.1% drop in the EA output growth within the first year and a half. A change in inflation is largely comparable between the US and other advanced economies (EA, the UK and Canada). In fact, the EA interest rate moves similarly to the interest rate in the US. These findings illustrate the importance of US policy for
Figure 4: Impacts of US Monetary Policy Uncertainty

Notes: Figure presents the response of US macroeconomy to an unexpected increase by 100 percent in the US interest rate volatility. Each entry shows the median (solid line), and the 68% intervals (shaded area).

The effect on the British economy is more limited when it comes to output and the interest rate. There is less gravity of the US shock to Japan but a very substantial impact on Canadian growth, which is expected, given geographical proximity and the size of the US economy. Interestingly, though exchange rates do not react significantly across countries, that is not true for the Chinese economy. Its reaction is modest when it comes to other macro variables; however, there is evidence that the renminbi has depreciated against the US dollar. This data-driven finding lends some support to claims that Chinese decision-makers are devaluing China’s currency in order to make its exports more attractive and increase competitiveness (see, among many others, Steinberg, 2015, Mattoo et al., 2017).

Figure 6 reports impulse responses for other groups of countries. We find evidence of important spillovers to Latin American output growth and interest rates. This finding echoes results though different time periods play a role, the original contribution of Dees et al. [2007] found the US interest rate shock on the EA output and inflation to be very small and statistically insignificant at all horizons. Our theoretically and intuitively more consistent results underscore, if only indirectly, the importance of volatility shocks in the empirical model for large and heavily interlinked economies.

An increase in the real exchange rate means depreciation.
from Fernández-Villaverde et al. [2011], who document that volatility shocks of the real interest rate have a substantial effect on output and act as an important driver of business cycle fluctuations in Latin America. The rest of Europe shows patterns similar to that of the EA, indicating strong integration inside the old continent. Other European economies get substantially - even if only indirectly - exposed to the US economy. As for Asia and Australia and New Zealand, we find smaller impacts except for the interest rate, which moves downwards, at least partly capturing the importance of US interest rate.

As the literature has focused solely on the US economy, one may wonder about the added value of having it incorporated into the global economy, while allowing for rich interactions across countries, variables and volatilities. In Figure 7, we demonstrate how our conclusions would have changed if we left aside spillovers and spillbacks from the rest of the world to the US economy. Absent global linkages and reactions, we would have documented a substantially smaller effect on output growth, barely any effect on inflation and a less pronounced reaction in interest rates.
We also observe that international linkages affect output and inflation volatilities but have little impact on the interest rate volatility. This finding is well aligned with Table 1 and Ludvigson et al. [forthcoming], demonstrating that macro volatility reacts endogenously to macroeconomic shocks while financial volatility is largely explained by its own shocks. Therefore, the ultimate objective of the analysis matters; clearly, if one was primarily interested in the mean effects of core macroeconomic variables or macroeconomic uncertainty, then one should have solved a global model even for the US economy.\footnote{Dees et al. [2014] treat the US as a closed economy, with no spillback effects, whereas the original version of the global VAR does not separate the impact of US variables from the cross-sectional averages of foreign economies (Dees et al., 2007). The latter approach cannot be refuted empirically employing, e.g., weak exogeneity assumption testing. For more on the changing importance of the US economy see Dees and Saint-Guilhem [2011].}
Figure 7: Impacts of US Monetary Policy Uncertainty: No spillovers and spillbacks

Notes: Figure presents the response of US macroeconomy to an unexpected increase by 100 percent in the US interest rate volatility if spillovers to and spillbacks from the rest of the world to the US economy are close: in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals in the case of no spillovers and spillbacks; the black dashed line is the corresponding response in the baseline.

5 Discussions

We explore the importance of channels through which monetary policy shocks get transmitted from and imported back to the US economy. First, we analyze goods (final and intermediate) trade, paying special attention to the structural break that occurred between the third quarter of 2008 and the second quarter of 2009. During that time the world trade plummeted by 10%, synchronically across the globe, while the world GDP shrank by 1%. This phenomenon, referred to as the “great trade collapse”, has stimulated a literature of its own.\footnote{Though the conclusive explanation for the “great trade collapse” has not been reached, as a number of factors played a role, many reasons have been suggested: Alessandria et al. \cite{2010} concentrate on the role of inventory adjustments, Altomonte et al. \cite{2012} emphasize magnification of the negative demand shock due to global value chains, Levchenko et al. \cite{2010} confirm the importance of intermediate inputs, with more nuanced sectoral effects where larger drops in domestic output lead to larger drops in trade. Most recently, Novy and Taylor \cite{2020}, by connecting different channels, argue that inventories, coupled with the uncertainty channel, produce a magnification effect, which is capable of replicating a sharp contraction of international trade flows followed by a swift recovery.} As one of the effects of that period is increased uncertainty, we thus explore how our results change if we use post-crisis data.
trade weights, which may reflect the post-trade-collapse rebalancing across economies.

