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By Swapnil Singh, Christian A. Stoltenbergz

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HOW MUCH DO HOUSEHOLDS REALLY KNOW ABOUT THEIR FUTURE INCOME?*

Swapnil Singh†
Christian A. Stoltenbergz ‡

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† Center for Excellence in Finance and Economic Research (CEFER), Bank of Lithuania. Email: ssingh@lb.lt.
‡ Macroeconomics and International Economics (MInt), Department of Economics, University of Amsterdam and Tinbergen Institute, 1018 WB Amsterdam, The Netherlands, email: c.a.stoltenberg@uva.nl.

Address
Totorių g. 4
LT-01121 Vilnius
Lithuania
Telephone (8 5) 268 0103

Internet
http://www.lb.lt

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Abstract

We develop a consumption-savings model that distinguishes households' perceived income uncertainty from income uncertainty as measured by an *econometrician*. Households receive signals on their future disposable income that can drive a gap between the two uncertainties. With an uncertainty gap that is consistent with direct estimates stemming from subjective income expectations, the model jointly explains three consumption inequality and insurance measures in US micro data that are not captured without the difference: (i) the cross-sectional variance of households' consumption, (ii) the covariance of current consumption and income growth and (iii) the income-conditional mean of household consumption.

JEL classification: E21, D31, D52
Keywords: Risk sharing, Advance information, Consumption insurance, Endogenous borrowing constraints, Limited commitment

1 Introduction

What is households' income uncertainty when they decide about their savings to insure against undesirable fluctuations of their consumption? The answer to this question is of central importance to understanding consumption risk sharing; only what households don't know yet constitutes uncertainty they seek to hedge. Typically, households' income uncertainty measures stem from aggregating earnings across household members and income types in the population. As Browning, Hansen, and Heckman (1999) point out, this procedure may create a disconnect between the uncertainty as assessed by an *econometrician* and income uncertainty as perceived by households. Quantifying households' perceived income uncertainty is, however, a prerequisite for evaluating the welfare effects of hotly-debated reforms such as changes in the progressivity of the tax system. In this paper, we argue that accounting for households' perceived income uncertainty is key to understand consumption inequality and insurance of households in the United States.

We consider an environment in which risk-averse households seek insurance against idiosyncratic fluctuations of their disposable income. As the new element here, we explicitly extend households' information set by signals that inform households about their income in the next period with certain precision. Due to the signals, households' expectations of future income are heterogeneous even when current income is the same. While the stochastic income process constitutes the income uncertainty as assessed by an econometrician, the joint process of signals and income represents households' income uncertainty. The difference between the two income uncertainties depends on the precision of the signals; the more precise are the signals, the smaller are households' forecast errors for income growth and the lower is households' perceived income uncertainty.

The extension of households' information set with signals is motivated by a mounting literature that finds that subjective expectations on future realizations of idiosyncratic risk have significant predictive power even when other information available to the econometrician is taken into account.[1] Controlling for current realizations of income, Dominitz (1998) estimates that conditioning additionally on households' reported subjective expectations significantly reduces the forecast error of income growth. Thus, households typically have more information

[1] Exemplary papers are Dominitz (1998) and Dominitz and Manski (1997) for income risk, Smith, Taylor, and Sloan (2001) for predicting mortality risk, Stephens (2004) for unemployment, or Campbell, Carruth, Dickerson, and Green (2007) for job insecurity.

than their current income to predict their future earnings. In our environment, we capture this advance information with informative signals. Correspondingly, the signals collect a wide spectrum of information relevant for future changes in disposable income that are already known to households before the actual change occurs. Examples of this type of foreknowledge are information on future performance bonuses, promotions, demotions or wage cuts, wage rises, changes in income taxes and transfers.

In reality, households can smooth income shocks in a variety of ways, involving progressive taxation, family transfers, informal networks or default. To capture these various insurance possibilities, we employ a general-equilibrium model with endogenous solvency constraints stemming from limited contract enforcement as proposed by Alvarez and Jermann (2000). In this model, households have access to a full set of securities to capture formal and informal insurance arrangements with the drawback that the arrangements are not enforceable under all circumstances.

Existing models of risk sharing without advance information have difficulties capturing consumption inequality and insurance of households in the United States. While standard incomplete markets models as pioneered by Aiyagari (1994) tend to predict too little models with endogenous solvency constraints tend to result in too much consumption smoothing. Employing US micro data to inform our theoretical model, we find that it can explain consumption heterogeneity better than existing models that do not account for households perceived income uncertainty. To the best of our knowledge, it is the first model that jointly matches three distinct key consumption inequality and insurance measures that are not captured without advance information: (i) the unconditional variance of households consumption in the cross-section, (ii) the covariance of current consumption growth and income growth and (iii) the income-conditional mean of household consumption for six income groups.

To explain these measures, we discover that households have advance information on their future income such that their income uncertainty is lower than what is typically considered in consumption risk sharing and insurance models; according to the theoretical model, we quantify that advance information reduces households' mean-squared forecast error for income by approximately 12%. This implies a systematic gap between the income uncertainty as perceived by households and the income uncertainty as estimated by an econometrician. The size of the uncertainty gap as quantified by the model is consistent with the direct estimates of Dominitz (1998) who finds that accounting for households' subjective income expectations reduces the econometrician's mean-squared forecast error between 12% and 21%.

We show that advance information improves the data fit of the model with endogenous solvency constraints because more precise signals reduce consumption risk sharing. The mechanism for this surprising result is that more precise signals decrease the value of insurance for high-income households which tightens the solvency constraints of low-income agents and thus limits opportunities for risk sharing. Consequently, advance information decreases risk sharing. Similar to Ábrahám and Cárceles-Poveda (2010), we also find that the endogenous solvency limits are consistent with US data on credit limits. In particular, high-income households face more generous credit limits than low-income households.

Further, we characterize cross-sectional long-run distributions of consumption, income, and wealth across households with advance information. As a methodological contribution, we develop a dynamic stochastic model with an explicit specification of the joint distribution of income and signals to consistently model the additional predictive power of informative signals on future realizations of idiosyncratic risk. Consistency implies that the distributions of expected income and income realizations are aligned. When income is persistent, we show that consistency requires non-trivial but intuitive assumptions on the stochastic process for signals. This methodological contribution is general and can be widely applied to individual decision problems under risk beyond consumption risk sharing. Empirically, Dominitz (1998) and more recently Attanasio and Augsburg (2016) find that expected income and realized income are indeed very similar.

For a given size of the uncertainty gap, we analyze the quantitative implications for several over-identifying restrictions. Blundell, Pistaferri, and Preston (2008) show that advance information of the type we consider can result in counterfactual non-zero correlations of current consumption growth with future income growth in a standard incomplete markets model. With endogenous solvency constraints, we find that advance information does not induce counterfactual correlations of current consumption with future income growth.

Advance information also improves the fit of the income-conditional distribution of consumption. The model (almost) perfectly tracks the income-conditional mean of consumption of low, medium and high-income earners. Further, the advance information helps to attenuate a non-linearity present in limited contract enforcement models without information but absent in the data. In the absence of information, the limited commitment model implies a variance of consumption conditional on a high income that is equal to zero. With informative signals, the conditional variance is positive, bringing the model closer to the data.

Related literature We are not the first to find that households know more than econometricians about their future earnings.[2] The main difference to these papers is that we point out that the quantitative importance of advance information for consumption risk sharing crucially depends on the structure of insurance markets. In a standard incomplete markets model with exogenous solvency constraints, the uncertainty gap has only modest implications for consumption risk sharing. With endogenous solvency constraints, however, the uncertainty gap matters to understand consumption heterogeneity.

Our paper is closely related to Heathcote, Storesletten, and Violante (2014), Kaplan and Violante (2010) and Guvenen and Smith (2014) who study the role of advance information in standard incomplete markets environments with a single non-state contingent bond. Heathcote, Storesletten, and Violante (2014) consider two different type of shocks, "uninsurable shocks" and "insurable shocks". The former shocks can be only partially smoothed while the latter type of shocks can be interpreted as perfectly forecastable and are completely insured (by construction). We consider signals on uncertain future income realizations without taking a stand a priori whether certain shocks are insurable or not. In particular, we highlight that when households use a large variety of insurance possibilities (and not only a single non-state contingent bond) perfectly forecastable shocks do not necessarily enhance but may actually restrict the degree of insurance.

Kaplan and Violante (2010) show that the resulting increase in consumption smoothing in the standard incomplete markets model with advance information is quantitatively not important enough to account for the cross-sectional dispersion of consumption in the data. With our paper, we clarify that the quantitative effects of advance information on risk sharing depend on the structure of insurance markets. While we can confirm the earlier findings on advance information in the standard incomplete markets model, a model with endogenous solvency constraints bridges the gap to several consumption insurance measures observed in the data. Guvenen and Smith (2014) study a different type of advance information. In a life-cycle model, households have initial knowledge about their individual deterministic part of income growth while we consider households that receive signals every period about future realizations of their stochastic part of income.

[2] Exemplary papers in that literature are Cunha and Heckman (2016), Cunha, Heckman, and Navarro (2005), Guvenen (2007), Guvenen and Smith (2014), Huggett, Ventura, and Yaron (2006), Primiceri and van Rens (2009).

Methodologically, our paper draws on Kehoe and Levine (1993), Alvarez and Jermann (2000) and Krueger and Perri (2006, 2011) who analyze the theoretical and quantitative properties of constrained efficient allocations with limited contract enforcement. Aiyagari (1994) pioneered in characterizing invariant distributions of consumption and assets in the standard incomplete markets model in general equilibrium. Building on these papers, Broer (2013) provides a thorough comparison of the quantitive implications of both consumption risk sharing models to the data. We extend the limited contract enforcement model and the standard incomplete markets model with a role for information to study how households' perceived income uncertainty – instead of the uncertainty assessed by an econometrician – affects consumption risk sharing of US households.

Hirshleifer (1971) shows that better information makes risk-averse agents ex-ante worse off if such information leads to evaporation of risks that otherwise could have been shared in a competitive equilibrium with full insurance and perfect contract enforcement. Schlee (2001) provides conditions under which better public information about idiosyncratic risk is undesirable. Similar to these authors, we also find that better public information can result in less risk sharing. The difference is that the negative effect relies on the importance of the limited enforceability of contracts and arises only when consumption insurance is not full but partial. If enforcement frictions are absent, information does not affect consumption allocations in the limited commitment model.

The remainder of the paper is organized as follows. In the next section, we start with a simple model to analytically show how advance information affects consumption risk sharing with limited commitment. In Section 3, we present the theoretical model that we take to the data. Section 4 describes the data and the calibration that we employ in Section 5 to study the quantitative implications of advance information for risk sharing of US households. The last section concludes.

2 A simple model with limited commitment

To understand the intuition behind the quantitative results derived later, we provide here analytical results on the effect of advance information on consumption risk sharing with limited commitment employing an illustrative example. As our main result here, we show that better information on future income realizations reduces risk sharing.

Consider a two-period, pure-exchange economy with a continuum of ex-ante identical agents and a single perishable consumption good. In each period, agent i receives a stochastic labor-income endowment that can be either high, $e_h = \bar{e} + \delta_e$, or low, $e_l = \bar{e} - \delta_e$, with $\delta_e > 0$ and $\bar{e}$ as the arithmetic mean of the income process. Both income states are equally likely and the income realizations are independent across time and agents. In the first period, agents also receive a public signal k^i that informs about their income realizations in the second period.[3] Signals are i.i.d. as well and can indicate either a high income ("good" or "high" signals) or a low income ("bad" or "low" signals) in the future. The signals' precision κ is defined as the probability that signal and future income coincide, $\kappa = \pi(e_2 = e_j | k = e_j)$, with $j \in \{h, l\}$ and $\kappa \in [1/2, 1]$. Uninformative signals are characterized by precision $\kappa = 1/2$, perfectly informative signals by $\kappa = 1$.

The preferences of agents are given by the following expected utility function:

$$\mathbb{E}[u(c_1) + u(c_2)], \tag{1}$$

where c_1 and c_2 are consumption in the first and in the second period, respectively, $u(c)$, is increasing and strictly concave. We measure social welfare according to (1), i.e., as agents' expected utility before any risk has been resolved.

