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**NETWORK-BASED MACRO FLUCTUATIONS:  
EVIDENCE FROM LITHUANIA**

## **NETWORK-BASED MACRO FLUCTUATIONS: EVIDENCE FROM LITHUANIA**

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## Abstract

Do inter-sectoral linkages of intermediate products affect the spread of sectoral shocks at the aggregate level in Lithuania, a small and open economy? We answer this question by: i) constructing the domestic sector-by-sector direct requirements table using the Lithuanian interindustry transactions tables, and ii) applying [Acemoglu et al. \(2012\)](#)'s network-based methodology and [Gabaix and Ibragimov \(2011\)](#)'s modified log rank-log size regression to analyse the nature of inter-sectoral linkages. Our results indicate that the direct and indirect inter-sectoral linkages cause aggregate volatility to decay at a rate lower than  $\sqrt{n}$  – the rate predicted by the standard diversification argument. Furthermore, indirect linkages play an important role in the above-mentioned process, supporting the findings of [Acemoglu et al. \(2012\)](#). These results suggest that the inter-sectoral network of linkages represent a potential propagation mechanism for idiosyncratic shocks throughout the Lithuanian economy.

**JEL** codes: C13, C46, C67, E00.

**Keywords:** Input-Output Linkages, Inter-sectoral Network, Aggregate Volatility, Small-Open Economy, Complexity Economics

# 1 Introduction

According to [Lucas \(1977\)](#), microeconomic shocks would average out and are less likely to have a significant impact on aggregate variables due to the diversification argument. The diversification argument indicates that in the presence of microeconomic/independent/sectoral shocks, aggregate output concentrates around its mean at a rate  $\sqrt{n}$ , where  $n$  is the number of sectors in the economy. This indicates that when  $n$  gets larger, micro shocks become less important at the macro level and their impact vanishes quickly.

However, a growing literature, for instance [Carvalho \(2008\)](#), [Acemoglu et al. \(2010\)](#), [Gabaix \(2011\)](#), [Acemoglu et al. \(2012\)](#), [Carvalho and Gabaix \(2013\)](#), [Johnson \(2014\)](#), [di Giovanni et al. \(2014\)](#), and [Atalay \(2017\)](#), has argued that micro and sectorial shocks may have a non-negligible impact at aggregate level under specific circumstances. For instance, according to [Gabaix \(2011\)](#), firm-level shocks can transform into aggregate fluctuations if firm size distribution has a heavy tail and firms unequally contribute to the final aggregate output. [Acemoglu et al. \(2012\)](#), by taking input-output linkages into consideration, provide a novel network-based methodological approach. By using the US data, the authors show that sectorial shocks are spreading at the aggregate level and have a non-trivial aggregate impact due to existing input-output intermediate production networks and their unbalanced structure within the economy.

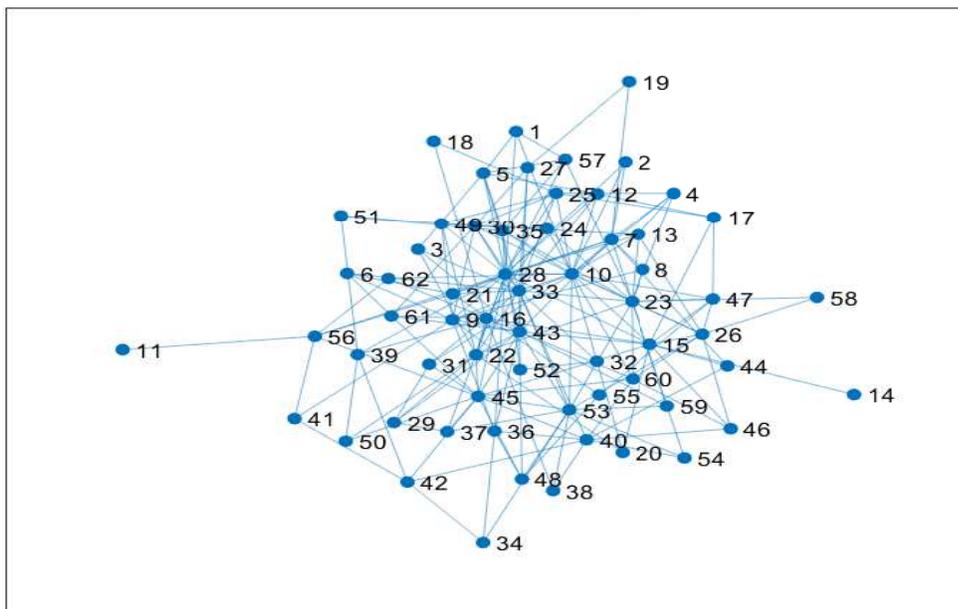
Network-based methodology has been used in analysing an increasing variety of economic and financial issues. One stream literature, such as [Gai and Kapadia \(2010\)](#), [Kali and Reyes \(2010\)](#), [Gourieroux et al. \(2012\)](#), [Elliott et al. \(2014\)](#), [Acemoglu et al. \(2015\)](#), analyzes contagions and cascades in financial networks, as well as the propagation of shocks through financial networks. Another stream, including [Kali and Reyes \(2010\)](#), [Chaney \(2014\)](#), [Reyes et al. \(2014\)](#), [Bosker and Westbrock \(2014\)](#) analyses the structure of networks of international trade and their impacts on the economies and world trade flows. Meanwhile, several studies such as [Oberfield \(2012\)](#), [Acemoglu et al. \(2012\)](#), [Caliendo et al. \(2014\)](#), [Carvalho et al. \(2014\)](#), [Johnson \(2014\)](#), analyse the relationship between different types of trade networks and the propagation of disaggregated shocks (such as changes of disaggregated productivity) to the rest of the economy. Furthermore, [Acemoglu et al. \(2016\)](#) and [Ozdagli and Weber \(2017\)](#) decompose the overall effect of various types of shocks into a direct effect and a

network effect and find that the later plays a larger role than the former.