5.1 The trade integration channel

Though trade volumes dropped substantially during the global financial crisis, that fact in itself does not say much about relative trade shares in the global trade network. We therefore start by documenting the shares of trade between the US and its main trading partners as well as changes in shares (see Figure 8, panels (a) and (b) respectively) between two sub-periods before the GFC (black dashed and magenta-circle lines), the most recent period after the GFC (blue line with squares), and a longer period before and after the GFC (red line). We concentrate on three stylized facts. First, trade gravity forces are clearly at play since Canada and Mexico, geographically the closest large trade partners, are also the most linked to the US economy through trade linkages. Second, the role of the US in global trade has been diminishing over time, with the black dashed line (referring to 1990-1998) being above the blue squared line (referring to 2009-2016), essentially across all trade partners. Third, a structural change seems to have been induced by the GFC (though other compounding effects, such as trade wars and a sustained growth of emerging countries, in particular China, which creates third-country effects (see, for instance, Rebucci et al., 2012), are also at play).

Adjustments were made before the GFC, but not on the scale that they have been made since. The only trade partner that has not suffered a steep decline in trade share is Switzerland, whereas Brazil, Canada, Japan, Malaysia, Mexico and Philippines are among the most decoupled from the US trade partners. The magnitudes of such changes in the historical context, as depicted in Figure 8’s panel (b), raise questions about changing global spillovers and, importantly, spillbacks to the US economy before and after the GFC, when emerging markets have been playing a more prominent role than ever before.

We start with exploring the effects of spillovers over the whole period and after the GFC on the US economy in Figure 9. Though the results are robust to both weighting schemes, there is evidence of the interest rate reaction becoming slightly more aggressive to the uncertainty shock using more recent trade linkages for the US economy, although global effects are slightly attenuated. It is therefore of interest to see how global spillovers change. The signs of global re-shuffling are clear from Figures 10 and 11. They both provide hints about somewhat less pronounced effects from the US monetary policy uncertainty (which is well aligned with a some-
what decreased US role in terms of international trade), though still preserving economic and statistical significance in many cases, especially among advanced economies. It also looks like inflation and interest rate channels have been quite mitigated (i.e. the reactions are somewhat subdued). The same conclusion holds for emerging economies, and Australia and New Zealand (see to Figure 10). Interestingly, neighboring Canada and fast-growing China react in a virtually identical way, whereas Japanese responses, at least judging from point estimates, are quite different. Observing from the angle of Figure 8, Japan has experienced the largest drop in the trade share with the USA (observe the difference between black-dashed line and blue-squared line in panel (a)). Though the US has diminished in importance to Canada, it still remains overwhelmingly dominant. Last, the US share with China has barely adjusted in the period
after GFC, confirming this stability in the effects of US monetary policy uncertainty shocks.

Figure 9: Impacts of US Monetary Policy Uncertainty: Post-crisis Trade Weight

Notes: Figure presents the response of US macroeconomy to an unexpected increase by 100 percent in the US interest rate volatility using the post-crisis trade weight: in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals; the black dashed line is the corresponding response in the baseline.

Though spillovers got somewhat smaller, also echoing results in Dees and Saint-Guilhem [2011] and demonstrating weaker US impacts during recent periods (but a stronger persistence of shocks), they may actually hide other linkages that play important roles. Indeed, even though real trade may appear to confirm the idea of Asian economies becoming important players worldwide, the role of the US in financial markets remains substantial, as we document when using financial weights.

5.2 The financial integration channel

Financial globalization has accelerated at an unprecedented pace, integrating financial markets of all countries, but particularly affecting emerging markets. We model financial linkages by taking the average of the main facets of financial integration, namely: outward portfolio investment, inward portfolio investment, outward foreign direct investment, inward foreign direct investment, outward claims of domestically headquartered banks and inward claims of foreign-headquartered
Figure 10: Global Impacts of US Monetary Policy Uncertainty: Post-crisis Trade Weight

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility using the post-crisis trade weight: in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals; the black dashed line is the corresponding response in the baseline.

It is instructive to compare trade weights with financial linkages, as graphically summarized in Figure 12. The red line, corresponding to financial linkages, dominates a dashed blue line, summarizing trade weights, for all except two countries, Canada and Mexico. For some, like Chile and Argentina from the emerging countries group, the difference is substantial. Even among advanced economies, like the Euro area, Japan or the UK, the US plays a key role when it comes to financial flows, considerably exceeding linkages through international trade. Given that monetary policy uncertainties are heavily linked to financial markets, it is of interest to compare the results of unexpected changes in the US interest rate volatility on the rest of the world spilling through financial conduits.