If the agents are able to commit before any endowments are realized, the efficient risk-sharing arrangement is perfect risk sharing. The commitment requirement is crucial because after observing current income an agent with a high income may have an incentive to deviate from the perfect risk-sharing agreement. To capture this rational incentive, we analyze risk-sharing possibilities with limited contract enforcement or voluntary participation. A risk-sharing arrangement is consistent with limited commitment if each agent in each possible state, after observing his first-period endowment and the signal on his future income realization, at least weakly prefers to follow the arrangement rather than to defect into autarky. For the second period, we assume that agents respect the commitments made in the first period. Otherwise, if voluntary participation were allowed in both periods, there would be no room for risk sharing because agents would always choose to consume their endowments.

Let $c_{i,1}^{j}$ be first-period consumption of agents with signal k^i and endowment e_j and $c_{i,2}^{jk}$ be second-period consumption of agents with first-period signal k^i and endowment e_j in the first

[3] As a robustness exercise, we also consider private signals (see Appendix A.3 for the details).

period and endowment e_k in the second period with $i, j, k \in \{l, h\}$. The incentives to deviate to autarky are represented by enforcement constraints that are given by the following expressions for high-income agents with good and bad signals

$$u(c_{h,1}^h) + \kappa u(c_{h,2}^{hh}) + (1-\kappa)u(c_{h,2}^{hl}) \geq u(e_{h,1}) + \kappa u(e_{h,2}) + (1-\kappa)u(e_{l,2}) \equiv V_{h,out}^h \tag{2}$$

$$u(c_{l,1}^h) + (1-\kappa)u(c_{l,2}^{hh}) + \kappa u(c_{l,2}^{hl}) \geq u(e_{h,1}) + (1-\kappa)u(e_{h,2}) + \kappa u(e_{l,2}) \equiv V_{l,out}^h \tag{3}$$

and for low-income agents with good and bad signals

$$u(c_{h,1}^l) + \kappa u(c_{h,2}^{lh}) + (1-\kappa)u(c_{h,2}^{ll}) \geq u(e_{l,1}) + \kappa u(e_{h,2}) + (1-\kappa)u(e_{l,2}) \tag{4}$$

$$u(c_{l,1}^l) + (1-\kappa)u(c_{l,2}^{lh}) + \kappa u(c_{l,2}^{ll}) \geq u(e_{l,1}) + (1-\kappa)u(e_{h,2}) + \kappa u(e_{l,2}). \tag{5}$$

The resource feasibility constraints in the first and second period are the following

$$\frac{1}{4}\left(c_{h,1}^h + c_{h,1}^l + c_{l,1}^h + c_{l,1}^l\right) = \frac{1}{2}\sum_{j\in\{l,h\}} e_{j,1} \tag{6}$$

$$\frac{1}{4}\left[\kappa\left(c_{h,2}^{hh} + c_{h,2}^{lh} + c_{l,2}^{hl} + c_{l,2}^{ll}\right) + (1-\kappa)\left(c_{h,2}^{hl} + c_{h,2}^{ll} + c_{l,2}^{hh} + c_{l,2}^{lh}\right)\right] = \frac{1}{2}\sum_{j\in\{l,h\}} e_{j,2} \tag{7}$$

An *efficient allocation* is a consumption allocation, $\{c_{i,1}^j, c_{i,2}^{jk}\}$, that maximizes ex-ante utility (1), subject to the enforcement constraints (2)-(5) and the resource constraints (6)-(7).

Efficient allocations may feature either perfect risk sharing (all agents consume $\bar{e}$ in all states), no insurance against income risk (autarky, all agents consume their endowments in all states) or partial risk sharing. Here we focus on the empirically relevant case of partial risk sharing. As summarized in the following proposition, better public signals lead to less risk sharing and higher consumption dispersion.

Proposition 1 (Information and risk sharing) *Consider an efficient allocation with partial risk sharing such that the enforcement constraints* (2)-(3) *are binding. Conditional on the income-signal pair in the first period, the consumption allocation is characterized by perfect smoothing across future income states and across periods, i.e.,*

$$c_{i,1}^j = c_{i,2}^{jh} = c_{i,2}^{jl} = c_i^j, \quad \forall i, j.$$

An increase in information precision has the following effects on the consumption allocation in *each period:*

1. *The conditional mean of consumption of high-income agents increases and the conditional mean of low-income agents decreases.*

2. *The conditional standard deviations of consumption of high-income and low-income agents increase.*

3. *The unconditional standard deviation of consumption increases.*

The proof is provided in Appendix A.1.

The main messages from the proposition are first that conditional on an income-signal pair in the first period efficient allocations feature perfect consumption smoothing across future income states and time periods. More importantly, more precise signals result in a more unequal consumption distribution when enforcement constraints matter.

To get intuition why better information on individual future income realizations increase consumption dispersion, consider an increase in the precision of signals. By (2) and (3), this results in an increase in the value of the outside option for high-income agents with a good signal and a decrease for agents with a bad signal. As captured by the changes in the outside option values, agents with a bad signal are more willing while the agents with a good signal are less willing to share their current high income. Thus, consumption of high-income agents spreads out and the conditional standard deviation of consumption of high-income agents increases. Thereby, the changes in the value of the outside option of high-income agents with a good signal ($V^h_{h,out}$) and with a bad signal ($V^h_{l,out}$) are symmetric:

$$\frac{\partial V^h_{h,out}}{\partial \kappa} = -\frac{\partial V^h_{l,out}}{\partial \kappa}.$$

For informative signals, the high-income agents with a good signal have a lower marginal utility of consumption and thus require more additional resources than the high-income agents with a bad signal are willing to give up. In sum, mean consumption of high-income agents increases which by resource feasibility reduces the risk-sharing possibilities for low-income agents. As a consequence, the consumption allocation becomes riskier ex ante and the unconditional standard deviation of consumption increases as well.

In this section, we have shown that more precise signals result in a riskier allocation ex ante such that the standard deviation of consumption increases. Further, better information results in higher consumption of high-income and lower consumption of low-income agents. In the next section, we present a more general model with an infinite time horizon and endogenous solvency constraints embedded into a production economy with capital.

3 Environment

Preferences and endowments Consider an economy with a continuum of households indexed by i. Time is discrete and indexed by t from zero onward. Households have preferences over consumption streams and evaluate them conditional on the information available at $t = 0$

$$U\left(\{c_t^i\}_{t=0}^{\infty}\right) = (1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i), \tag{8}$$

where the instantaneous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave and satisfies the Inada conditions.

Household i's disposable labor income in period t is given by $w_t y_t^i$, where w_t is the real wage per unit of effective labor and y_t^i are individual effective labor unit endowments. Effective labor unit endowments are generated by a stochastic process $\{y_t^i\}_{t=0}^{\infty}$, where the set of possible realizations in each period is time-invariant and finite $y_t^i \in Y \equiv \{y_1, ..., y_N\} \subseteq \mathbb{R}_{++}$, ordered. The history $(y_0, ...y_t)$ is denoted by y^t. Effective labor units are independent across households and evolve across time according to a first-order Markov chain with time-invariant transition matrix $\pi_{jk} > 0$ for all j, k whose elements are the conditional probabilities of next period's endowment y_k given current period endowment y_j. There is no aggregate risk, and the Markov chain induces a unique invariant distribution of income $\pi(y)$ such that the aggregate labor endowment is constant and equal to $L_t = \bar{y} = \sum_y y\pi(y)$.

Information Each period $t \geq 0$, household i receives a public signal $k_t^i \in Y$ that informs about endowment realizations in the next period. The signal has as many realizations as endowments states and its precision κ is captured by the probability that signal and future endowment coincide, $\kappa = \pi(y_{t+1} = y_j | k_t = y_j)$, $\kappa \in [1/N, 1]$. Uninformative signals are characterized by precision $\kappa = 1/N$, perfectly informative signals by $\kappa = 1$. Hence, at each point in time the agents can find themselves in one of the states $s_t = (y_t, k_t)$, $s_t \in S$, where S is the Cartesian product $Y \times Y$ and $s^t = (y^t, k^t) = (s_0, ...s_t)$ is the history of the state.

The realizations of the signal follow an exogenous Markov process with transition probabilities $\pi(k_{t+1} = y_i | k_t = y_j)$ that are chosen such that the resulting joint distribution of endowments and signals satisfies the following two consistency requirements.

Consistency Requirement I: The marginal distribution of the joint invariant distribution $\pi(s) = \pi(y, k)$ with respect to income equals the invariant distribution of endowments $\pi(y)$, i.e.,

$$\hat{\pi}(y) = \sum_{k \in Y} \pi(y, k) \doteq \pi(y).$$

Consistency Requirement II: The conditional distribution of endowments $\pi(y'|y)$ follows from integrating $\pi(y'|y, k)$ with respect to signals, i.e.,

$$\hat{\pi}(y'|y) = \sum_{k \in Y} \pi(y'|y, k)\pi(k|y) \doteq \pi(y'|y),$$

where y' denotes a future realization and y a current realization of the variable y. In Appendix A.2, we show that consistency with these requirements yields signal transition probabilities that depend in general on the properties of the Markov process for endowments and on the precision of signals. In case of a symmetric transition matrix for endowments, signal transition probabilities are independent of κ and consistency requires the signals to follow the same stochastic process as endowments. Otherwise, the joint distribution of endowments and signals does violate at least one of the consistency requirements. If signal were for example i.i.d. but endowments are persistent, we show that at least one of the two consistency requirements is violated. Thus, when income is persistent, signals are persistent as well. This implies that the effect of a signal realization today does not only affect expectations for income in the next period but can have long-lasting effects for future expectations.

Using the assumptions on endowments and signals, the probabilities for the distribution of future endowments conditional on today's state s is given by[4]

$$\pi(y'|s) = \pi\left(y' = y_j | k = y_m, y = y_i\right) = \frac{\pi_{ij} \kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N} \pi_{iz} \kappa^{\mathbf{1}_{m=z}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \quad , \tag{9}$$

where $\mathbf{1}_{m=j}$ is an indicator function and equals one if the signal and the actual realization of the endowment coincide. The formula follows from Bayes law and its logic is a signal extraction with

[4] Appendix A.4 provides details on the derivation of the formulas for the joint distribution of endowments and signals.

two independent signals on future endowment realizations, current endowments and the public signal. Both "signals" enter the signal extraction weighted with their precision, endowments with transition probability π_{ij} and signals with precision κ.[5]

For example, with uninformative signals ($\kappa = 1/N$) the conditional probability of endowment y_j tomorrow given today's signal k_j and endowment y_i can be computed as

$$\pi\left(y' = y_j | k = y_j, y = y_i\right) = \frac{\pi_{ij}\frac{1}{N}}{\frac{1}{N}\sum_{z=1}^{N}\pi_{iz}} = \pi_{ij}.$$

With signals following an exogenous process, the conditional distribution of signals and endowments can be combined to a time-invariant Markov transition matrix P_s with conditional probabilities $\pi(s'|s)$ as elements

$$\begin{aligned}\pi(s'|s) = \pi\left(y' = y_j, k' = y_l | k = y_m, y = y_i\right) \\ = \pi(k' = y_l | k = y_m)\pi\left(y' = y_j | k = y_m, y = y_i\right). \quad (10)\end{aligned}$$

Production A representative firm hires labor L_t and capital K_t at rental rates w_t and r_t to maximize profits. Capital depreciates at rate δ and the production of consumption goods Y_t takes place via a linear homogenous production function

$$Y_t = AF(L_t, K_t),$$

with A as a productivity parameter that is constant in the stationary equilibria that we focus on in the following. Aggregate labor endowments L_t are normalized to unity.