This paper is closely related to [Acemoglu et al. \(2012\)](#) and [Gabaix \(2011\)](#). So far, the existing literature such as [Acemoglu et al. \(2012\)](#), [Carvalho and Gabaix \(2013\)](#), [Acemoglu et al. \(2016\)](#), [Ozdogli and Weber \(2017\)](#), has provided evidence based primarily on US data. This study investigates whether inter-sectoral linkages of intermediate products affect the spread of the sectoral shocks at the aggregate level in Lithuania. Specifically, we analyse whether industry-level shocks affect aggregate output via the network of intermediate products as argued by [Acemoglu et al. \(2012\)](#).

The main motivation for investigating the network effect in Lithuania, a small and open economy, comes from the unbalanced structure of its inter-sectoral network. [Fig. 1](#) represents the network of inter-sectoral linkages in 2010. Each node represents one of the 62 Lithuanian sectors (see [Appendix A](#) for the full list of sectors). If a sector purchases intermediate inputs from another sector for more than 1% of the value of its final output, the link between that producing sector and the sector of intermediate inputs is drawn. As presented in [Fig. 1](#), there are several sectors in Lithuania that are connected with a large number of other sectors via the production of intermediate inputs; such as *Wholesale trade services, except of motor vehicles and motorcycles* (sector 28), *Warehousing and support services for transportation* (sector 33), *Computer, electronic and optical products* (sector 16), and *Real estate activities excluding imputed rents* (sector 43), etc. On the other hand, at the same time there are some sectors that are weakly connected with other sectors via the production of intermediate inputs; such as *Basic pharmaceutical products and pharmaceutical preparations* (sector 11), *Basic metals* (sector 14), *Motor vehicles, trailers and semi-trailers* (sector 19) or *Creative, arts and entertainment services* (sector 58).

**Figure 1** – Intersectoral Network in Lithuania in 2010



Note: Figure presents the network of intersectoral linkages in Lithuania in 2010. Each node represents one of the 62 Lithuanian sectors. If a sector purchases intermediate inputs from another sector for more than 1% of the value of its final output, the link between that producing sector and the sector of intermediate inputs is drawn.

Our paper contributes to the literature by investigating the potential of inter-sectoral network to act as a propagation mechanism for idiosyncratic shocks in a small and open economy. It is worth mentioning that the US data, i.e. commodity-by-commodity direct requirements tables, used for this type of analysis, is derived from the commodity-by-commodity total requirements tables available from the Bureau of Economic Analysis. However, this type of data is not available for Lithuania, therefore we need to construct the domestic sector-by-sector direct requirements table using the Lithuanian interindustry transactions table.

Our results show that due to the first-order and second-order inter-sectoral connections, the aggregate volatility decays at the rate lower than  $\sqrt{n}$  – the one predicted by the standard diversification argument. Analysing this in more detail, due to the first-order inter-sectoral connections, the aggregate volatility decays at the rate smaller than  $n^{0.41}$  while taking into account the second-order connections, the aggregate volatility decays at the rate smaller

than  $n^{0.22}$ . Our results are in line with the argument that indirect linkages play a more important role in the propagation of shocks. Due to these connections and the unbalanced structure of the input-output intermediate production networks, sectoral shocks to the one of the dominant sectors would propagate through its downstream sectors and thus cause fluctuations at the aggregate level.

The paper is structured as follows. Section 2 presents the research methodology, based on [Acemoglu et al. \(2012\)](#) and [Gabaix and Ibragimov \(2011\)](#). Here we also present the calculation of domestic direct requirements table using Lithuanian interindustry transactions table at basic prices. Data availability allows us to analyse the inter-sectoral linkages between 62 industries in Lithuania. Section 3 presents the main empirical results. Lastly, section 4 concludes.

## 2 Methodology

### 2.1 [Acemoglu et al. \(2012\)](#)'s network-based methodology

In this section we briefly present [Acemoglu et al. \(2012\)](#)'s network-based methodology together with the modified log rank–log size regression introduced by [Gabaix and Ibragimov \(2011\)](#) to analyse the inter-sectoral linkages.

The theoretical model of [Acemoglu et al. \(2012\)](#) is based on the real business cycle's multi-sectoral model of [Long and Plosser \(1983\)](#). In this model, the representative household has inelastic one unit of labour and Cobb-Douglas preferences for different  $n$  goods as in Eq. (1):

$$u(c_1, c_2, \dots, c_n) = A \prod_{i=1}^n (c_i)^{1/n}, \quad (1)$$

where  $c_i$  presents consumption of good  $i$  and  $A$  is a normalisation constant.<sup>1</sup>

Competitive sectors produce goods in the economy that can be used as intermediate inputs by sectors for their production or consumed by final users. The output of sector  $i$ ,  $x_i$ , is given by:

$$x_i = z_i^\alpha l_i^\alpha \prod_{j=1}^n (x_{ij})^{(1-\alpha)w_{ij}} \quad (2)$$

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<sup>1</sup>In this model, the normalisation constant  $A$  affects only the mean of aggregate output without affecting aggregate volatility or any other distributional parameters. For further analysis regarding normalisation constant, see [Acemoglu et al. \(2012\)](#).

where  $l_i$  is labour input in sector  $i$ ,  $\alpha$  is a share of labour,  $x_{ij}$  presents the amount of good  $j$  used in the production of good  $i$ ,  $z_i$  is idiosyncratic productivity shock to sector  $i$ ,  $w_{ij}$  is the share of goods of sector  $j$  needed in the production of  $i$  goods.