Figure 13 summarizes uncertainty shocks on the US variables when using financial weights. Three findings stand out: first, the real economy, measured by output growth, is affected in a
Figure 11: Global Impacts of US Monetary Policy Uncertainty: Post-crisis Trade Weight (cont)

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility using the post-crisis trade weight: in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals; the black dashed line is the corresponding response in the baseline.

more limited way than using trade weights, with a quicker rebounding effect; second, financial variables, as captured by inflation and interest rate, deliver considerably different responses (US experiences more deflationary pressures, whereas the interest rate declines more moderately over the medium run); finally, volatilities are more robust to different weights, though there is some evidence of uncertainty changes transpiring more quickly under financial linkages. It is interesting to note that the US interest rate volatility shock is more deflationary but less growth-reducing, thereby altering a trade-off between economic development and price stability under financial (rather than trade) spillovers (this is in line with the argument of important spillback effects which alter the policy cost of attaining a given price path, emphasized by Obstfeld, 2020). Though, traditionally, an emphasis has been placed on emerging markets and their strengthened effects on advanced economies, especially after the GFC (BIS, 2016, Agenor and da Silva, 2018, Carney, 2019), we can also explore how spillovers to both emerging and advanced economies...
Figure 12: Financial versus Trade Linkages with US

Notes: Figure shows the financial and trade linkage with US for each country. The financial weight is the average of main facets of financial integration, namely: outward portfolio investment, inward portfolio investment, outward foreign direct investment, inward foreign direct investment, outward claims of domestically headquartered banks and inward claims of foreign-headquartered banks. The data are from Eickmeier and Ng [2015].

Figure 13: Impacts of US Monetary Policy Uncertainty: Financial weight

Notes: Figure presents the response of the US macroeconomy to an unexpected increase by 100 percent in the US interest rate volatility using the financial weight: in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals; the black dashed line is the corresponding response in the baseline. The effects vary under different transmission channels and thus result in different spillback effects on the US economy.
Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility using the financial weight: in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals; the black dashed line is the corresponding response in the baseline.

We will now analyze how financial linkages impact spillover magnitudes of the US monetary policy shock across the globe in comparison to the more standard, international trade, channel (Figures 14 and 15). Though the conclusions are largely intact, there are a few interesting insights. First, the shock is more deflationary almost across the board, if transmitted by financial flows (except for China). The transmission of monetary policy uncertainty into prices is stronger in the global financial, rather than the goods trade, network, where the US plays a more prominent role (indirectly corroborating the vast literature on global banks, international capital markets, and a tight link between the global financial cycle and the US monetary policy, e.g. Shin, 2012, Cetorelli and Goldberg, 2012, Miranda-Agrippino and Rey, 2020). Second, the policy uncertainty shock induces larger downward shifts in interest rates across all groups except for Japan (marginally and over the longer run) and Canada (compared to the trade weights baseline). The latter is much more integrated with the US with respect to real rather than financial...
Third, geographically more distant economies seem to experience larger spillovers from financial flows, which makes good intuitive sense and confirms evidence in Figure 12 as well as the trade gravity literature of large costs across space despite overwhelming globalization during the past few decades (Anderson and van Wincoop, 2004).

Another crucial aspect of how globalization and monetary policy transpire into the real economy is the exchange rate channel. Our model confirms that the renminbi tends to get devalued, despite the nature of linkages, even if most discussions about foreign currency interventions revolve around real trade imbalances and tariff wars. Moreover, we find that Chinese output growth is substantially more vulnerable to monetary policy shock if it was transmitted through financial flows. Interestingly, this relates to debates about the US dollar status as the reserve currency and huge reserve accumulation by China, making the US and Chinese economy heavily intertwined through capital flows. Figure 15 shows that inflation and interest rates in

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility using the financial weight: in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals; the black dashed line is the corresponding response in the baseline.
the emerging markets react more strongly to the shock under financial interdependencies. We thus confirm the prominent role of the US as a financial center in transmitting shocks globally, particularly to the emerging markets.

6 Conclusions

Motivated by unprecedented uncertainty within a global economy, we set out to evaluate the role of monetary policy uncertainty. Since uncertainty plays a prominent role in agents’ decision-making and the resulting time-variation in the second moments, we expand a standard macroeconomic model to allow for stochastic volatility, cross-country interdependencies and policy shocks. We contribute to the literature by shedding new light on the global impact of uncertainty shocks (second-moment shocks), covering 32 countries, which account for 90% of the world output. We find that macroeconomic uncertainties are state-dependent and require a dynamic interaction between endogenous variables and time-varying volatility. Our results indicate significant recessionary and deflationary effects as well as jumps in output and inflation volatilities in response to the unexpected change in the US interest rate volatility. We also document strong spillovers, which cause the rest of the world to move in a highly synchronous manner, providing a new global channel and rationalizing increased co-movements during recessions.

We also expand the coverage of international connections (trade, post-financial crisis trade, financial linkages) and find helpful nuances (though the existence of strong global impacts remains intact). For instance, we document slightly less pronounced spillovers after the “great trade collapse”, hinting at a possible rebalancing in the global trade network. The US economy nevertheless remains very impactful once financial flows are used, also providing new evidence about the dependence of Chinese economy (its output gets depressed only when financial interlinkages are used) and currency adjustments in response to the shock (independently of weights or shock identification schemes). The advanced economies, particularly the euro area, remain heavily impacted by changes in the US. Given higher-order effects, it seems vital to allow for cross-country interlinkages when evaluating macroeconomic fluctuations. In fact, we establish that the global dimension is critical even for the US economy via the non-trivial spillback effects (thereby not only quantitatively corroborating the idea of substantial spillbacks on the US real economy, discussed by Obstfeld [2020], but also exploring the role of trade and financial linkages in a large set of both emerging and advanced economies).
Our model can be used for many different applications on global uncertainty, well beyond those we covered in the paper. The framework would be particularly well suited for the volatility shocks in the macro-uncertainty, which, as documented in our paper and recently evidenced in Ludvigson et al. [forthcoming], is not only a cause but also a consequence of macroeconomic shocks. Rich dynamic interactions between the endogenous variables and the time-varying volatility as well as spatial linkages will prove particularly valuable for explorations of uncertainty about trade policy and fiscal policy, political uncertainty, COVID-19-induced uncertainty or other macro-level risks. We are exploring not only new applications but also new ways to deal with non-linearities, such as using time-varying parameters\(^{18}\) or endogenizing weight matrix. All these directions would further enhance our understanding of how uncertainty, global interlinkages, and the real economy interact - crucial knowledge at a time when sources of uncertainty and forces of deglobalization are on the upswing.