Endogenous solvency constraints Following Alvarez and Jermann (2000), there is no restriction on the type of insurance contracts that can be traded but the contracts suffer from limited commitment because every period agents have the option to default to autarky. Households can buy or sell state-contingent assets $a(s^t, s_{t+1})$ priced at $q(s^t, s_{t+1})$. The state-contingent asset $a(s^t, s_{t+1})$ prescribes one unit of the consumption good in state s_{t+1} to or from an agent that experiences the history s^t. Households trade the asset with financial intermediaries that live for one period and can also invest into capital. Households face state-contingent endogenous credit limits $A(s^t, s_{t+1})$ that are not "too tight", i.e., credit limits that only ensure that

[5] At the end of this section, we elaborate on our modelling choices regarding the specification of signals.

households have no incentive to default to autarky but do not constrain insurance contracts otherwise

$$a(s^t, s_{t+1}) \geq A(s_{t+1}) = \min \left\{ a(s_{t+1}) : V\left[a(s_{t+1}), s_{t+1}\right] \geq U^{Aut}(s_{t+1}) \right\}, \quad \forall s_{t+1}, \tag{11}$$

with U^{Aut} as the value of the outside option and $V(a,s)$ as the continuation value of a household with asset holdings a and state s (see the recursive problem (12)-(14) below). In case of defaulting to the outside option and consistent with US bankruptcy law, households lose all their assets. Further, access to financial markets is restricted. While agents can save unlimited amounts in a non-state contingent bond with gross return R, they cannot borrow. Thus, the value of the outside option is a solution to an optimal savings problem that can be written in recursive form as follows

$$v(a,s) = \max_{0 \leq a' \leq y + aR} \left[(1-\beta) u(aR + y - a') + \beta \sum_{s'} \pi(s'|s) v(s', a') \right],$$

such that the value of the outside option is given by

$$U^{Aut}(s) = v(0,s).$$

Given asset holdings a, state $s = (y,k)$, and prices $w, \{q(s,s')\}$, households' problem can be written recursively as

$$V(a,s) \quad = \quad \max_{c, \{a'(s')\}} \left\{ (1-\beta) u(c) + \beta \sum_{s'} \pi(s'|s) V\left[a'(s'), s'\right] \right\} \tag{12}$$

subject to a budget constraint and solvency constraints

$$c + \sum_{s'} q(s,s') a'(s') \quad \leq \quad wy + a \tag{13}$$

$$a'(s') \quad \geq \quad A(s'), \quad \forall s'. \tag{14}$$

The result of the utility maximization problem are policy functions $c(a,s), \{a'(a,s;s')\}$. In period zero, households differ with respect to initial asset asset holdings and initial shocks where the heterogeneity is captured by the invariant probability measure $\Phi_{a,s}$. In an economy with one non-state contingent asset, Ábrahám and Cárceles-Poveda (2010) show that the endogenous credit limits derived according to (11) share some realistic features with credit limits observed in the Survey of Consumer Finances (SCF). As in the data, credit limits in the model become looser as labor income increases. While agents with a higher income have more incentives to default because higher income shocks lead to a higher autarky value, this does not necessarily lead to tighter credit limits. In our quantitative results, we confirm the results of Ábrahám and Cárceles-Poveda (2010) for a complete set of state contingent assets.

Equilibrium The stationary recursive competitive equilibrium with solvency constraints is summarized in the following definition.

Definition 1 *A stationary recursive competitive equilibrium with solvency constraints comprises a value function $V(a,s)$, a price system $R, w, q(s,s')$, an allocation $K, c(a,s), \{a'(a,s;s')\}$, a joint probability measure of assets and exogenous state $\Phi_{a,s}$, and endogenous credit limits $A(s')$ such that*

(i) $V(a,s)$ is attained by the decision rules $c(a,s)$, $\{a'(a,s;s')\}$ given $R, w, q(s,s')$

(ii) Endogenous credit limits are determined according to (11)

(iii) The joint distribution of assets and state $\Phi_{a,s}$ induced by $\{a'(a,s;s)\}$ and P_s is stationary

(iv) No arbitrage applies

$$q(s,s') = \frac{\pi(s'|s)}{R}$$

(v) Factor prices satisfy

$$R - 1 = AF_K(1,K) - \delta$$
$$w = AF_L(1,K)$$

(vi) The asset market clears

$$R'K' = \int \sum_{s'} a'(a,s;s')\pi(s'|s)\,\mathrm{d}\,\Phi_{a,s}.$$

We conclude this section with a discussion of some features of the information environment outlined at the beginning of this section. With the signals, we collect a wide spectrum of information such as foreknowledge of future performance bonuses, promotions, demotions wage cuts or wage rises. In formula (9), we model signals with a "hit or miss specification" in the following sense. The probability that the signal indicates the correct endowment realization is κ, and $1-\kappa$ is the probability that signal and future endowment realization differ. This probability is then allocated equally to the endowment states not indicated by the signal. Thus, conditional on the signal being wrong, the transition probability is exclusively driven by the endowment transition probabilities $\pi(y'|y)$. For the type of information we seek to model this is a reasonable specification, in particular when endowment shocks are persistent as in reality. To see this, suppose that an agent receives a signal that he will likely get a bonus in the next period and according to the formula his probability to receive an endowment rise increases compared to the case without the signal. With some probability, however, he might not get the bonus. In this case, the probability to transit to a particular endowment state should not be affected by the signal anymore but solely be conditional on his current endowment state. This is exactly what is captured in the formula by allocating $1-\kappa$ equally over the states without a bonus.

The examples in the introduction include both private and public information. We opted for public signals for the following reasons. First, with the signals, we seek to model the predictive power of publicly available subjective income expectations as estimated in the empirical literature and public signals serve exactly this purpose. The subjective expectations elicited in studies such as Dominitz (1998) are probably the result of both, private and public information on future earnings. To the best of our knowledge, there is no paper yet that disentangles the public and private sources underlying the subjective expectations. Second, an advantage of public signals compared to private signals is that we can specify a fully decentralized version in which households engage in unmonitored trade of ordinary securities. Atkeson and Lucas (1992) show that such a decentralization is not feasible with private information. Thus, with private signals we have to consider a social planner problem which is less realistic. Third, we also don't want to be silent about the fact that public signals are more tractable than private signals which allows us to employ realistic income processes with both transitory and persistent shocks. In our quantitative exercise, we employ a signal-income distribution with 36 states. With public signals alone, that means that at each node (a, y, k) we have to check 36 occasionally binding constraints resulting from the option to default to autarky which is already more than the 14 states typically considered in related studies. With private information, we would have to additionally consider 5 constraints that capture households' incentives to reveal the true realization of the private signal. More than the sheer increase in occasionally binding constraints at each node, the interaction of enforcement and truth-telling constraints results in additional challenges in the computation of allocations. For this reason, existing studies such as Broer, Kapička, and Klein (2017) focus on the theoretical implications of private information and consider stylized endowment processes with only two states. Finally, we find that increases in signal precision with private signals have qualitatively similar effects on risk sharing in a two-period model (see Appendix A.3).

In the following, we study the quantitative implications of advance information in this model for consumption risk sharing.

4 Quantitative exercise

In this section, we describe the data employed in the quantitative exercise and the calibration of parameters. Further, we explain how we measure consumption risk sharing and how we assess the difference in income uncertainty measured by econometricians and households.

4.1 Data and calibration

Data To quantitatively evaluate the model, one would like to employ a household panel data set with a large number of observations that contains detailed information on households' income, their consumption expenditures and their subjective expectations of future income. To the best of our knowledge such a data set does not exist for the US. For this reason, we opt for the following strategy. To facilitate comparison with related studies in particular to Krueger and Perri (2006) and Broer (2013), we also use as a first step the Consumer Expenditure Interview Survey (CEX) for information on households' income and consumption expenditures. Starting from the calibration used in these papers, we investigate how informative signals affect consumption inequality and insurance in the model by varying signal precision to find our preferred value for the parameter κ. Afterwards, we relate the value of the parameter to information on the predictive power of subjective expectations elicited in the special edition of the Survey of Economic Expectations from 1993–1994 that contains information on US households' income realizations and their corresponding income expectations.

For the CEX, we exactly follow Krueger and Perri (2006) and Broer (2013) in their methodology. In particular, we decompose consumption and income inequality in between and within group inequality. Between-group inequality are differences in household income and consumption attributable to observable characteristics for example education, region of residence, etc., and assume that households cannot insure against these observable characteristics. Income inequality devoid of between group inequality component is called within group inequality. This residual measure of inequality is the focus of this paper as it is caused by the idiosyncratic income shocks and hence, depending on the insurance available against these shocks, consumption inequality will not exactly mirror income inequality.

As measure of household consumption, we employ non-durable consumption (ND+) which also includes an estimate for service flows from housing and cars. For households' disposable income, we use after-tax labor earnings plus transfers (LEA+). Consistent with voluntary participation, we thus take the mandatory public insurance as given and focus on private insurance. LEA+ comprises the sum of wages and salaries of all household members, plus a fixed fraction of self-employment farm and non-farm income, minus reported federal, state, and local taxes (net of refunds) and social security contributions plus government transfers.

We drop the households who report zero or only food consumption, whose head is older than 64 years or younger than 21 years, with negative or zero labour income or have negative working hours, which have positive labour income but no working hours, that live in the rural area or their weekly wage is below half the minimum wage and which are not present in all interviews. To facilitate a comparison between households of different size, the consumption and income measures are divided by adult equivalence scales as in Dalaker and Naifeh (1997).

To compute within group inequality, we follow Krueger and Perri (2006) and Blundell et al. (2008), and regress the logs of household consumption and income on a cubic function of age and a set of dummies that include region, marital status, race, education, experience, occupation and sex. The residuals of the regression are treated as consumption and income shock.

Model parameters Our annual calibration is designed to highlight the differences between a standard limited commitment model without information as entertained in Broer (2013) and a model with information. Therefore, we set a number of corresponding parameters to the same values. In particular, we consider a period utility function that exhibits constant relative risk aversion with parameter $\sigma = 1$. The discount factor β is chosen to yield an annual gross interest rate of $R = 1.025$ in general equilibrium. We employ a Cobb-Douglas production function $AF(K, L)$ with a capital-production elasticity of 0.30. Given R, we choose the depreciation of the capital stock δ and the technology parameter A to yield a real wage rate of unity and an aggregate wealth-to-income ratio of 2.5 as for example estimated by Kaplan and Violante (2010) based on the Survey of Consumer Finances (SCF). With a wage rate of unity, labor income is $wy = y$ and we use the terms individual endowment and individual income interchangeably.

Following the practice in the literature, the income specification comprises persistent and transitory income components. Log income of household i is modelled as

$$\ln(y_{it}) = z_{it} + \epsilon_{it}, \qquad z_{it} = \rho z_{it-1} + \eta_{it},$$

where ϵ_{it} and η_{it} are independent, serially uncorrelated and normally distributed with variances σ_ϵ^2 and σ_η^2, respectively.[6] The persistence parameter ρ is set to 0.9989 which is the value originally found by Storesletten, Telmer, and Yaron (2004). Given the persistence parameter, we identify the variances $\sigma_\epsilon^2, \sigma_\eta^2$ from the cross-sectional within-group income variance and auto-covariance in the CEX data as the averages of the years 1999–2003. The method proposed by Tauchen and Hussey (1991) is used to approximate the persistent part of income by a Markov process with three states and time-invariant transition probabilities, and the transitory part is captured with two exogenous states of equal probability. We normalize the value of all income states such that mean income (or aggregate labor endowment) is equal to unity. For each of the six income states, there are therefore six public signals such that the joint income-signals state S is approximated by 36 states which is higher than the 14 states typically considered in related studies (Broer, 2013, Krueger and Perri, 2006). The increase in the number of states leads to a numerical challenge for computing consumption allocations in general equilibrium.[7]

4.2 Insurance and risk-sharing measures and uncertainty gap

Insurance and risk-sharing measures To measure the extent of consumption smoothing and inequality from the data, we focus on two measures: (1) the covariance of consumption and income growth as an insurance measure, and (2) the relative variance of log-consumption with respect to log-income to capture consumption inequality. The first measure captures the

[6] We consider a signal that informs about total future disposable income, i.e., about the combination of transitory and persistent shocks. An alternative is to specify two signals, one that informs about future realizations of transitory and one that informs about future realizations of persistent shocks.

[7] In Appendix A.5, we describe our algorithm for computing allocations in the endogenous-solvency constraints model in more detail. With 500 points on the asset grid, we solve in each iteration step for 666,000 variables. In a standard model without information and 14 income states as in Broer (2013), the corresponding number of variables is 105,000.