The input-output table is used in this Cobb-Douglas function as  $w_{ij}$ 's, where it shows the needed expenditure on input  $j$  per dollar of output of sector  $i$ . Assumption  $\sum_{j=1}^n w_{ij} = 1$  in this model implies that sectoral production functions have constant returns to scale. Productivity shocks  $z_i$  are independent with  $\varepsilon_i = \log(z_i) \sim F_i$ . An economy is defined as  $\xi = (I, W, \{F_i\}_{i \in I})$ , where  $I$  denotes the set of sectors,  $W$  denotes the input-output matrix and  $F_i$  is described above.

With this specification, normalised aggregate output can be derived as:<sup>2</sup>

$$y \equiv \log(GDP) = v' \varepsilon, \quad (3)$$

where  $\log(GDP)$  is aggregate output, sectoral shocks  $\varepsilon \equiv [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]'$  and  $v$  is the  $n$ -dimensional influence vector.

The influence vector  $v$  is related to Bonacich centrality vector corresponding to the intersectoral network.<sup>3</sup> Sectors with higher centrality in the network are more important in determining aggregate output as these sectors have more connections, and shocks to these sectors might propagate to other sectors in the economy. On the other hand, sectors with low influence have little or no connections with other sectors. Therefore, shocks to these sectors might weakly influence other sectors in the economy. In detail, the influence vector  $v$  is written as:

$$v \equiv \frac{\alpha}{n} [I - (1 - \alpha)W']^{-1}. \quad (4)$$

Equations (3) and (4) imply that aggregate output depends on the network of intersectoral linkages via the Leontief inverse  $[I - (1 - \alpha)W']^{-1}$ . This term captures how idiosyncratic productivity shocks propagate downstream to other sectors through the input-output matrix.

In order to derive the aggregate volatility, we need the following assumptions regarding the sectoral level shocks:

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<sup>2</sup>Without normalisation constant  $A$ , the aggregate output would be equal to  $y = v' \varepsilon + \mu$ .

<sup>3</sup>According to [Bonacich \(1987\)](#), the most central sectors in the network have the most connections within the network. A number of connections within the network presents number of sectors that one particular sector is connected with.

**a.**  $\mathbb{E}(\varepsilon_{in}) = 0,$

**b.**  $\mathbb{E}(\varepsilon_{in}, \varepsilon_{jn}) = 0,$

**c.**  $\text{var}(\varepsilon_{in}) = \sigma_{in}^2 \in (\underline{\sigma}^2, \bar{\sigma}^2),$  where  $0 < \underline{\sigma} < \bar{\sigma}.$

Assumption (a) is for normalisation (the mean of shocks is equal to zero). Assumption (b) implies that all idiosyncratic productivity shocks are independent of each other. Assumption (c) implies that variance of idiosyncratic productivity shocks is bounded from zero when  $n \rightarrow \infty.$  While using assumptions (a) and (b) with Eq. (3), we can derive that:

$$(\text{var } y_n)^{1/2} = \sqrt{\sum_{i=1}^n \sigma_{in}^2 v_{in}^2} \quad (5)$$

where  $v_{in}$  denotes  $i^{\text{th}}$  element of  $v_n.$  With assumptions (b) and (c), we obtain:<sup>4</sup>

$$(\text{var } y_n)^{1/2} = \Theta(\|v_n\|_2), \quad (6)$$

where  $\|v_n\|_2 = \sqrt{\sum_{i=1}^n v_{in}^2}.$

Intuitively, Eq. (6) implies that aggregate volatility might decrease at different (slower) rate than  $\sqrt{n} -$  the rate predicted by the standard diversification argument. At the same time, a lower rate of decay suggests that sectoral shocks might lead to more persistent fluctuations at the aggregate level.

These effects may come from several sources: the first-order connections between sectors or higher-order inter-connectivities. Specifically, the first-order connections between sectors capture how shocks propagate from sector  $i$  to other sectors that are directly connected with the sector  $i$  and use  $i$ 's goods as inputs in their production. The larger number of sectors that use  $i$ 's goods as inputs, the larger the first-order effect. Meanwhile, the higher-order inter-connectivity captures how shocks propagate from sector  $i$  to those sectors that are using inputs of the sectors using  $i$ 's goods as inputs in their production. Such higher-order inter-connectivities are referred to as the second-order connections between sectors. In the next two sections we describe the first-order and second-order connections in detail.

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<sup>4</sup> $y_n = \Theta(x_n)$  if  $\limsup_{n \rightarrow \infty} y_n/x_n < \infty$  and  $\liminf_{n \rightarrow \infty} y_n/x_n > 0,$  when  $\{y_n\}_{n \in \mathbb{N}}$  and  $\{x_n\}_{n \in \mathbb{N}}$  are sequences of real positive numbers.

## 2.2 First-order degree interactions

The influence of the first-order degree inter-sectoral network on the sectoral shocks impact on the aggregate volatility depends on the asymmetry between sectors, which is measured by the coefficient of variation ( $CV_n$ ) of the degrees of the inter-sectoral network. The degree (or weighted out-degree) of sector  $i$ ,  $d_i$ , shows the share of sector  $i$ 's output (normalised by the constant  $1 - \alpha$ ) in the input supply of the entire economy presented in Eq. (7):

$$d_i \equiv \sum_{j=1}^n w_{ji}. \quad (7)$$

For each economy  $\xi_n$  with sectoral degrees  $\{d_1^n, d_2^n, \dots, d_n^n\}$ , let the coefficient of variation ( $CV_n$ ) be defined:

$$CV_n \equiv \frac{1}{\bar{d}_n} \left[ \frac{1}{n-1} \sum_{i=1}^n (d_i^n - \bar{d}_n)^2 \right]^{1/2} \quad (8)$$

where  $\bar{d}_n = (\sum_{i=1}^n d_i^n)/n$  devotes for the average degree of the economy  $n$ .