\(^{18}\)We experiment with modeling the VARX models with time-varying parameters, similarly to Primiceri [2005]. Granger [2008] shows, via the White’s Theorem, that a nonlinear model can be approximated by a time-varying parameter linear model, thus increasing the richness of the model even further.
Appendix

A  Global Solution

After estimating the individual country models, we collect all the \( k = \sum_{i=0}^{N} k_i \) endogenous variables of the global economy \( x_t = [x_{1t}', ..., x_{Nt}]' \) and solve the solve the global model. To ease notation and arrive at the global model, we shall ignore the deterministic components, so express the VARX model in (3.1) as

\[
x_{it} = \sum_{\ell=1}^{p_i} \Phi_i \ell x_{i,t-\ell} + \sum_{\ell=0}^{q_i} \Lambda_i \ell x_{i,t-\ell}^* + \sum_{\ell=0}^{s_i} \Psi_i \ell h_{i,t-\ell} + u_{it}, \quad (A.1)
\]

and denote by \( z_{it} = (x_{it}', x_{it}^*)' \). Equation (A.1) can be rewritten as

\[
B_{i0} z_{it} = \sum_{\ell=1}^{p_i} B_i \ell z_{i,t-\ell} + \sum_{\ell=0}^{s_i} \Psi_i \ell h_{i,t-\ell} + u_{it}, \quad (A.2)
\]

such that \( B_{i0} = (I_{k_i}, -\Lambda_{i0}) \) and \( B_{i\ell} = (\Phi_i \ell, \Lambda_i \ell) \), \( p_z = \max_i (p_i, q_i) \). Given invertibility of \( B_{i0} \), we arrive at

\[
z_{it} = \sum_{\ell=1}^{p_z} B_{i0}^{-1} B_{i\ell} z_{i,t-\ell} + \sum_{\ell=0}^{s_i} B_{i0}^{-1} \Psi_i \ell h_{i,t-\ell} + B_{i0}^{-1} u_{it}. \quad (A.3)
\]

Following the idea in (3.5), combine domestic and foreign variables into \( z_{it} \) and express as

\[
z_{it} = (x_{it}', x_{it}^*)' = W_i x_t, \quad (A.4)
\]

such that \( W_i = (E_i', \bar{W}_i') \), where \( E_i \) is a selection matrix \( (x_{it} = E_i x_t) \). Plugging equation (A.4) into equation (A.3) yields

\[
B_{i0} W_i x_t = \sum_{\ell=1}^{p_z} B_{i0} W_i x_{t-\ell} + \sum_{\ell=0}^{s_i} \Psi_i \ell h_{i,t-\ell} + u_{it}.
\]

Let \( G_{i0} = B_{i0} W_i \) and \( G_{i\ell} = B_{i\ell} W_i \),

\[
G_{i0} x_t = \sum_{\ell=1}^{p_z} G_{i0} x_{t-\ell} + \sum_{\ell=0}^{s_i} \Psi_i \ell h_{i,t-\ell} + u_{it}.
\]
Stacking for all countries, \( i = 1, 2, \ldots, N \), delivers

\[
G_0 \mathbf{x}_t = \sum_{\ell=1}^{p_x} G_\ell \mathbf{x}_{t-\ell} + \sum_{\ell=0}^{s} \Psi_\ell h_{t-\ell} + \mathbf{u}_t. \tag{A.5}
\]

Inverting \( G_0 \) delivers

\[
\mathbf{x}_t = \sum_{\ell=1}^{p_x} G_0^{-1} G_\ell \mathbf{x}_{t-\ell} + \sum_{\ell=0}^{s} G_0^{-1} \Psi_\ell h_{t-\ell} + G_0^{-1} \mathbf{u}_t.
\]

Or,

\[
\mathbf{x}_t = \mathbf{x}_0 + \sum_{\ell=1}^{p_x} K_\ell \mathbf{x}_{t-\ell} + \sum_{\ell=0}^{s} \Theta_\ell h_{t-\ell} + \mathbf{v}_t.
\]

where \( K_\ell = G_0^{-1} G_\ell, \Theta_\ell = G_0^{-1} \Psi_\ell, \) and \( \mathbf{v}_t = G_0^{-1} \mathbf{u}_t. \)

Similar to the logic above, the stochastic volatility equation can be expressed, after stacking, as

\[
h_t = \sum_{\ell=1}^{m} \Upsilon_\ell h_{t-\ell} + \sum_{\ell=1}^{q} \Xi_\ell \mathbf{x}_{t-\ell} + \eta_t,
\]

where \( \mathbf{h}_t = [h_{1t}', \ldots, h_{Nt}']' \) is the stack of volatilities of country models into the global economy.