Table 1: Baseline parameters and CEX moments

	Parameter	Value
σ	Risk aversion	1
α	Elasticity of capital in production function	0.3000
R	Gross interest rate	1.0250
ρ	Auto-correlation	0.9989
σ_η	Standard deviation innovation persistent shocks	0.0245
σ_ϵ	Standard deviation innovation transitory shocks	0.3390
S	Income-signal states	36
var_y	Variance log income	0.3654
var_c	Variance log consumption	0.1462
$\beta_{\Delta y}$	Regression coefficient	0.1078
RS	Risk-sharing ratio	0.5999

sensitivity of consumption growth to income growth. Following Mace (1991), the sensitivity is captured by the coefficient $\beta_{\Delta y}$ in the following regression equation

$$\Delta c_{it} = \psi + \beta_{\Delta y}\Delta y_{it} + v_t + \nu_{it} \tag{15}$$

where ψ is a constant, v_t a vector of time dummies and ν_{it} a residual; Δc_{it} and Δy_{it} are the growth rates of consumption and income of individual i in period t. When the coefficient $\beta_{\Delta y}$ is zero, then consumption growth is perfectly insured against changes in income growth. The higher is the coefficient, the less insurance is achieved.

The second measure is defined as one minus the ratio of the cross-sectional unconditional variance of logged consumption over logged income:

$$RS = 1 - \frac{\mathrm{var}_c}{\mathrm{var}_y} \tag{16}$$

On one extreme, if $\mathrm{var}_c = \mathrm{var}_y$, then $RS = 0$, and there is no private risk sharing of fluctuations in disposable income. On the other hand, if $\mathrm{var}_c = 0$ then $RS = 1$ implying full risk sharing with respect to income shocks and the absence of consumption inequality. In Table 1, we summarize the calibrated parameters in the upper part and unconditional moments of consumption and income from the CEX data in the lower part. The value of $\beta_{\Delta y}$ is equal to 0.11 with a standard error of 0.0035; the ratio RS is $1 - \frac{\mathrm{var}_c}{\mathrm{var}_y} = 0.60$ which implies 40% of income shocks transfer to consumption.

Measuring the uncertainty gap To interpret the effects of an increase in information precision κ, we compute the percentage reduction of households' perceived income uncertainty $\tilde{\kappa}$ as measured by the reduction in the mean-squared forecast error resulting from conditioning expectations on signals

$$\tilde{\kappa}(\kappa) = \frac{\text{MSFE}_y - \text{MSFE}_s}{\text{MSFE}_y}, \tag{17}$$

with

$$\text{MSFE}_y = \sum_y \pi(y) \sum_{y'} \pi(y'|y) \left[y' - \text{E}(y'|y)\right]^2$$

$$\text{MSFE}_s = \sum_s \pi(s) \sum_{y'} \pi(y'|s) \left[y' - \text{E}(y'|s)\right]^2,$$

and $\pi(s)$ as the joint invariant distribution of endowments and signals. Thus, $\tilde{\kappa}$ captures the difference in income uncertainty as measured by an econometrician in the aggregate and the uncertainty as perceived by households stemming from subjective expectations. For this reason, we refer to $\tilde{\kappa}$ as the uncertainty gap.

5 Quantitative results

In this section, we provide quantitative results on the effect of advance information on consumption insurance and inequality of households in the United States. First, we employ the model with endogenous solvency constraints (ESC model) presented in Section 3 to quantify advance information through the lens of this model. We further discuss how the quantified value for advance information relates to direct estimates of the predictive power of subjective expectations and what this values implies for various “over-identifying restrictions”. We also study the quantitative effects of advance information on consumption insurance and inequality in a standard incomplete markets model (SIM).

5.1 Endogenous solvency constraints model

Quantifying advance information To discipline the only free parameter $\tilde{\kappa}$, we choose the parameter such that the consumption inequality and insurance predicted by the model matches two distinct measures observed in the data. The risk-sharing ratio as the first measure characterizes the cross-sectional dispersion of consumption. As the second measure, we employ the regression coefficient of current consumption growth with respect to income growth as a measure to determine the sensitivity of consumption with respect to changes in income (insurance). In general, we therefore expect to pin down two values for the reduction in households' perceived income uncertainty $\tilde{\kappa}_1, \tilde{\kappa}_2$ that yield insurance measures in the model that are consistent with the value of the measures observed in the CEX.

For the first measure, we use the cross-sectional variance of consumption in the invariant distribution. For the second insurance measure, we employ stationarity and simulate the model for 300,000 time periods and discard the first 100,000 periods to ensure convergence. Then we estimate covariances of consumption and income growth using the simulated data.

Our main quantitative findings are summarized in Table 2 that displays the two measures in the data and for various values of $\tilde{\kappa}$. As can be seen in the first row, without signals, $\tilde{\kappa} = 0$, consumption is almost perfectly smooth such that the risk-sharing ratio equals 0.99.[8] Consistent with the third part of Proposition 1, the risk-sharing ratio decreases in the precision of signals or equivalently in $\tilde{\kappa}$. For $\tilde{\kappa}_1 = 0.124$, the insurance ratio of 0.60 in the data is also explained in the model.

The third row shows how the uncertainty gap affects the regression coefficient $\beta_{\Delta y}$. While in the absence of information consumption growth is nearly perfectly guarded against changes in income growth, the sensitivity of consumption increases with the size of the uncertainty gap. For $\tilde{\kappa}_2 = 0.116$, the model matches the regression coefficient observed in the data. In that sense, both measures are jointly explained by the model for an uncertainty gap of 12% (rounded). This result is remarkable because in general the two measures have to coincide only in the extreme cases when risk sharing (and insurance) is either perfect or absent.

Discussion and robustness How can we interpret a reduction of perceived income risk of 12%? If income was i.i.d., the mean-squared forecast error for a variance of logged income of 0.37 is 0.42. With persistent income alone, the mean-square forecast error is reduced to 0.21; with signals alone the mean-squared forecast error amounts to 0.30. Thus, current income is a more important predictor for future income than signals. Considering both predictors of future income jointly, the signals reduce the mean-squared forecast error of conditioning on income

[8] The result of the (almost) absence of consumption inequality in the standard model without signals seems to be surprising but it corresponds to the consumption inequality that would prevail in the invariant distribution in Krueger and Perri (2006) with the income inequality equal to the value at the end of their sample in the year 2003. Krueger and Perri (2006) do not report the stationary level of consumption inequality in the paper because their focus is on understanding the relationship between changes in income and consumption inequality (see Figure 5 in their paper). Further, in the year 2003 the model economy they consider is not in the stationary equilibrium associated with the income inequality of 2003 but converges to this equilibrium.

Table 2: Insurance measures and advance information ESC model

	Risk-sharing ratio, RS				
	$\tilde{\kappa} = 0.00$	$\tilde{\kappa}_1 = 0.124$	$\tilde{\kappa} = 0.138$	$\tilde{\kappa}_{14} = 0$	**Data**
Baseline	0.99	0.60	0.47	0.94	**0.60**
Aguiar and Bils (2015)	0.99	0.60	0.47	0.94	**0.47**
	Regression coefficient, $\beta_{\Delta y}$				
	$\tilde{\kappa} = 0$	$\tilde{\kappa}_2 = 0.116$	$\tilde{\kappa} = 0.123$	$\tilde{\kappa}_{14} = 0$	**Data**
Baseline	0.00	0.11	0.16	0.01	**0.11**
Gervais and Klein (2010)	0.00	0.11	0.16	0.01	**0.16**

Notes: ESC model. Insurance ratio and regression coefficient in the data and in the model for different values of $\tilde{\kappa}$. No signals is $\tilde{\kappa} = 0.00$.

only by 12%.

A natural question is how the value of $\tilde{\kappa}$ that we indirectly quantify using the model with endogenous solvency constraints compares with direct estimations of the predictive value of subjective expectations. In the Spring and Fall of 1993, households in the Survey of Economic Expectations (SEE) were asked in to report their weekly earnings expectations for 1994. To elicit households' income expectations, Dominitz (1998) analyzes the relationship between the expectations reported in 1993 and the actual realizations of earnings in 1994. In particular, he estimates best linear predictors for 1994 earnings. He finds that even after controlling for earnings realizations in 1993, the reported subjective expectations have predictive value. Conditioning not only on the earnings realizations in 1993 but additionally on reported subjective earnings expectations from Spring 1993 decreases the mean-squared forecast error by 0.118, conditioning on the Fall expectations reduces the error by 0.214. Given that we employ one-year ahead earnings forecasts in the model, the values of $\tilde{\kappa}_1 = 0.124$ and $\tilde{\kappa}_2 = 0.116$ we find in the model are consistent with the direct evidence of 0.118 stemming from the Spring 1993 forecast.

Aguiar and Bils (2015) and Attanasio, Hurst, and Pistaferri (2012) argue that the consumption expenditures reported in the CEX Interview Survey may suffer from non-classical measurement error, resulting in biased estimates of cross-sectional consumption inequality measures. In particular, Aguiar and Bils (2015) find that consumption inequality (measured as the cross-sectional variance of logged consumption) has not increased by less than income inequality (measured as the cross-sectional variance of logged income) but moved hand-in-hand with income inequality from 1980-2003. The uncertainty gap we identify is not very sensitive with respect to a potentially noisy estimate for consumption inequality for two reasons. First, even if the correct insurance ratio is different than the number computed directly from the CEX, the identified uncertainty gap would only be mildly affected. Suppose that consumption inequality has mirrored income inequality between 1980 and 2003 which results in an risk-sharing ratio of 0.47 instead of the 0.60 we report in Table 1[9]. As can be seen in the second row of Table 2, the model can capture the modified risk-sharing ratio with a slightly higher value for the size of the uncertainty gap, $\tilde{\kappa} = 0.138$ instead of 0.124 as in the baseline calibration exercise. Second, using the regression coefficient as an alternative insurance measure yields very similar numbers for advanced information. This measure is less prone to measurement error because the regression coefficient employs growth rates as a ratio and therefore corrects for time-invariant multiplicative measurement error. Additionally, we find that the regression coefficient in the CEX of 0.11 is very close to the corresponding coefficient that we estimate using PSID data, 0.12, as an alternative data source.

Gervais and Klein (2010) argue that the standard estimator $\beta_{\Delta y} = 0.11$ tends to overstate the degree of insurance in CEX data, and propose an alternative estimator. Using the same data as we do but employing the procedure proposed by Gervais and Klein (2010), Broer (2013) estimates a value of $\hat{\beta}_{\Delta y} = 0.16$ (see Row 7, Column 6 of Table 3 on page 132), implying that consumption growth reacts more sensitively to income changes than in our baseline estimation. As displayed in the fourth row of Table 2, alternatively matching this value of the regression coefficient, we need more precise signals: $\tilde{\kappa} = 0.123$ instead of $\tilde{\kappa} = 0.116$, a value that is very close to $\tilde{\kappa} = 0.124$ which yields an insurance ratio of 0.60 in the model.

The high degree of insurance in the standard model without signals could be an artefact of employing an income grid with 6 states. Applying alternatively a finer income grid with 14 states – 7 states for the persistent shocks and 2 states for the transitory shocks – only mildly reduces consumption smoothing (see the forth column of Table 2, $\tilde{\kappa}_{14} = 0$); the risk-sharing ratio decreases from 0.99 to 0.94 and the regression coefficients increases from 0.00 to 0.01.

In Figure 3 in Appendix A.3, we compare the effects of increases in signal precision of public and private signals on consumption insurance. While increases in precision in both cases increase consumption dispersion, consumption risk sharing and insurance react more sensitively

[9] Assume that consumption inequality increases with the same rate as income inequality (0.5012) such that consumption inequality is 0.1938 instead of 0.1462. Thus, the insurance ratio is 0.47 which requires $\tilde{\kappa} = 0.1376$ to capture the modified risk-sharing ratio.

to private information. If subjective expectations were exclusively based on private information, matching the insurance measures from the data in the model would thus require lower values of $\tilde{\kappa}$ than the 12% found for public signals, resulting also in a lower predictive power of subjective expectations than the 12-21% estimated in Dominitz (1998).

Similar to Ábrahám and Cárceles-Poveda (2010) for a non-state contingent asset, we also find that credit limits become looser as labor income increases with a full set of state-contingent assets. Further, we find that the difference in credit limits between low-income and high-income earners increases in $\tilde{\kappa}$. In the absence of signals, low-income households face credit limits that are 14% stricter than for high-income households. For $\tilde{\kappa}_2 = 0.116$ and averaging across signals, the credit limits for low-income agents are 30% stricter than for high-income agents.