Based on Eq. (6) we obtain:<sup>5</sup>

$$(var y_n)^{1/2} = \Omega \left( \frac{1}{n} \sqrt{\sum_{i=1}^n (d_i^n)^2} \right) \quad (9)$$

and

$$(var y_n)^{1/2} = \Omega \left( \frac{1 + CV_n}{\sqrt{n}} \right) \quad (10)$$

Equations (8)-(10) show that an increase in the weighted out-degree leads to an increase in the coefficient of variation, causing aggregate volatility to decay at the rate slower than  $\sqrt{n}$ . Moreover, high  $CV_n$  means that a small part of sectors in the economy provides the main inputs for most of the sectors in the economy. A shock to one of those dominant sectors would propagate through all the downstream sectors.

At the same time, Eq. (9) describes the aggregate volatility condition on the tail. In other words, the fluctuations of aggregate volatility are larger if the order of the degrees in the economy has a heavier tail, which can be captured by the power law degree distribution of intersectoral network.

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<sup>5</sup> $y_n = \Omega(x_n)$  if  $\liminf_{n \rightarrow \infty} y_n/x_n > 0$ , when  $\{y_n\}_{n \in \mathbb{N}}$  and  $\{x_n\}_{n \in \mathbb{N}}$  are sequences of real positive numbers.

A sequence of economies  $\{\xi_n\}_{n \in \mathbb{N}}$  has power law degree sequence if the following assumptions are satisfied:

- a.** There exists a constant  $\beta > 1$  showing that the tail of the empirical degree distribution has scaling behavior. The lower the value of  $\beta$ , the heavier the tail of the empirical degree distribution that leads to the higher differences between the degrees of different sectors in the economy.
- b.** There exists a slowly varying function  $L(\cdot)$  that satisfies the following:

$$\begin{cases} \lim_{t \rightarrow \infty} L(t)t^\delta = \infty \\ \lim_{t \rightarrow \infty} L(t)t^{-\delta} = 0 \end{cases} \quad (11)$$

for all  $\delta > 0$ .

- c.** A sequence of positive numbers  $c_n = \Theta(1)$  that for all  $n \in \mathbb{N}$  and all  $k < d_{max}^n = \Theta(n^{1/\beta})$ , where  $d_{max}^n$  is the maximum degree in the economy  $\xi_n$ .

From these assumptions we get the following Eq. (12) where  $P_n(k)$  is the empirical counter-cumulative distribution function (CCDF):<sup>6</sup>

$$P_n(k) = c_n k^{-\beta} L(k) \quad (12)$$

Taking into account the first-order degree intersectoral network, the aggregate volatility is defined as Eq. (13), when economies have a power law degree sequence and shape parameter  $\beta \in (1, 2)$ , that shows scaling behavior of the tail of the empirical degree distribution:

$$(var y_n)^{1/2} = \Omega(n^{-(\beta-1)/\beta-\delta}) \quad (13)$$

where  $\delta$  is arbitrary. Eq. (13) suggests that if the heavy tail of the first-order intersectoral network degree sequence is captured in the economy, the aggregate volatility may decay at the rate smaller than  $n^{(\beta-1)/\beta}$ , which is lower than  $\sqrt{n}$ .

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<sup>6</sup>The empirical counter-cumulative distribution function (CCDF) shows how many data points (sectors) in the economy are at or below given point of the weighted out-degree. In other words, the CCDF presents the probability of particular weighted out-degree level in the economy.

## 2.3 Second-order degree interactions

It is important to mention that sectors with identical first-order degrees might have different impact on the aggregate volatility. This effect depends on the second-order inter-connectivity that indicates how sectors are related indirectly with downstream sectors in the economy. For example, two sectors  $r$  and  $u$  are selling their products (as intermediate products) to two other sectors in the economy ( $r$  sells to  $l$  and  $m$  (both small sectors) while  $u$  sells to  $m$  and  $g$  (the later having the highest degree in the economy)). Even if both  $r$  and  $u$  have the same first-order degrees,  $u$  will affect the economy much more, since it is selling products to  $g$  that is connected with many other sectors in the economy.

The second-order inter-connectivity coefficient of the economy  $\xi_n$  is defined as follows:

$$\tau_2(W_n) = \sum_{i=1}^n \sum_{j \neq i} \sum_{k \neq i, j} w_{ji}^n w_{ki}^n d_j^n d_k^n \quad (14)$$

The coefficient  $\tau_2$  measures how highly-connected sectors are related in the economy through the same suppliers of inputs. This coefficient is higher if the same supplier is being shared between two highly connected sectors (when  $d_j^n$  and  $d_k^n$  are both comparably high).

The coefficient  $\tau_2$  affects aggregate volatility in the following way:

$$(var y_n)^{1/2} = \Omega \left( \frac{1}{\sqrt{n}} + \frac{CV_n}{\sqrt{n}} + \frac{\sqrt{\tau_2(W_n)}}{n} \right) \quad (15)$$

Comparing Eq. (15) with Eq. (10) we can see that even if the first-order degrees of economies are the same, the second-order relations between different sectors play an important role in creating the fluctuations of aggregate volatility. At the same time, Eq. (15) captures the possibility of cascade effects in the economy, when shock in one sector leads to negative impact not only to its downstream sectors but also to all other interconnected sectors in the economy.

The same as for the first-order degree, the second-order intersectoral network effect can be presented as condition on the tail of the distribution. Here, the second-order degree of sector  $i$  is defined as Eq. (16):

$$q_i^n = \sum_{j=1}^n d_j^m w_{ji}^n \quad (16)$$

where second-order degree of sector  $i$  is calculated as weighted sum of degrees of those sectors that demand inputs from sector  $i$ .