## B Prior distributions and starting values

### B.1 VARX coefficients

The initial conditions for the VARX coefficients \( \mathbf{F}_0 \) and the covariance around these initial conditions \( \mathbf{P}_0 \) (to be used in the Kalman filter described below) are obtained via dummy observations, similar to the construction of prior in Bańbura et al. [2010]. The dummy observations \( \mathbf{Y}_D \) and \( \mathbf{X}_D \) are defined as follows:

\[
\mathbf{Y}_D = \begin{pmatrix} \text{diag}(\chi_1 \sigma_1, \ldots, \chi_N \sigma_N) \\ 0_{(N \times (L-1)) \times N} \\ 0_{EX \times N} \end{pmatrix}
\]

\(^{19}\)To ease the presentation, in this section, we ignore the notation \( i \) that indicates the country-\( i \) model.
\[ X_D = \left( \begin{array}{ccc} J_L \otimes \text{diag}(\sigma_1, \ldots, \sigma_N) & 0_{NL \times EX_1} & 0_{NL \times 1} \\ 0_{EX_1 \times NL} & I_{EX_1} \times 1/c_1 & 0_{EX_1 \times 1} \\ 0_{1 \times NL} & 0_{1 \times EX_1} & 1/c_2 \end{array} \right) \]

where \( \chi_j \) represents the initial value for the coefficient on the first lag of \( j \)-th endogenous variable, which is obtained from an AR(1) regression for the corresponding variable, \( \sigma_j \) is the standard deviation of error term from the above AR(1) regression. \( L \) is the lag length and \( J_L = \text{diag}(1, 2, \ldots, L) \). \( EX \) denotes the number of exogenous and pre-determined regressions in each equation, including the contemporaneous and lagged coefficients of log volatilities and of foreign variables \( (EX_1) \) and the intercept, so \( EX = EX_1 + 1 \). \( \tau \) controls the tightness of the prior on the VAR coefficients. \( c_1 \) measures the tightness of the prior of log volatilities and of foreign variables regressors and \( c_2 \) controls the tightness of the prior on intercepts. We set \( \tau = 0.1 \), \( c_1 = \sqrt{0.1} \), \( c_2 = 1000 \), which are common in the literature, for instance Mumtaz and Theodoridis [2019]. Appendix G performs a robustness check with a more loosening value of \( \tau \). The VAR error covariance matrix is time-varying in our setting, so we do not directly implement a prior belief on the covariance matrix.

We therefore obtain \( F_0 \) and \( P_0 \), as follows:

\[ F_0 = (X_D'X_D)^{-1}(X_D'Y_D) \quad \text{and} \quad P_0 = S \otimes (X_D'X_D)^{-1} \]

where \( S \) is a diagonal matrix with components are the variance of endogenous variables obtained from the AR(1) process above.

### B.2 Elements of \( A \)

We set the prior for the off-diagonal elements of \( A \), defined as \( A_0 \sim N(\hat{\alpha}, V(\hat{\alpha})) \) by using a training sample (of observations 1979Q2-1989Q2), following Cogley and Sargent [2005] and Mumtaz and Theodoridis [2015]. Let \( \hat{\Sigma}_{pre} \) denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data. We then set \( \hat{a} \) to the off-diagonal elements of the inverse of Cholesky decomposition of \( \hat{\Sigma}_{pre} \), with each row scaled by the corresponding element on the diagonal. \( V(\hat{\alpha}) \) is assumed to be diagonal with the elements set equal to 100. A tighter prior is set for elements where sign restrictions are imposed.

\[ ^{20}\text{In our application, the country VARX model’s specification is not large, so it does not require substantial shrinkage on coefficients. In addition, with the GVAR approach, the parametric restrictions are imposed through the linkage matrix. For an application with a large (B)VAR, one may consider more flexible shrinkage priors proposed by, e.g., Huber and Feldkircher [2019], George et al. [2008].} \]
B.3 Elements of $H_t$

The prior for $h_t$ at $t = 0$ is given by $\ln h_0 \sim N(\ln \mu_0, 10 \times I)$ where $\mu_0$ are the diagonal elements of $\hat{\Sigma}_{\text{pre}}$ from the training sample.

B.4 Parameters of the transition equation

In order to set the prior for the parameters of the transition equation, following Mumtaz and Zanetti [2013], in the first step we estimate a simpler version of the benchmark model where the stochastic volatility does not enter the mean equations and log volatility follows an AR(1) process as in Primiceri [2005]. This also provides the initial estimates of the log volatility series. Then we set the prior on the coefficients and error covariance of the transition equation via dummy variables as in Ba˘nbura et al. [2010], shrinking each equation towards an AR process, with priors being estimates from the simplified model. The prior on $Q$ is inverse Wishart with degrees of freedom $v_0 = N + 1$ and the prior scale parameter scale parameter $G_0 = \text{diag}(s_1 v_0, \ldots, s_N v_0)$ where $s_i$ is the variance of errors in the transition equation from the simplified model. Similar to Mumtaz and Theodoridis [2019], the prior tightness parameter measuring the strength of the prior on the coefficients on the lagged volatilities is set equal to 0.05. The parameter that controls the prior tightness on the lagged pre-determined variables is set to $\sqrt{0.05}$.