Kaplan and Violante (2010) conclude that advance information cannot reconcile insurance ratios or regression coefficients in a life-cycle standard incomplete markets model with the measures observed in the data. We find that the picture changes when we alternatively employ a model with endogenous solvency constraints. Here, advance information on future income shocks can bridge the gap to the data. Correspondingly, we can quantify households' advance information by matching the insurance ratio or, alternatively, by capturing the regression coefficient of current consumption growth on current income growth. Our main quantitative finding is that both insurance measure can be jointly explained when households' perceived income uncertainty is reduced by 12%. In the following, we fix information precision at this value, and analyze the model's performance for various "over-identifying restrictions".

5.2 Over-identifying restrictions

The goal of this section is to further test the model with the amount of advance information as quantified in the previous section. Throughout this section, we compare the standard model without signals to the case of informative signals.

Consumption-income growth correlations with advance information Blundell, Pistaferri, and Preston (2008) argue that including advance information in standard incomplete markets model (SIM) may lead to correlations of current consumption with future income growth that are not consistent with the data.[10] To test for a potential role of advance

[10] Guvenen and Smith (2014) consider households with initial knowledge about their individual deterministic part of income growth. This type of advance information does not result in the counterfactual consumption-income growth correlations in a SIM-model.

Table 3: Income and consumption growth regression: *ESC* model

	$\tilde{\kappa} = 0.00$	$\tilde{\kappa}_2 = 0.116$	$\tilde{\kappa}_1 = 0.124$	**Data**
$\beta_{\Delta y_t}$	0.00	0.11	0.16	**0.11**
β_2	-0.00	0.01	0.01	–
T-cov$(\Delta c_t, \Delta y_t)$	0.00	0.05	0.05	**0.00**
T-cov$(\Delta c_t, \Delta y_{t+1})$	0.00	0.89	0.92	–

Notes: In the table, we provide regression coefficients and their p values for the regression $\Delta c_t^i = \beta_0 + \beta' \Delta y^i + \epsilon_t^i$, with $\beta = [\beta_{\Delta y_t}, \beta_2]'$ and $\Delta y^i = [\Delta y_t^i, \Delta y_{t+1}^i]'$ for different precisions of signals. No signals is $\tilde{\kappa} = 0.00$. T-cov$(\Delta c_t, \cdot)$ reports p-values for the covariances to be significantly different from zero.[11]

knowledge of future income shocks, the authors employ household panel data from the Panel Study of Income Dynamics (PSID) to estimate correlations of current consumption growth $\Delta c_{i,t} = \log(c_t^i) - \log(c_{t-1}^i)$ with future income growth $\Delta y_{i,t+j} = \log(y_{t+j}^i) - \log(y_{t+j-1}^i)$ for $j \geq 1$. Through the lens of a *SIM*-model, if there was advance knowledge of income shocks, the correlation in the data should be significantly different from zero because consumption should adjust before the shock has occurred. However, Blundell, Pistaferri, and Preston (2008) estimate correlations that are not significantly different from zero with p-values larger than 25%.

The endogenous-solvency constraints model with information is consistent with that evidence. As reported in the first column of Table 3, the correlation of current consumption growth with future income growth is not significantly different from zero for the standard model with $\tilde{\kappa} = 0$. This pattern does not change for informative signals. As displayed in the second and third column, for $\tilde{\kappa}_1 = 0.124$ (yields the insurance ratio from the data) and for $\tilde{\kappa}_2 = 0.116$ (yields the regression coefficient from the data), only the correlation of current income growth and current consumption growth is significantly different from zero. Consistent with Blundell, Pistaferri, and Preston (2008), the correlations of current consumption growth with future income growth are not significantly different from zero with p-values larger than 89%. For the precision of information necessary to capture consumption insurance in the data, advance information in the *ESC* model does not induce counterfactual correlations of current consumption growth with future income growth.[12]

The logic for this result can be rationalized within the limited commitment endowment economy presented in Section 2. As summarized in Proposition 1, one key feature of the (constrained) efficient allocation is that conditional on a particular income-signal pair in the

[12] This result also applies for more precise signals. In an endowment economy, we compute p-values larger than 60% even when information precision is as high as $\kappa = 0.90$.

first period consumption is perfectly smoothed across both income states in the second period but also across time, decoupling consumption from future income realizations. Thus, the planner encourages high-income agents with binding enforcement constraints to transfer resources today in exchange for insurance of income shocks in the future. When signals become more precise, the outside option becomes more attractive for agents with a high income and it becomes more difficult for the planner to generate transfer to less fortunate agents. The efficient way to facilitate these transfers is to continue to promise consumption smoothing across time and states and to increase the level of consumption for high-income agents but not to break up the decoupling of consumption from future income realizations. Consequently, current consumption remains decoupled from future income realizations, and current consumption growth is not correlated with future income growth even when signals become more precise.

Income-conditional distribution of consumption To compare conditional moments from the data and models, our procedure is the following. We start with the stationary distribution of income implied by the Tauchen and Hussey (1991)'s procedure and compute the conditional mean and variance corresponding to this stationary distribution in each model. For the data, we employ the percentiles from the stationary income distribution and compute the moments for the percentiles, accordingly. For our calibration, this corresponds to the following percentiles: $[17th, 33th, 50th, 67th, 83th]$. For example, households with a high income represent the top 17% of income earners.

Insurance is close to perfect in the endogenous-solvency constraints model without signals. To facilitate a fair comparison, we employ the results derived in the endowment economy for the standard model.[13]

In Figure 1, we plot the conditional mean of log consumption in the data, in the standard model without signals and for informative signals of precision $\tilde{\kappa}_2 = 0.116$. In the absence of signals, the average consumption of low-income households is too high compared to the data while the consumption of high-income agents is too low. Further, indicating also too much insurance for low-income states, average consumption is constant for the two low-income groups in the absence of information; in the CEX data, average consumption is increasing for all income states. With informative signals, household consumption becomes more dispersed. Consistent with the first part of Proposition 1, we find that average consumption of low-income

[13] Alternatively employing the standard model with the possibility to return from autarky to insurance as in Broer (2013) yields similar conditional consumption moments and are available on request.

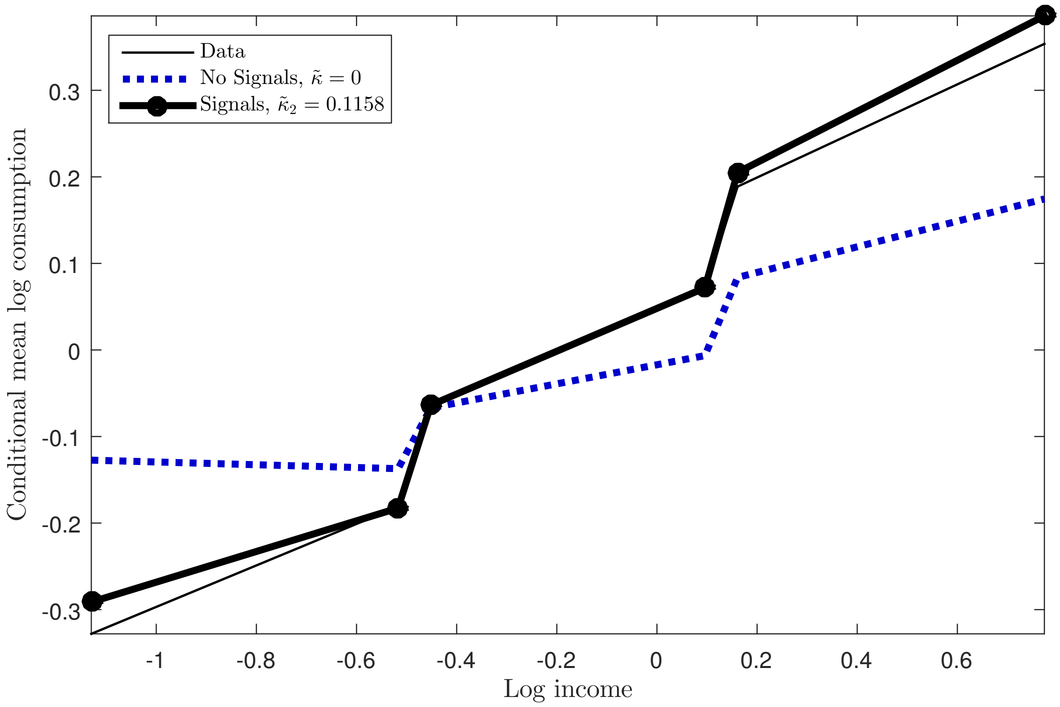

Figure 1: *ESC* model. Conditional mean of logged consumption with respect to logged income for different precisions of signals. The $x-$axis captures the log income and $y-$axis represents the conditional mean of log consumption. Income steps represent percentiles: $[17th, 33th, 50th, 67th, 83th]$. Solid line captures the conditional means for the years 1999–2003 in the CEX.

Table 4: Conditional moments of consumption: *ESC* model

	$\tilde{\kappa} = 0$	$\tilde{\kappa}_2 = 0.116$	$\tilde{\kappa}_1 = 0.124$	**Data**
MSE, $\mathrm{E}\left[\log(c)\vert y\right], no$	34.15	1	4.67	–
$\mathrm{E}\left[\log(c)\vert y_h\right] - \mathrm{E}\left[\log(c)\vert y_l\right]$	0.30	0.68	0.80	**0.68**
MSE, $\mathrm{STD}\left[\log(c)\vert y\right], no$	3.48	1	0.91	–
$\frac{\mathrm{STD}[\log(c)\vert y_h]}{\mathrm{STD}[\log(c)\vert y_l]}$	0	0.38	0.36	**0.95**

Notes: The table provides the mean squared deviations of model and data for the conditional means and standard deviations of consumption expressed relative to signals with $\tilde{\kappa}_2 = 0.116$, normalized *no*; the table also provides spreads between average consumption and the standard deviation of low-and high income households. No signals is $\tilde{\kappa} = 0$.

households decreases while consumption of high-income households increases, leading to a more dispersed consumption distribution and a better fit to the data. Further and as in the data, the conditional mean of consumption is increasing in income over all incomes states. Overall, the conditional mean of consumption is tracked in an almost perfect way for informative signals over all six income groups.

The second part of Proposition 1 suggests that more precise signals increase the income-conditional standard deviations of consumption. As displayed in Figure 2, advance information indeed results in a higher conditional standard deviation for all income groups. In particular,

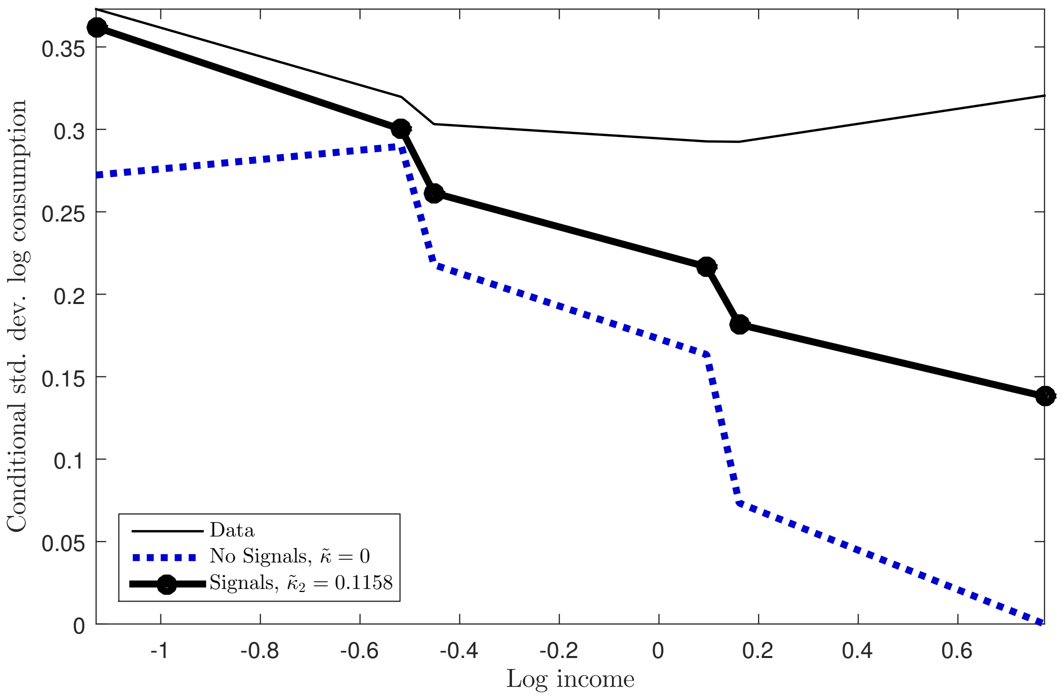

Figure 2: *ESC* model. Conditional standard deviation of logged consumption with respect to logged income for different precisions of the signals. The $x-$axis captures the log income and the $y-$axis the conditional standard deviation of logged consumption. Income steps represent percentiles: $[17th, 33th, 50th, 67th, 83th]$. Solid line captures the conditional standard deviations for the years 1999–2003 in the CEX.

information leads to an increase in consumption dispersion conditional on a high income; in the absence of information, the standard deviation is zero while with informative signals it is positive and increasing in precision. With informative signals, the conditional standard deviation is tracked reasonably well for low-and middle-income earners but the distance to the data increases for the high-income groups.