Taking into account the second-order degrees, aggregate volatility follows Eq. (17) if economies, again, have power law degree sequence and shape parameter  $\zeta \in (1, 2)$ , that shows scaling behavior of the tail of the empirical degree distribution:

$$(\text{var } y_n)^{1/2} = \Omega(n^{-(\zeta-1)/\zeta-\delta}) \quad (17)$$

where  $\delta > 0$  is arbitrary. Eq. (17) shows that if the heavy tail of intersectoral network second-order degree sequence is captured, the aggregate volatility may decay at the rate smaller than  $n^{(\zeta-1)/\zeta}$  which is lower than  $\sqrt{n}$  – the one predicted by the standard diversification argument.

## 2.4 Estimation of shape parameters

As shown above, the first and the second orders are linked to a power law degree sequences that can be generally defined as:

$$P(Z > s) \sim C s^{-\vartheta}, \quad C, s > 0 \quad (18)$$

where  $\vartheta$  is a tail index (shape parameter), sequence  $\{Z_1, Z_2, \dots, Z_n\}$  stands for observations satisfying the power law and  $C$  is a positive constant.

By estimating the Pareto exponent, we obtain the first-order and the second-order shape parameters ( $\beta$  and  $\zeta$  accordingly). The OLS log rank–log size regression is one of the most popular tools for estimation of Pareto exponent:

$$\log(\text{Rank}) = a - b \log(\text{Size}) \quad (19)$$

where  $b$  is the estimate of the tail index. However, as [Gabaix and Ibragimov \(2011\)](#) mentioned, such procedure is strongly biased in small samples. Due to this problem, [Gabaix and Ibragimov \(2011\)](#) introduced the modified log rank–log size regression. The regression is presented in Eq. (20) and is used for estimation of  $\beta$  and  $\zeta$  shape parameters.

$$(\log(\text{Rank} - 1/2) = a - b \log(\text{Size}) \quad (20)$$

where  $b$  is the estimate of the tail index ( $\beta$  and  $\zeta$  accordingly),  $\log(\text{Rank})$  stands for empirical log-CCDF, and  $\log(\text{Size})$  stands for the log-outdegree sequence. According to the [Gabaix and Ibragimov \(2011\)](#), the shift by  $1/2$  is optimal and reduces the bias.

Due to the small sample size of Lithuanian data ( $n = 62$ ), we use a larger cut-off value than the one used in [Acemoglu et al. \(2012\)](#) where  $n \approx 480$ , 60% instead of 20%. In other words, we take the tails of the counter cumulative distributions equal to 60% of the sectors with the largest first-order and second-order degrees ( $d$  and  $q$  accordingly). The full set of sectors (100%) is not optimal since shocks to the sectors with low degrees, those with only a few or zero downstream connections, will not propagate to the economy.

## 2.5 Data

Network-based methodology requires having the direct requirements input-output table. According to the [Miller and Blair \(2009\)](#), the direct requirements table is filled with technical coefficients and can be expressed by:

$$\mathbf{A} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}, \quad (21)$$

where  $a_{ij}$ 's are technical coefficients that show the flow of products from industrial sectors ( $i$ 's), called producers, to the same sector and all others ( $j$ 's), called consumers. Technical coefficient  $a_{ij}$  can also be explained as follows: if industry  $j$  wants to produce output worth 1 Euro, it has to buy inputs from industry  $i$  for the price equal to  $a_{ij}$  Euro. These direct requirements tables  $A$  are prepared according to the formula:

$$A = Z\hat{x}^{-1}, \quad (22)$$

where  $Z$  is the statistical interindustry transactions table which is given by:

$$\mathbf{Z} = \begin{bmatrix} z_{11} & \dots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \dots & z_{nn} \end{bmatrix}, \quad (23)$$

where  $z_{ij}$  shows the monetary values of transactions of intermediate products from sectors  $i$  (rows) to sectors  $j$  (columns).

The matrix  $\hat{x}^{-1}$  presents the inverse diagonal matrix with elements of the vector of the total outputs along the main diagonal:

$$\hat{\mathbf{x}}^{-1} = \begin{bmatrix} 1/x_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1/x_n \end{bmatrix}, \quad (24)$$

where  $x_i$  shows the total output of each sector  $i$ . The total output of each sector is equal to the sales to sectors as intermediate products plus sales of the production for the final demand.

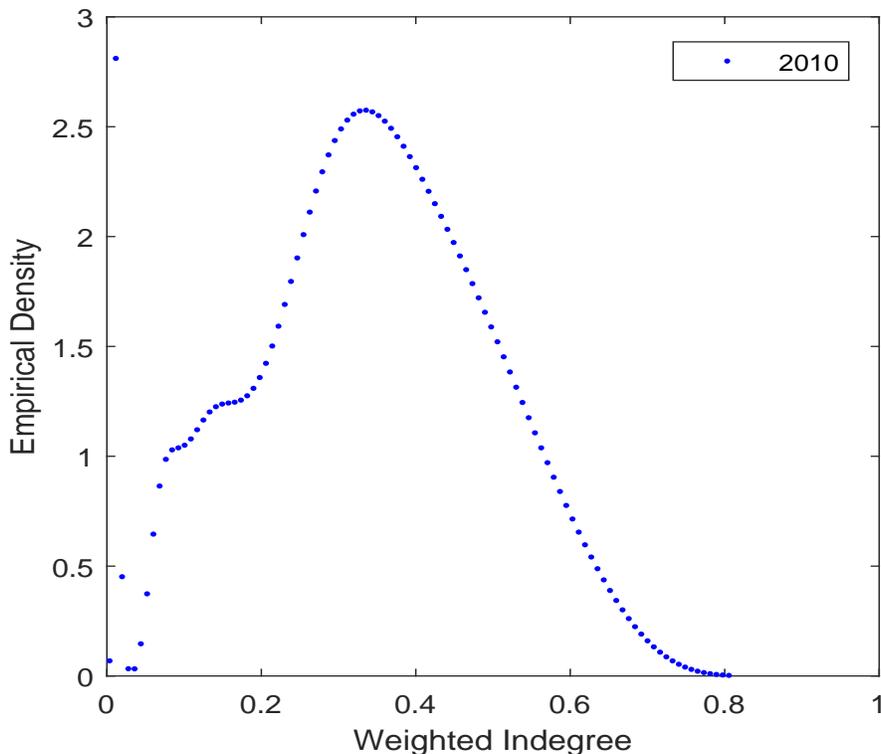
For the input–output intermediate production network’s model, we use the data from Lithuanian interindustry transactions table at the basic prices for the year 2010. The data is available from the Lithuanian Statistics database for the year 2005 and 2010, therefore we perform a robustness check with the 2005 dataset.