C Simulating the posterior distributions

The posterior distributions are simulated based on the Gibbs sampling.

C.1 Element of $A$

We draw the elements of $A$ in a similar manner with the approach proposed Cogley and Sargent [2005]. Given a draw of VARX coefficients $F = (a, \Phi, \Lambda, \Psi)$ and $h_t$, we obtain that $A\hat{u}_t = e_t$ where $\hat{u}_t$ are the known residuals of the VARX model:

$$\hat{u}_t = x_t - a_o - \sum_{l=1}^{p} \Phi_j x_{t-l} - \sum_{l=0}^{q} \Lambda_j x_{t-l}^* - \sum_{l=0}^{s} \Psi_j h_{t-l}$$
and \( \text{var}(e_t) = H_t \). The system \( A \hat{u}_t = e_t \) is a system of linear equations with a known form of heteroscedasticity. The \( j - th \) equation of this system is given as:

\[
\hat{u}_{jt} = -\alpha \hat{u}_{-jt} + e_{jt}
\] (C.1)

where \( \hat{u}_{-jt} \) denotes the residuals 1 to \( j - 1 \). To obtain the homoscedastic errors, we use a simple GLS transformation by dividing both sides of (C.1) by \( \sqrt{\text{exp}(h_{jt})} \) resulting in:

\[
\hat{u}_{jt}^* = -\alpha \hat{u}_{-jt}^* + e_{jt}^*
\]

where \( \hat{u}_{jt}^* = \hat{u}_{jt} / \sqrt{\text{exp}(h_{jt})} \), \( \hat{u}_{-jt}^* = \hat{u}_{-jt} / \sqrt{\text{exp}(h_{jt})} \) and \( e_{jt}^* = e_{jt} / \sqrt{\text{exp}(h_{jt})} \) and \( \text{var}(e_{jt}^*) = 1 \).

The conditional posterior for \( \alpha \) is normal with mean \( M^* \) and variance \( V^* \) as follows:

\[
V^* = (V(\hat{\alpha})^{-1} + \hat{u}_{-jt}^* \hat{u}_{-jt}^* - 1)^{-1}
\]

\[
M^* = V^* \times (V(\hat{\alpha})^{-1} \hat{\alpha} + \hat{u}_{-jt}^* \hat{u}_{jt}^*)
\]

In the identification scheme with sign restrictions, as in the baseline model, we draw the elements of \( A \) till the sign restrictions are satisfied.

### C.2 VARX coefficients

Conditional on other parameters and stochastic volatility \( h_t \), the distribution of the VARX coefficients \( F \) is linear and Gaussian: \( N(F_T|T, P_T|T) \). The posterior mean and variance are obtained via the Kalman filter by following the Carter and Kohn [1994] algorithm. This is equivalent to a GLS transformation of the VAR model with heteroscedasticity. Note that in our analysis on the global impacts of US monetary policy uncertainty shocks, the country VARX model is of a small size (i.e. a maximum of four endogenous variables) and the GVAR approach imposes parametric restrictions through the linkage matrix, so the usage of Kalman filter is fast.

In order to use the Kalman filter, we write the VAR in state space form as

\[
x_t = Z_t F_t + \Omega_t^{1/2} e_t
\]

\[
F_t = F_{t-1}
\]
The Kalman filter is initialized at the prior values $F_0$ and $P_0$, and the recursion for $t = 1, 2, ..., T$ is described as follows

$$
F_{t|t-1} = F_{t-1|t-1}
$$

$$
P_{t|t-1} = P_{t-1|t-1}
$$

$$
\kappa_{t|t-1} = x_t - Z_t F_{t|t-1}
$$

$$
f_{t|t-1} = Z_t P_{t|t-1} Z_t^\prime + \Omega_t
$$

$$
K_t = P_{t|t-1} Z_t^\prime f_{t|t-1}^{-1}
$$

$$
F_{t|t} = F_{t|t-1} + K_t \kappa_{t|t-1}
$$

$$
P_{t|t} = P_{t|t-1} - K_t Z_t P_{t|t-1},
$$

where $\Omega_t = A^{-1} H_t A^{-1}$. From this procedure, we obtain $F_{T|T}$ and $P_{T|T}$. We then draw the coefficients for VARX from $N(F_{T|T}, P_{T|T})$ and keep those that satisfy the stability condition.

C.3 Volatility $H_t$

In this step, we draw the elements of $H_t$ conditional on the VARX coefficients and the parameters of the transition equation. The model is first written in a multivariate non-linear state-space representation. Given the presence of pre-determined variables in the transition equation, the intercept term is time-varying. Then, following Carlin et al. [1992], the conditional distribution of the state variables in a general state-space model can be derived as the product of the three terms: $h_t \mid x_t, \Psi \propto f(h_t \mid h_{t-1}) \times f(h_{t+1} \mid h_t) \times f(x_t \mid h_t, \Psi)$, where $\Psi$ is the set of all other parameters.