Quantitatively, the fit of the conditional consumption distribution to the data is substantially improved by advance information. As displayed in Table 4, the mean-squared deviations of the conditional mean of consumption between model and data are approximately 34 times as large in the standard model than for $\tilde{\kappa}_2 = 0.116$; for $\tilde{\kappa}_1 = 0.124$, the mean deviations are 4.5 times higher than for $\tilde{\kappa}_2 = 0.116$ but still over 7 times lower than in the standard model. Further, the spread between average consumption of high-and low income households in the CEX data of 0.68 is perfectly captured by signals with $\tilde{\kappa}_2 = 0.116$.

There is also some improvement in fit for the conditional standard deviation of consumption but the improvement is not as striking as for the conditional mean. Relative to the standard model, the mean-square error is 3.5 times smaller for $\tilde{\kappa}_2 = 0.116$, and approximately 4 times smaller for $\tilde{\kappa}_1 = 0.124$. Further, the ratio of the conditional standard deviations for high-and low-income households increases from 0 in the standard model to 0.4 with advance information. This increase is however too small to capture the ratio of almost 1 observed in the CEX.

5.3 Standard incomplete markets model

In the following, we consider advance information in a standard incomplete markets (SIM) model.

Environment While preferences and endowments are as described in Section 3, households in the standard incomplete markets economy can only trade in a single non-state contingent bond with gross return R and face an exogenous borrowing limit $\bar{a}$. There are no enforcement frictions and we directly focus on stationary allocations. The model we consider is similar to Huggett (1993) and relies on a market structure with a continuum of households as in Aiyagari (1994). Given asset holdings a, state $s = (y, k)$, and an interest rate R, households' problem can be written recursively as

$$V(a, s) = \max_{c, a'} \left[(1 - \beta) u(c) + \beta \sum_{s'} \pi(s'|s) V(a', s') \right]$$

subject to a budget and a borrowing constraint

$$\begin{aligned} c + a' &\leq wy + Ra \\ a' &\geq -\bar{a}. \end{aligned}$$

Here, households differ with respect to initial asset asset holdings and initial shocks where the heterogeneity is captured by the probability measure $\Psi_{a,s}$. The state space is given by $M = A \times S$, where $A = [-\bar{a}, \infty)$.

The stationary recursive competitive equilibrium is summarized in the following definition.

Definition 2 *A stationary recursive competitive equilibrium in the standard incomplete markets economy comprises a value function $V(a, s)$, prices R, w, an allocation $c(a, s), a'(a, s), K$ a joint probability measure of assets and the state $\Psi_{a,s}$, and an exogenous borrowing limit $\bar{a}$ such that*

(i) $V(a, s)$ is attained by the decision rules $c(a, s)$, $a'(a, s)$ given R

(ii) The joint distribution of assets and state $\Psi_{a,s}$ induced by $a'(a, s)$ and P_s is stationary.

(iii) Factor prices satisfy

$$R - 1 = AF_K(1, K) - \delta$$
$$w = AF_L(1, K)$$

(iv) The bond market clears

$$\int a'(a, s)\,\mathrm{d}\,\Psi_{a,s} = K'.$$

Households are restricted to trading a single non-state contingent asset. For this reason, one convenient feature of the (SIM) model is that the distinction between public or private information is irrelevant.

Quantitative results As emphasized by Blundell, Pistaferri, and Preston (2008) and Kaplan and Violante (2010), in a SIM model better information on future income realizations allows households to improve on their consumption-savings decisions, and risk sharing improves. Thus, better information has a positive effect by improving individual decision which is referred to as a Blackwell (1953) effect of information. For generating the quantitative results, we employ for the common parameters the same parameter values as in the corresponding limited commitment economy. Wolff (2011) finds that 19% of all US households are borrowing constrained. For this reason, we choose an exogenous borrowing limit $\bar{a}$ to yield in equilibrium 19% borrowing-constrained households in the standard model without information.

In line with earlier findings by Kaplan and Violante (2010), we find that insurance ratios improve monotonically in information precision but the improvement is too small to capture the risk-sharing ratio of 0.60 observed in the data even for very informative signals. In the absence of signals, the model implies that households share about 40% of all fluctuations in their after-tax income. As an extreme case, if information precision amounts to $\kappa = 0.99$ – corresponding to a reduction of income uncertainty $\tilde{\kappa}$ of 97% – the risk-sharing ratio reaches 0.51. Thus, the increase in risk sharing by better information is quantitatively too small to capture the insurance observed in CEX data.

The simulation results for the regression coefficient displayed in Table 5 confirm the findings from the first insurance measure. For the standard case of uninformative signals, current consumption growth reacts with a coefficient of 0.32 too sensitively to changes in current income. With better information, the sensitivity decreases to 0.29 for $\tilde{\kappa} = 0.21$ as upper value estimated

Table 5: Income and consumption growth regression: SIM model

	$\tilde{\kappa} = 0.00$	$\tilde{\kappa} = 0.12$	$\tilde{\kappa} = 0.21$	$\tilde{\kappa} = 0.78$	**Data**
$\beta_{\Delta y_t}$	0.32	0.31	0.29	0.20	**0.11**
β_2	-0.03	-0.01	0.01	0.12	–
T-cov$(\Delta c_t, \Delta y_t)$	0.00	0.00	0.00	0.01	**0.00**
T-cov$(\Delta c_t, \Delta y_{t+1})$	0.74	0.91	0.94	0.10	–

Notes: SIM model. In the table, we provide regression coefficients and their p values for the regression $\Delta c_t^i = \beta_0 + \beta' \Delta y^i + \epsilon_t^i$, with $\beta = [\beta_{\Delta y_t}, \beta_2]'$ and $\Delta y^i = [\Delta y_t^i, \Delta y_{t+1}^i]'$. No signals is $\tilde{\kappa} = 0.00$. T-cov$(\Delta c_t, \cdot)$ reports p-values for the covariances to be significantly different from zero.

by Dominitz (1998). Even for a very high $\tilde{\kappa} = 0.97$, the coefficient $\beta_{\Delta y_t}$ is with a value of 0.17 too high compared to the data. As signals become informative, the SIM model predicts that current consumption growth is counter-factually correlated with future income growth. For uninformative signals and informative signals with precisions below $\tilde{\kappa} = 0.78$, current consumption growth is uncorrelated with income growth one period ahead on a 10% significance level (see the first three columns). However, the regression coefficient of current consumption with current income growth of 0.20 is still too high compared to the 0.11 estimated in the data. From $\tilde{\kappa} = 0.78$ onwards, the correlation of current consumption growth with income growth one period in the future is statistically significantly different from zero and with a coefficient of $\beta_2 = 0.12$ also economically significant (see the fourth column). The non-zero correlation is inconsistent with the evidence provided in Blundell, Pistaferri, and Preston (2008) who find correlations of current consumption growth with future income growth not significantly different from zero.

The logic behind the non-zero correlation of current consumption with future income growth in the SIM model can be rationalized as follows. In the SIM model, better information reduces directly the income fluctuations households want to insure and thus decreases their incentives for precautionary savings. Knowing more about future income further allows households to better self-insurance their income risk. As a consequence, before the income shock realizes, households' consumption today reacts to the part of the future income shock that is known in advance, and consumption today is correlated with future income when signals become precise enough.

For $\tilde{\kappa} = 0.78$ as the highest value for $\tilde{\kappa}$ that yields no counterfactual correlation of current consumption with future income growth, the risk-sharing ratio falls with 0.50 however short compared to the 0.60 observed in the data.

6 Conclusions

In this paper, we have developed a framework to address the issue of a potential disconnect between households' income uncertainty and the income uncertainty as measured by an econometrician raised by Browning, Hansen, and Heckman (1999) and Cunha and Heckman (2016). To that end, we have developed a consumption risk-sharing model that can distinguish between the two types of uncertainties in a systematic and consistent way.

To quantify the difference in the perception of uncertainty, we have employed a general equilibrium model with endogenous borrowing constraints. Using US micro data, we have found that there is a systematic uncertainty gap: households' perceived income uncertainty is 12% lower than the uncertainty estimated by an econometrician that is typically used in consumption risk sharing models. For this uncertainty gap, the model jointly explains three distinct consumption risk-sharing and insurance measures that are not captured in the absence of advance information: (i) the cross-sectional variance of consumption, (ii) the covariance of consumption with income growth, and (iii) the income-conditional mean of household consumption. Further, the model performs well across several over-identifying restrictions test and the uncertainty gap of 12% is also consistent with direct estimates on the predictive value of subjective expectations in forecasting earnings.

With their recent paper, Heathcote, Storesletten, and Violante (2016) contribute to a lively debate on the optimal progressivity of taxes in the United States. One of the main arguments in favor for a progressive tax system is that it helps to insure idiosyncratic earnings uncertainty when private insurance is limited. Thereby, a higher tax progressivity reduces the earnings risk after taxes. Computing the optimal tax progressivity requires a precise estimate for households' earnings uncertainty. In particular, if there is a systematic uncertainty gap as suggested in this paper and income uncertainty is actually lower than what is typically considered, less tax progressivity might be desirable than conventional wisdom suggests.

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A Appendix

A.1 Proof of Proposition 1

The first order conditions for agents with a low income and a high signal in the first period are

$$\frac{u'(c_{h,1}^l)}{4} - \frac{\lambda_1^{rs}}{4} = 0, \quad \kappa\frac{u'(c_{h,2}^{lh})}{4} - \kappa\frac{\lambda_2^{rs}}{4} = 0, \quad (1-\kappa)\frac{u'(c_{h,2}^{ll})}{4} - (1-\kappa)\frac{\lambda_2^{rs}}{4} = 0$$

Dividing through by κ and $(1-\kappa)$ implies that $c_{h,1}^l, c_{h,2}^{lh}, c_{h,2}^{ll}$ have the identical marginal effect on social welfare. Thus, as long as the amount of resources is identical in both periods, we get $\lambda_1^{rs} = \lambda_2^{rs}$ and thus $c_{h,1}^l = c_{h,2}^{ll} = c_{h,2}^{lh} = c_h^l$. The first order conditions for consumption of agents with a high income and a high signal in the first period can be written

$$\begin{aligned}
&\frac{u'(c_{h,1}^h)}{4} + \lambda_h^{h,pc}u'(c_{h,1}^h) - \frac{\lambda^{rs}}{4} = 0\\
&\kappa\frac{u'(c_{h,2}^{hh})}{4} + \lambda_h^{h,pc}\kappa u'(c_{h,2}^{hh}) - \kappa\frac{\lambda^{rs}}{4} = 0\\
&(1-\kappa)\frac{u'(c_{h,2}^{hl})}{4} + \lambda_h^{h,pc}(1-\kappa)u'(c_{h,2}^{hl}) - (1-\kappa)\frac{\lambda^{rs}}{4} = 0
\end{aligned}$$

It follows that $c_{h,1}^h = c_{h,2}^{hl} = c_{h,2}^{hh} = c_h^h$. In a similar way, we get $c_{l,1}^h = c_{l,2}^{hl} = c_{l,2}^{hh} = c_l^h$, $c_{l,1}^l = c_{l,2}^{ll} = c_{l,2}^{lh} = c_l^l$, and consumption of high-income agents follows directly from the binding participation constraints

$$2u(c_h^h) = u(e_{h,1}) + \kappa u(e_{h,2}) + (1-\kappa)u(e_{l,2}) \geq$$
$$2u(c_l^h) = u(e_{h,1}) + (1-\kappa)u(e_{h,2}) + \kappa u(e_{l,2}).$$

1. The conditional mean of consumption of high-income agents is $c^h = (c_h^h + c_l^h)/2$ such that the derivative of it with respect to κ is

$$\frac{\partial c^h}{\partial \kappa} = \frac{u(e_{h,2}) - u(e_{l,2})}{2} \left(\frac{1}{u'(c_h^h)} - \frac{1}{u'(c_l^h)} \right) \geq 0$$

From resource feasibility it follows immediately that the conditional mean of consumption for low-income agents decreases in κ.