By applying Eq. (22), we obtain the domestic direct requirements table for Lithuania. Existing data allows forming a  $62 \times 62$  matrix. In other words, with this data we can analyse existing intersectoral linkages between 62 industries in Lithuania.

### 3 Empirical Results

#### 3.1 Weighted in-degrees and out-degrees

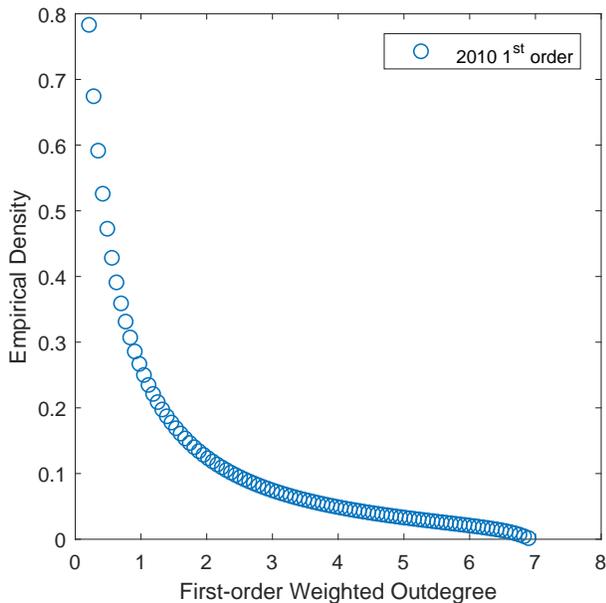
**Figure 2** – Weighted in-degree for Lithuanian industries



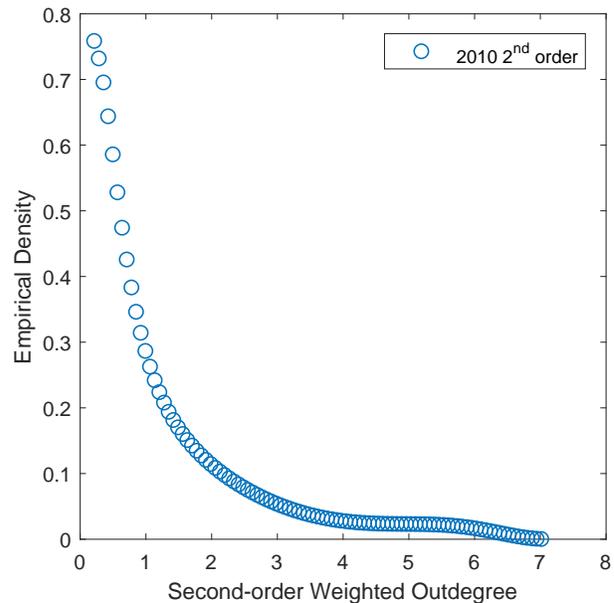
Note: Figure presents weighted indegrees of industries in Lithuania in 2010. It shows the importance of intermediate products in production of final goods in different sectors.

We first present the total intermediate input shares within industries, known as weighted in-degree for each of the industries, in Lithuania in 2010. Fig. 2 presents the nonparametric estimate of the empirical density of the intermediate input shares for 2010. In other words, it presents the importance of the intermediate products for production in different sectors. The average share of the intermediate inputs in the production of the final products in Lithuania in 2010 is equal to 0.337 (33.7%). Though some industries have more interindustrial connections than others, around 70% of industries are within one standard deviation of the mean input share.<sup>7</sup>

Fig. 3 and 4 present the nonparametric estimates of the empirical densities of weighted first-order and second-order out-degrees in 2010, suggesting that first-order ( $d_i$ ) and second-order ( $q_i$ ) outdegree empirical distributions are skewed with right tails. According to [Acemoglu et al. \(2012\)](#), this type of distributions indicates that: i) some sectors produce “general purpose” products used as inputs in many other industries, or ii) some sectors produce inputs to other sectors that produce “general purpose” inputs.