An analytical expression for complete conditional $h_t \mid x_t, \Psi$ is not available because of the non-linearity of the observation equation, we follow Mumtaz and Theodoridis [2019] and Jacquier et al. [1994] to draw $h_t \mid x_t, \Psi$ using a date-by-date independence metropolis step in two steps: i) draw a candidate for $h_t^{\text{new}}$ from the density $f(h_t \mid h_{t-1}, h_{t+1}, \Psi)$ and ii) update $h_t^{\text{old}} = h_t^{\text{new}}$ with acceptance probability $\frac{f(e_t \mid h_t^{\text{new}}, \Psi)}{f(e_t \mid h_t^{\text{old}}, \Psi)}$. Repeat the two steps for the entire time series delivers a draw of the stochastic volatilities. Note that the initial estimates for $h_t^{\text{old}}$ is the volatility estimated from a simpler model as mentioned above.
C.4 Parameters of the transition equation

The transition equation can be re-written as: \( H = Yb + \eta \), where \( H \) is a \( T \times N \) matrix of values of \( N \) variables \( h_{jt} \) over \( T \) periods and \( Y \) is a \( T \times K \) matrix with \( K \) regressors. Conditional on a draw for \( h_t \), the conditional posterior for the coefficients can be derived easily. Specifically, the conditional posterior of the coefficients is normal:

\[
G(b|Q,h_t) \sim N(b^*, Q \otimes (Y^*Y^*)^{-1})
\]

where \( b^* = vec((Y^*Y^*)^{-1}(Y^*H^*)) \). \( H^* = [H' \ H_D']' \), and \( Y^* = [Y' \ Y_D']' \) are the data appended with dummy observations.

The conditional posterior for covariance \( Q \) is inverse Wishart

\[
G(Q|h_t) \sim IW(S^*, T^*)
\]

where the degrees of freedom \( T^* = T + v_0 \) and the scale parameter \( S^* = ((H^* - Y^*b^*)'Y^*b^*+G_0)^{-1} \).

The MCMC algorithm is applied using 100,000 iterations with the first 90,000 as burn-in. We estimate the VARX model for each country/region and then solve the global model as discussed in A.

D Assessing Convergence

This section diagnoses the convergence of the Gibbs iterations by checking mixing and stationarity for each parameter in the VARX model of each country. In the model of USA, there are a total of 78 parameters in the VARX: 48 from the VARX model with 3 endogenous variables, 21 from 3 stochastic volatility processes corresponding to 3 endogenous variables, 3 from identification matrix, and 6 from the lower triangular of variance-covariance matrix \( Q_i \). For other economies/regions but Saudi Arabia, in each VARX there are a total of 128 parameters: 76 from the VARX model with 4 endogenous variables, 36 from the four stochastic volatility processes, 6 from identification matrix, and 10 from the lower triangular of variance-covariance matrix \( Q_i \). For the Saudi Arabia model, there are three endogenous variables resulting in a total of 78
parameters.\footnote{Data of short-term interest rate is not available for Saudi Arabia.}

We follow the procedure suggested by Brooks and Gelman [1998] and Gelman et al. [2013]. In the first step, we obtain the second set of Gibbs iterations, also using 100000 iterations and retaining only the last 10000 draws for analysis. As a result, for each parameter, there are two chains of iterations. We then take each of these chains and split into the first and second half, resulting in a total of $m = 4$ chains, each of length $n = 5000$.

For each scalar estimand $\beta$, we label the iterations as $\hat{\beta}_{ij}$ where $i = 1, \ldots, n$ and $j = 1, \ldots, m$. We then calculate the between- and within-sequence variances as followed:

- **Between variances**
  \[
  B = \frac{n}{m - 1} \sum_{j=1}^{m} (\hat{\beta}_j - \hat{\beta})^2
  \]
  where $\hat{\beta}_j = \frac{1}{n} \sum_{i=1}^{n} \beta_{ij}$ and $\hat{\beta} = \frac{1}{m} \sum_{j=1}^{m} \hat{\beta}_j$

- **Within-sequence variances**
  \[
  W = \frac{1}{m} \sum_{j=1}^{m} s_j^2
  \]
  where $s_j^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\beta_{ij} - \hat{\beta}_j)^2$

We can estimate the marginal posterior variance of the estimand $\text{var}(\beta | y)$, by a weighted average of $W$ and $B$:

\[
V = \frac{n-1}{n} W + \frac{1}{n} B
\]

Then the convergence of iterations is evaluated by estimating the potential scale reduction factor (PSRF), namely

\[
R = \sqrt{\frac{V}{M}}
\]

which declines to 1 as $n \to \infty$. If the PSRF is not close to one, then the chains might not have been converged yet. Thus, further simulations may help to improve the inference about the target distribution of the associated scalar estimand. A rule of thumb for convergence is that the PSRF is below 1.1 (Gelman et al., 2013).