2. The conditional mean of consumption of high-income agents increases because c_h^h increases by more than c_l^h decreases. Thus, the conditional mean increases by less than c_h^h such that $(c_h^h - c^h)^2$ and $(c_h^l - c^h)^2$ increase, and therefore also the conditional standard deviation of consumption of high-income agents. For low-income agents there are two cases, either both enforcement constraints are slack or the enforcement constraints of low-income agents with a high signal bind (for sufficiently high precision). In the first case, the conditional standard deviation is zero because consumption of low-income agents is independent from signal realizations, i.e., $c_h^l = c_l^l = c^l$. In the second case, it follows from the enforcement constraints that c_h^l is increasing in κ. From the first part, we get that the conditional mean of consumption of low-income agents decreases which implies that c_l^l decreases by more than c_h^l increases such that also the conditional standard of consumption for low-income agents increases in this case.

3. The unconditional mean of consumption in both periods equals $\bar{e} = (e_h + e_l)/2$ such that the unconditional variance is

$$\frac{1}{4} \left[(c_h^h - \bar{e})^2 + (c_l^h - \bar{e})^2 + (c_h^l - \bar{y})^2 + (c_l^l - \bar{e})^2 \right].$$

The first two terms increase in κ because c_h^h increases by more than c_l^h decreases (see the first part), irrespectively whether c_l^h is larger or smaller than $\bar{e}$. When enforcement constraints of low-income agents are all slack, $c_h^l = c_l^l = c^l$ decreases in κ (see the first part) such that the last two term collapse and increase in κ. Mean consumption of high-income agents is always larger than the income mean: enforcement constraints of high-income agents bind, for uninformative signals, $c_l^h = c_h^h = c^h > \bar{e}$, and increases in κ further increase c^h. Thus, $(c_h^l + c_l^l)/2 < \bar{e}$, and only $c_h^l > \bar{e}$ is possible. When enforcement constraints of low-income agents with a high signal bind, their consumption increases in

κ. However, only if $c_h^l < 0$, one of the last terms can decrease when κ increases. From the previous part, we get that c_l^l decreases by more than c_h^l increases such that the sum of the two last terms increases even when $c_h^l < 0$, and as a result the unconditional standard deviation of consumption increases in κ.

A.2 Signal processes and consistency

In this section, we analytically characterize the Markov process of signals that satisfies the two consistency requirements outlined in Section 3. The main messages in this section are first that in general the consistent stochastic process of signals depends on the stochastic process of endowments and on the precision of signals. In case of a symmetric transition matrix for endowments, consistent signal transition probabilities are independent of signal precision and consistency requires the signals to follow the same stochastic process as endowments.

A.2.1 Symmetric endowment transition matrix

In this subsection, we show that if and only if signals follow the same stochastic process as endowments, the two consistency requirements are satisfied and that this result does not depend on signal precision. If signals were to follow a different process then at least one of the requirements is violated. For the analytical results, we consider an endowment process with two values y_l and y_h and a symmetric transition between endowment states. The transition matrix for these two endowment states is given as

$$P = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

where rows represent the present endowment state and columns represent the future endowment states. For $p = 0.5$, endowment states are i.i.d.

Proposition 2 *Consider a Markov endowment process with transition matrix P and informative signals with $\kappa \in (0.5, 1)$.*

(i) If signals follow the same stochastic process as endowments then both consistency requirements are satisfied.

(ii) *Consider a Markov process for signals with transition matrix* $\tilde{P}$

$$\tilde{P} = \begin{bmatrix} \tilde{p} & 1-\tilde{p} \\ 1-\tilde{p} & \tilde{p} \end{bmatrix}$$

and $0 < \tilde{p} < 1$, $\tilde{p} \neq p$. *Then Consistency Requirement II is violated.*

Proof.

(i) When signals follow the same transition probabilities as endowments, the transition probabilities of s can be computed and are then summarized in the transition matrix P_s. For example, the probability of a low endowment and a low signals conditional on a low endowment and signal is

$$\pi\left(y' = y_l, k' = y_l | k = y_l, y = y_l\right) = p\frac{\kappa p}{(1-\kappa)(1-p) + p\kappa}.$$

The unique stationary distribution corresponding to the transition matrix P_s is given by

$$\pi(y,k) = \begin{bmatrix} \pi(y_l, k_l) \\ \pi(y_l, k_h) \\ \pi(y_h, k_l) \\ \pi(y_h, k_h) \end{bmatrix} = \begin{bmatrix} \kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} \\ \frac{\kappa}{2} + \frac{p}{2} - \kappa p \\ \frac{\kappa}{2} + \frac{p}{2} - \kappa p \\ \kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} \end{bmatrix}$$

Adding the first two and last two rows show that Consistency Requirement I is satisfied. Further, the probabilities of signals conditional on endowments can be computed from the invariant distribution. For example, the probability of a low signal conditional on a low endowment can be computed as

$$\pi(k = y_l | y = y_l) = \frac{\kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2}}{\kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} + \frac{\kappa}{2} + \frac{p}{2} - \kappa p} = 2\kappa p - \kappa - p + 1.$$

To check for the Consistency Requirement II, we consider present endowment $y = y_l$ and future endowment $y' = y_l$ (the other transitions can be computed in the same way and are omitted here)

$$
\begin{aligned}
&\hat{\pi}(y' = y_l | y = y_l) = \sum_{k \in Y} \pi(y' = y_l | y = y_l, k) \pi(k | y = y_l) \\
&= \frac{\kappa p}{\kappa p + (1-\kappa)(1-p)} (2\kappa p - \kappa - p + 1) + \frac{p(1-\kappa)}{\kappa(1-p) + p(1-\kappa)} (\kappa + p - 2\kappa p) \\
&= p
\end{aligned}
$$

which is also satisfied. From the other side, for the transition from low endowment today to low endowment in the future, Requirement II calls for

$$
p \doteq \sum_{k \in Y} \pi(y' = y_l | y = y_l, k) \hat{\pi}(k | y = y_l),
$$

which has as unique solution $\hat{\pi}(k = y_l | y = y_l) = 2\kappa p - \kappa - p + 1$ which completes the proof of part (i).

(ii) The general symmetric transition matrix for signals $\tilde{P}$ results in a joint transition matrix for signals and endowments $\tilde{P}_s$ and in a unique invariant distribution for endowment and signals $\tilde{\pi}(y, k)$ with a unique conditional probability $\tilde{\pi}(k = y_l | y = y_l)$. If an only if $\tilde{p} = p$, it is $\tilde{\pi}(k = y_l | y = y_l) = \hat{\pi}(k = y_l | y = y_l) = 2\kappa p - \kappa - p + 1$. Thus, Requirement II is violated for $\tilde{p} \neq p$. Requirement I is satisfied because $\sum_k \tilde{\pi}(y_l, k) = 1/2 = \sum_k \tilde{\pi}(y_h, k)$ for any $0 < \tilde{p} < 1$.

■

As an immediate implication of the proposition, i.i.d. signals violate Requirement II when endowments are persistent.

A.2.2 Non-symmetric endowment transition matrix

We continue our analysis with considering the case of non-symmetric endowment transitions. As before, we consider a two-state endowment process but now the endowment transition matrix is more general and given by

$$
P_g = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}
$$

where rows represent the present endowment state and columns represent the future endowment states, $0 < p_{11}, p_{22} < 1$, and $p_{11} \neq p_{22}$.

Proposition 3 *Consider a Markov endowment process with transition matrix P_g and informative signals with $\kappa \in (0.5, 1)$.*

(i) The transition matrix for signals that satisfies Consistency Requirement II is

$$P_k = \begin{bmatrix} p_{11}^k & 1 - p_{11}^k \\ 1 - p_{22}^k & p_{22}^k \end{bmatrix}$$

with $p_{11}^k = p_{22}(1-\kappa) + \kappa p_{11} \quad p_{22}^k = p_{11}(1-\kappa) + \kappa p_{22}$.

(ii) Signals that follow the transition P_k also satisfy Consistency Requirement I.

Proof.

(i) The logic of the proof is to treat the signal transition probabilities p_{11}^k, p_{22}^k as unknown and use the two equation imposed by second consistency requirement to solve for these probabilities. To satisfy the second consistency requirement the following two equations must be satisfied

$$p_{11} = \sum_{k \in Y} \pi(y' = y_l | y = y_l, k) \pi(k | y = y_l) \tag{18}$$

$$p_{22} = \sum_{k \in Y} \pi(y' = y_h | y = y_h, k) \pi(k | y = y_h). \tag{19}$$

The conditional probabilities $\pi(k|y = y_l), \pi(k|y = y_h)$ are functions of the signal transition probabilities, the conditional probabilities of a high and low income are given by the formulas in the text and does not depend on the signal transition probabilities. Solving first for the invariant distribution of income and signals as a function of p_{11}^k, p_{22}^k. From there, the conditional probabilities $\pi(k|y = y_l), \pi(k|y = y_h)$ can be computed in several steps resulting in tedious expressions that are not reported here. Substituting these expressions in (18) and (19) and solving for p_{11}^k, p_{22}^k eventually gives

$$p_{11}^k = p_{22}(1-\kappa) + \kappa p_{11} \quad p_{22}^k = p_{11}(1-\kappa) + \kappa p_{22}$$

if the following two regularity conditions hold

$$p_{22}(1-\kappa)+\kappa p_{11}<1 \quad p_{11}(1-\kappa)+\kappa p_{22}<1$$

which are satisfied for $\kappa \in [0,1]$ and $0<p_{11},p_{22}<1$.

(ii) The invariant distribution of income (y_l, y_h) is given by

$$\pi(y_l,y_h)=\left(\frac{1-p_{22}}{2-p_{11}-p_{22}},\frac{1-p_{11}}{2-p_{11}-p_{22}}\right).$$

Using the expressions for p_{11}^k, p_{22}^k and from part (i), results in the following invariant signal-income distribution

$$\pi(y,k)=\begin{bmatrix}\pi(y_l,k_l)\\ \pi(y_l,k_h)\\ \pi(y_h,k_l)\\ \pi(y_h,k_h)\end{bmatrix}=\begin{bmatrix}-\frac{(1-p_{22})(\kappa+p_{11}-2\kappa p_{11}-1)}{2-p_{11}-p_{22}}\\ \frac{(1-p_{22})(\kappa+p_{11}-2\kappa p_{11})}{2-p_{11}-p_{22}}\\ -\frac{(1-p_{11})(\kappa+p_{22}-2\kappa p_{22}-1)}{2-p_{11}-p_{22}}\\ \frac{(1-p_{11})(\kappa+p_{22}-2\kappa p_{22})}{2-p_{11}-p_{22}}\end{bmatrix}.$$

Adding the first two and the last two rows produces $\pi(y_l, y_h)$ such that the first consistency requirement is satisfied as well.

■

The results summarized in the proposition generalize the findings for symmetric transitions. The signal transition matrix depends in general on the precision of signals. Only when the income transition is symmetric, the transition probabilities for signals are independent of κ and are given by the corresponding income transition probabilities.

Unlike in the case of symmetric income transition, i.i.d. signals now neither satisfy the first nor the second consistency requirement. The rationale why now also the first requirement is violated is as follows. Without loss of generality, consider $p_{11}>p_{22}$ such that the ergodic distribution is characterized by $\pi(y_l)>\pi(y_h)$. With $p_{11}>p_{22}$, a larger fraction of households with a low income should receive a low signal than households with a high income receive a high signal. For i.i.d. signals, the fractions are equal. As a consequence, households underestimate the fraction of people with a low income and over estimate the fraction of households with a high income.