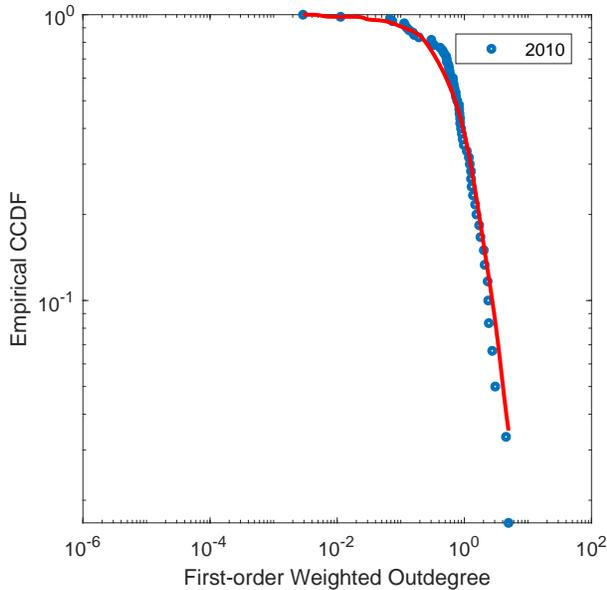
**Figure 3** – First-order weighted out-degree for Lithuanian industries



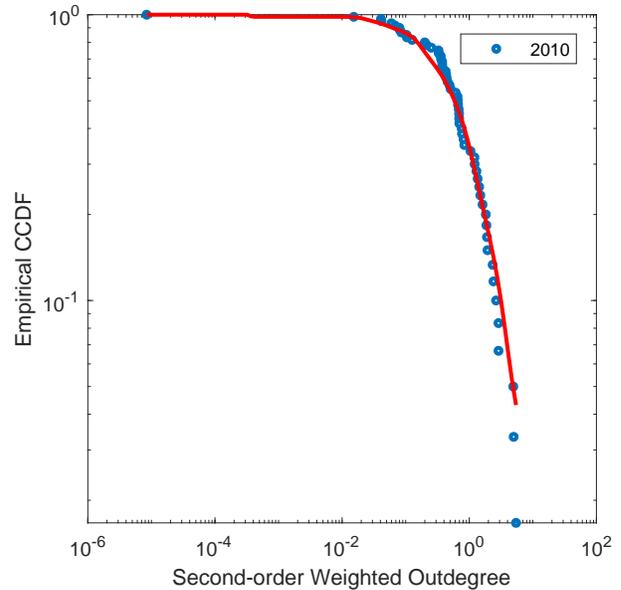
**Figure 4** – Second-order weighted out-degree for Lithuanian industries

Note: Figures present the nonparametric estimates of empirical densities of the weighted first-order (left figure) and second-order (right figure) out-degrees in 2010. Both of them are skewed with right tails.

<sup>7</sup>In this model the intermediate input share is constant and equal to  $1 - \alpha$ .



**Figure 5** – CCDF of the first-order degree



**Figure 6** – CCDF of the second-order degree

Note: Figures present the empirical CCDFs of the first-order (left figure) and second-order (right figure) degrees on a log–log scales together with nonparametric estimates for the empirical counter-cumulative distributions by Nadaraya-Watson kernel regression (solid lines in both figures). The tails of both distributions are well approximated by a power law distribution, as shown by the approximate linear relationships.

The first-order and second-order heavy-tailed distributions are also indicated in Fig. 5 and 6. These figures present the empirical CCDFs (i.e. 1 minus the empirical cumulative distribution function) of the first-order and second-order degrees on a log–log scales. At the same time, using the [Nadaraya \(1964\)](#) and [Watson \(1964\)](#)'s kernel regression with the least squares cross-validation bandwidth, nonparametric estimates for the empirical counter-cumulative distributions are obtained. The tails of both the first-order and second-order distributions are well approximated by a power law distribution, as shown by the approximate linear relationships.

### 3.2 Estimation of the shape parameters $\beta$ and $\zeta$

For the estimation of the shape parameters, we use the modified log rank–log size regression as described in Section 2.4.

In Table 1, we present the OLS estimates of the first-order and second-order degree parameters  $\beta$  and  $\zeta$  respectively, with corresponding standard errors (in the brackets), while  $n$  denotes the total number of sectors, and the cut-off value presents the number of sectors used in estimating the shape parameters. It is worth mentioning that  $\zeta$  is smaller than  $\beta$  (1.28 and 1.70 accordingly), which is inline with the argument that second-order connections in the economy play a more important role in creating fluctuations at the aggregate level than the first-order connections.

**Table 1 – Estimation of  $\beta$  and  $\zeta$**

year	$\beta$	$\zeta$	cut-off value	n
2010	1.70 (0.40)	1.28 (0.30)	37	62

Note: Table presents OLS estimates of the first-order and second-order degrees ( $\beta$  and  $\zeta$  accordingly) with standard errors in the brackets. The cut-off value presents the number of sectors used in the estimation of the shape parameters and  $n$  denotes the total number of sectors in the economy.

When it comes to the first-order degree, the estimated shape parameter  $\hat{\beta} = 1.70$ , suggesting that the aggregate volatility decays at a rate smaller than  $n^{(1.70-1)/1.70} = n^{0.41}$ . Regarding the second-order degree, the estimated shape parameter  $\hat{\zeta} = 1.28$ , indicating that the aggregate volatility decays at a rate smaller than  $n^{(1.28-1)/1.28} = n^{0.22}$ .

Consequently, both the first-order and second-order connections imply that the aggregate volatility decays at the rate lower than  $\sqrt{n}$  – the one predicted by the standard diversification argument ( $n^{0.22} < n^{0.41} < \sqrt{n}$ ), and the second-order connections play more important role. Due to the second-order connections and unbalanced structure of the input-output intermediate production networks, sectoral shocks to the one of the dominant sectors would propagate through all the downstream sectors by creating substantial fluctuations at the aggregate level.