Figure 16 shows the potential scale reduction factors for each estimand of country models providing evidence for convergence of the algorithm. Therefore, we proceed with these retained iterations for impulse response analyses.
Figure 16: Potential scale reduction factor: each estimand of country VARX models
E Alternative Identifications

E.1 Cholesky Ordering I

In the baseline, we adopt the sign restrictions to identify US monetary volatility shocks. We perform a robustness check by using the Cholesky decomposition as an alternative identification scheme. This requires to specify the ordering of endogenous variables. Specifically, we order the variables as in the baseline in which the interest rate is ranked first. Regarding the US VARX model, the ordering follows: interest rate, output growth, and inflation; meanwhile, for other economies, interest rate, output growth, inflation, and exchange rate growth. For the covariance matrix of the volatility equations $Q$, we use a Cholesky decomposition with volatilities ordered in the same manner as the endogenous variables in the VARX. As presented in Figures 17 - 19, we obtain similar responses with our baseline, therefore corroborating our results.

Figure 17: Impacts of US Monetary Policy Uncertainty: Cholesky I

Notes: Figure presents the response of US macroeconomy to an unexpected increase by 100 percent in the US interest rate volatility using the Cholesky ordering I (interest rate is ranked first): in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals.
Figure 18: Global Impacts of US Monetary Policy Uncertainty: Cholesky I

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility using the Cholesky ordering I (interest rate is ranked first): in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals.
Figure 19: Global Impacts of US Monetary Policy Uncertainty: Cholesky I (cont)

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility using the Cholesky ordering I (interest rate is ranked first): in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals.
E.2 Cholesky Ordering II

We consider an alternative ordering of variables in the VARX in which we place interest rate third instead of first as in the baseline and cholesky ordering I. Specifically, regarding the VARX model for US, the ordering follows: Output growth, inflation, and interest rate; meanwhile, for other economies, output growth, inflation, interest rate, and exchange rate growth. For the covariance matrix of the volatility equations $Q$, we use a Cholesky decomposition with volatilities ordered in the same manner as the endogenous variables in the VARX. As presented in Figures 20 - 22, we obtain similar responses with our baseline, therefore corroborating our results.

Figure 20: Impacts of US Monetary Policy Uncertainty: Cholesky II

Notes: Figure presents the response of US macroeconomy to an unexpected increase by 100 percent in the US interest rate volatility using the Cholesky ordering II (interest rate is ranked third after output and inflation): in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals.
Figure 21: Global Impacts of US Monetary Policy Uncertainty: Cholesky II

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility using the Cholesky ordering II (interest rate is ranked third after output and inflation): in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals.
Figure 22: Global Impacts of US Monetary Policy Uncertainty: Cholesky II (cont)

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility using the Cholesky ordering II (interest rate is ranked third after output and inflation): in each entry, the red solid line and the shaded area are the median response and the 68 percent intervals.
In Figure 23, we present the estimates of volatility for economies in our GVAR model, i.e. in each country, the measure is the first principal component of three (standardized) measures of interest rate volatility, output volatility, and inflation volatility. Exchange rate is notoriously volatile, so we exclude the exchange rate volatility from the calculation. This is equivalent to the measure of US economy in Figure 3. As it can be seen, our estimates capture reasonably well key periods of high uncertainty in these economies, relating to the crises in Argentina, Brazil, Chile, and Peru (in the late 1980s and early 1990s), in Mexico (1994-1995), in East Asian countries (Indonesia, Malaysia, Thailand, and South Korea) in the end of 1990s, in Turkey (1994), in euro area countries in the first half of 1990s, and in many economies during the financial crisis 2007-2009, among other instances.

Figure 23: Volatility of countries in the GVAR

Notes: Figure presents the first principal component of three (standardized) measures of interest rate volatility, output volatility and inflation volatility.

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22For Saudi Arabia, it is the first principal component of output volatility and inflation volatility because the interest rate data are not available.
G Robustness Check with Different Priors

Regarding the prior for VAR coefficients, the parameter that controls the overall tightness of the prior on the VAR coefficients is set to a common value in the literature $\tau = 0.1$. In this exercise, we set $\tau = 0.5$ - five times larger than the benchmark value, therefore reducing the tightness of the prior. With such a loosening setting, as shown in Figures 24-26, the uncertainty of impulse responses slightly increases; however, our results remain similar, confirming that the interest rate uncertainty driving output and inflation volatilities as well as causing output slump, deflation, and a drop in the interest rate (Figure 24). In addition, we also find strong global impacts as presented in Figures 25 and 26.

Figure 24: Impacts of US Monetary Policy Uncertainty: Different priors

Notes: Figure presents the response of US macroeconomy to an unexpected increase by 100 percent in the US interest rate volatility. Each entry shows the median (solid line), and the 68% intervals (shaded area).
Figure 25: Global Impacts of US Monetary Policy Uncertainty: Different priors

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility. Each entry shows the median (solid line), and the 68% intervals (shaded area).
Figure 26: Global Impacts of US Monetary Policy Uncertainty: Different priors (cont)

Notes: Figure presents the global spillover effects of an unexpected increase by 100 percent in the US interest rate volatility. Each entry shows the median (solid line), and the 68% intervals (shaded area).
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