	Consistency requirements	
	I, $\max(\lvert\widehat{\pi}(y) - \pi(y)\rvert)$	II, $\max(\lvert\widehat{\pi}(y'\lvert y) - \pi(y'\lvert y)\rvert)$
i.i.d. signals	0.0115	0.1620
consistent signals	$3.70e-16$	$2.11e-16$

Table 6: Consistency requirement results with persistent endowments

For $N > 2$, we apply a numerical procedure. For each κ, we use the $N^2 - N$ restrictions imposed by Consistency Requirement II to solve for the transition probabilities p_{ij}^k. Then we check whether the first consistency requirement is satisfied given the probabilities p_{ij}^k. In Table 6, we also compare both signal processes using the endowment process employed for computing the quantitative results in the main text for $\kappa = 0.99$ as an extreme case. As displayed in the first row of the table, i.i.d. signals fail both consistency requirements. The inconsistency following from i.i.d. signals is not negligible. On average, i.i.d. signals imply a perceived transition that differs from the true transition by 16%. When we compute signal transition probabilities according to the numerical procedure, the second requirement is satisfied by construction, while the first requirement – in line with the second part of Proposition 3 – is satisfied, too.

A.3 Risk sharing with private signals

Consider the two-period exchange economy described in Section 2 but with signals on agents' future income realizations that are only observed by the agents.[14] Let $c_{i,1}^j$ be first-period consumption of agents with reported private signal n^i and endowment e_j and $c_{i,2}^{jk}$ second-period consumption of agents with reported private signal n_i and endowment e_j in the first period and endowment e_k in the second period with $i, j, k \in \{l, h\}$. We focus on allocations with truthfully reported private signals. Let ν denote the precision of private signals.

The enforcement and resource feasibility constraints are again given by (2)-(5) and the resource constraints (6)-(7) with $\kappa = \nu$. Private information gives rise to another set of incentive constraints, truth-telling constraints that are given by the following expressions for high-income agents with a good and bad private signal

$$u(c_{h,1}^h) + \nu u(c_{h,2}^{hh}) + (1-\nu)u(c_{h,2}^{hl}) \geq u(c_{l,1}^h) + \nu u(c_{l,2}^{hh}) + (1-\nu)u(c_{l,2}^{hl}) \tag{20}$$

$$u(c_{l,1}^h) + \nu u(c_{l,2}^{hl}) + (1-\nu)u(c_{l,2}^{hh}) \geq u(c_{h,1}^h) + \nu u(c_{h,2}^{hl}) + (1-\nu)u(c_{h,2}^{hh}) \tag{21}$$

[14] Broer, Kapička, and Klein (2017) consider a limited commitment model in which household income is unobservable.

and for low-income agents with a good and bad private signal

$$u(c^{l}_{h,1}) + \nu u(c^{lh}_{h,2}) + (1-\nu)u(c^{ll}_{h,2}) \geq u(c^{l}_{l,1}) + \nu u(c^{lh}_{l,2}) + (1-\nu)u(c^{ll}_{l,2}) \tag{22}$$

$$u(c^{l}_{l,1}) + \nu u(c^{ll}_{l,2}) + (1-\nu)u(c^{lh}_{l,2}) \geq u(c^{l}_{h,1}) + \nu u(c^{ll}_{h,2}) + (1-\nu)u(c^{lh}_{h,2}) \tag{23}$$

An *efficient allocation* is a consumption allocation, $\{c^{j}_{i,1}, c^{jk}_{i,2}\}$, that maximizes ex-ante utility (1), subject to the enforcement constraints (2)-(5) and the resource constraints (6)-(7) with $\kappa = \nu$, and truth-telling constraints (20)-(23).

With private information, consumption cannot be perfectly smoothed across states and both time periods conditional on the income-signal pair in the first period because of truth-telling. Agents with a low private signal are discouraged to report a high-signal type by threatening them with a particular low consumption for high-private signal households in case of a low income in the second period. To compensate for this lack of insurance, efficient allocations prescribe a high consumption in case of a high income in the second period to high-signal households. This however makes smoothing across states and time impossible.

As illustrated in Figure 3, we find that numerically increases in private-signal precision lead to qualitatively similar changes in unconditional moments and welfare as summarized in Proposition 1 for public signals. While welfare decreases, volatility of consumption increases when signals become more precise. Compared to public information, private information introduces additional welfare costs for informative signals. For this reason, welfare is lower and consumption is more dispersed with private than with public signals.

A.4 Joint distribution of endowments and signals

In this subsection, we explain how to derive the formulas (9) and (10) stated in the main text. Further, we explain the logic behind the assumption that the stochastic process for signals shares the transition probabilities with the process for individual endowments of effective labor units.

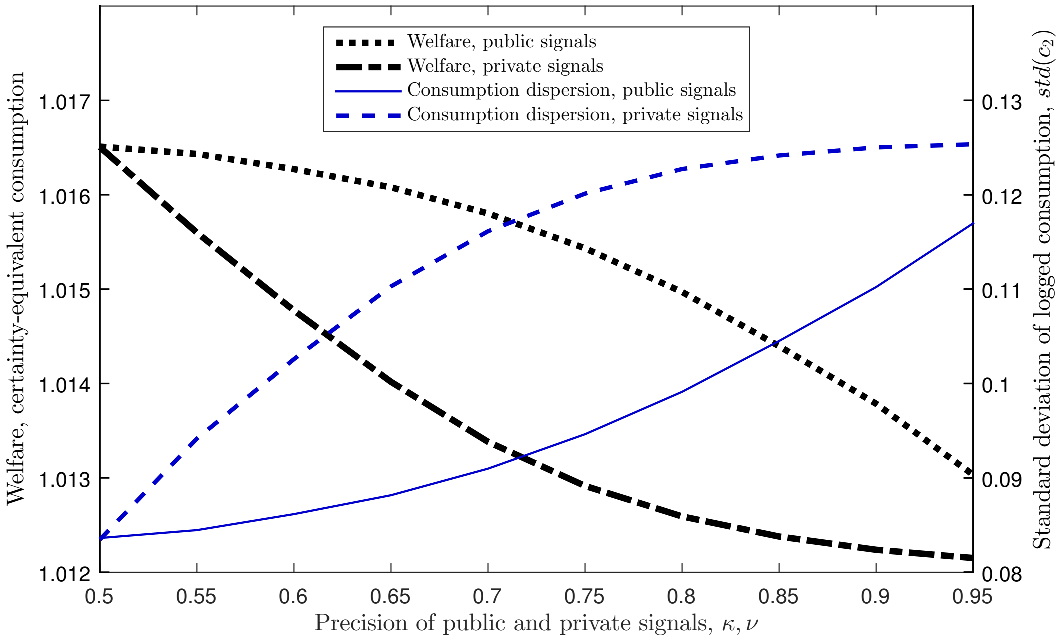

Figure 3: Two-period model. Welfare and consumption dispersion as functions of public and private information.

A.4.1 Formulas on the joint distribution of endowments and signals

We start with the derivation of the conditional probability of future endowments. Using the general formula for calculating conditional probabilities, we receive

$$\pi\left(y' = y_j | k = y_m, y = y_i\right) = \frac{\pi\left(y' = y_j, k = y_m, y = y_i\right)}{\pi\left(k = y_m, y = y_i\right)}.$$

The conditional probability can be simplified using the identity

$$\sum_{z=1}^{N} \pi\left(y' = y_z | k = y_m, y = y_i\right) = 1$$

to replace the denominator with the following expression

$$\pi\left(k = y_m, y = y_i\right) = \sum_{z=1}^{N} \pi\left(y' = y_z, k = y_m, y = y_i\right).$$

The joint probability in the numerator is

$$\pi\left(y' = y_j, k = y_m, y = y_i\right) = \pi_{ij} \kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}},$$

where π_{ij} is the Markov transition probability for moving from endowment i to endowment z. For all endowment states that are not indicated by the signal, $j \neq m$, we assume here that their probability of occurrence conditional on the signal is identical and therefore equals $(1-\kappa)/(N-1)$. For the conditional probability of endowments, the general formula can then be written as

$$\pi\left(y' = y_j | k = y_m, y = y_i\right) = \frac{\pi_{ij}\kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N}\pi_{iz}\kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \tag{24}$$

which resembles (9) in the main text. For example, with two equally likely persistent endowment states, the conditional probability of receiving a low endowment y_l in the future conditional on a high signal $k = y_h$ and a low endowment today is given according to (24) by

$$\pi\left(y' = y_l | k = y_h, y = y_l\right) = \frac{(1-\kappa)\pi_{11}}{(1-\kappa)\pi_{11} + (1-\pi_{11})\kappa}.$$

The joint transition probability $\pi(s'|s) = \pi\left(y', k'|k, y\right)$ can be computed by combining the conditional probability of income with an assumption on the signal process. With signals following an exogenous first-order Markov process, the conditional probability $\pi\left(y', k'|k, y\right)$ is given by

$$\pi\left(y' = y_j, k' = y_l | k = y_m, y = y_i\right) = \pi_{ml}\frac{\pi_{ij}\kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N}\pi_{iz}\kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \quad \forall k', \tag{25}$$

where compared to (10), we used $\pi(k' = y_l | k = y_m) = \pi_{ml}$ because the signal process is characterized by the same transition probabilities as endowments. In the following, we argue why we choose signals that share the transition probabilities with individual endowments.

A.5 Numerical algorithm

Given initial wealth a, state $s = (y, k)$, and an interest rate R, households' problem can be written recursively as

$$V(a,s) \quad = \quad \max_{c,\{a'\}}\left\{(1-\beta)u[c(a,s)] + \beta\sum_{s'}\pi(s'|s)V'[a'(a,s;s'),s']\right\}$$

subject to a budget and a borrowing constraint

$$c + \sum_{s'} \frac{\pi(s'|s)a'(a,s;s')}{R} \leq y + a \tag{26}$$

$$a'(a,s;s') \geq A(s'), \quad \forall s'. \tag{27}$$

The borrowing limits satisfy the following equations

$$U^{Aut}(s') = V'[A(s'), s'], \quad \forall s'. \tag{28}$$

The first order conditions are

$$u'[c(a,s)](1-\beta) = \lambda = V_a(a,y) \tag{29}$$

$$\beta V_a'[a'(a,s;s'), s'] \leq \frac{u'[c(a,s)](1-\beta)}{R}, \quad \forall s', \tag{30}$$

where $V_a'[a'(a,s;s'),s']$ denotes the derivative of the value function with respect to $a'(a,s;s')$. Consider N income states such that $s \in S = (s_1, s_2, ...s_{N^2})$. Consider a grid for a. Start with a guess of the value function V_0 and for the derivative $V_{a,0}$. From the guess of the value function, back out the state-dependent borrowing limits $A_0(s')$ from (28).

1. For each pair a, s, solve for the policy functions $c_0(a,s)$, $\{a_0'(a,s;s')\}$ using the N^2+1 first order conditions (30) and (26). Start with the strict equality for all s' and solve. Check borrowing constraints. If not satisfied in some state s', set $a_0'(a,s;s') = A_0(s')$ and solve again for $c_0(a,s)$ and the remaining $a_0'(a,s;s')$ until no borrowing constraint is violated.

2. Update the derivative of the value function with respect to a using the envelope condition and the policy function for consumption

$$V_{a,1}(a,s) = u'[c_0(a,s)](1-\beta)$$

3. Update the value function according to the Bellman equation to receive V_1

$$V_1(a,s) = (1-\beta)u[c_0(a,s)] + \beta \sum_{s'} \pi(s'|s) V_0[a_0'(a,s;s'), s']$$

4. Continue until convergence in the policy functions, the derivative of the value function

and in the value function $V_n(a,s) = V_{n+1}(a,s) = V(a,s)$ is achieved.

5. Then update the borrowing limits solving the following equation for A_1

$$V[A_1(s'), s)] = U^{aut}(s').$$

6. Continue until convergence in the policy functions, in the value function (and its derivative) and in the borrowing limits is achieved.

In the next step, use the policy functions $\{a'(a,s;s')\}$ and transition probabilities $\pi(s'|s)$ to define an operator T that maps the current probability measures for assets and the income-signal state into future measures. In the next step, compute the unique fixed point of the operator T and denote it by $\Phi_{a,s}$, the invariant distribution of assets and income-signal states. Using the invariant distribution compute the excess demand

$$d_K(\beta) = \int c(a,s)\,\mathrm{d}\,\Phi_{a,s} + K' - K(1-\delta) - AF(L,K).$$

and check whether it is satisfied. If not, decrease β if $d_K(\beta)$ is in surplus and increase β if it is in deficit, and go back to Step 1. We use a Ridder algorithm until convergence on the discount factor is achieved and excess demand equals zero.