### 3.3 Robustness check

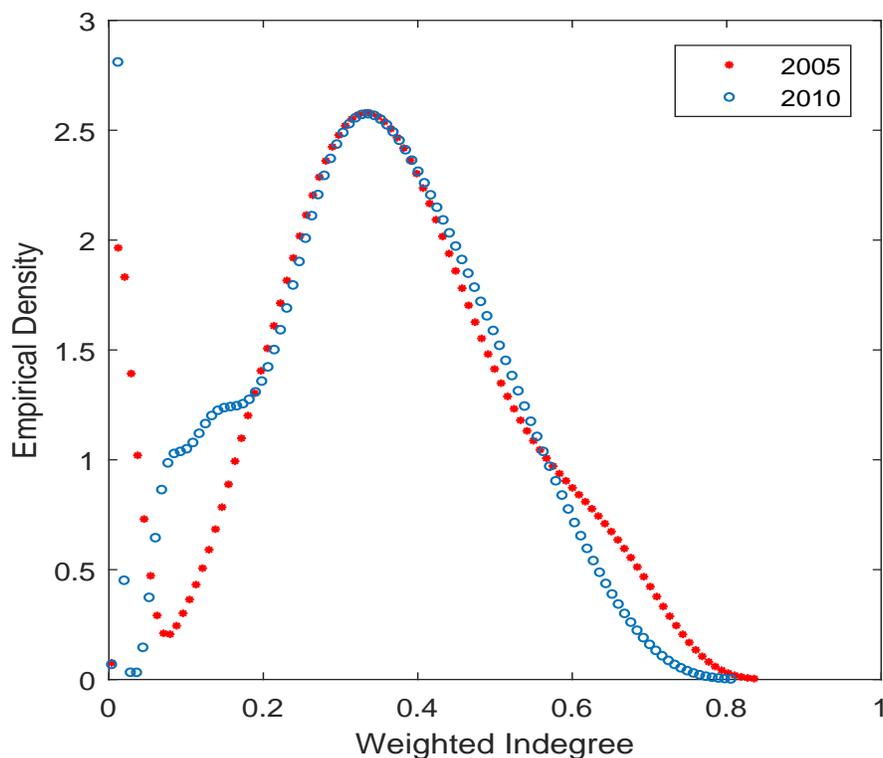
As for robustness check, we calculate the first-order and the second-order shape parameters for another data set, Lithuanian input-output intermediate production network for the year 2005, and compare it with the results of 2010. The domestic direct requirements table for Lithuania in 2005 is computed using the same procedure as described in Section 2.5. Existing data set allows analysing intersectoral linkages between 54 industries in Lithuania in 2005 which is more aggregated comparing with the data set for 2010 ( $n = 62$ ).

First we compare the total intermediate input shares within industries (weighted in-degrees for each of the industry) in Lithuania in 2005 and 2010. Fig. 7 presents the importance of the intermediate products for production in different sectors in 2005 and 2010. The average share of the intermediate inputs in the production of the final products in Lithuania in 2005 and 2010 is almost the same and equal to 0.354 (35.4%) and 0.337 (33.7%) accordingly. Though some industries have more interindustrial connections than others, around 70% of industries are within one standard deviation of the mean input share in 2005, the same as in 2010.<sup>8</sup>

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<sup>8</sup>In this model the intermediate input share is constant and equal to  $1 - \alpha$ .

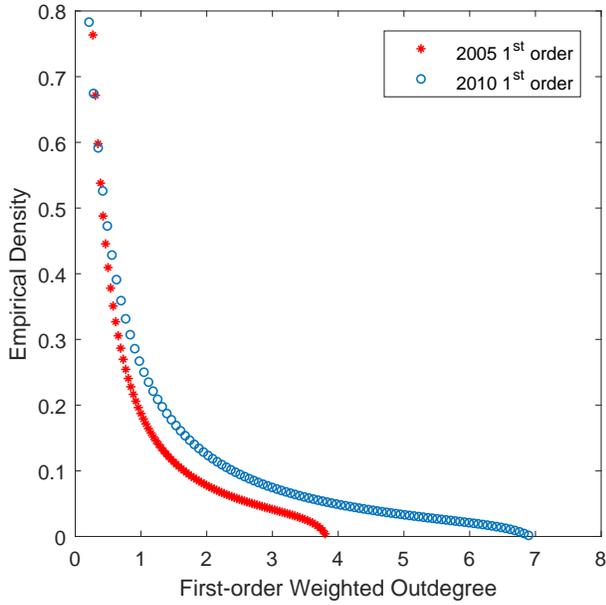
**Figure 7** – Weighted in-degree for Lithuanian industries in 2005 and 2010



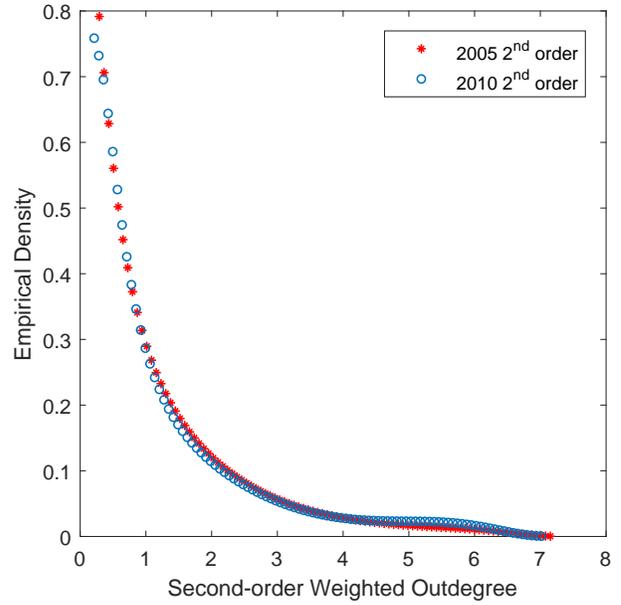
Note: Figure presents weighted indegrees of industries in Lithuania in 2005 and 2010. It shows the importance of intermediate products in production of final goods in different sectors.

Fig. 8 and 9 present the nonparametric estimates of the empirical densities of weighted first-order and second-order out-degrees in 2005 and 2010, suggesting that all first-order ( $d_i$ ) and second-order ( $q_i$ ) outdegree empirical distributions are skewed with right tails. However, Fig. 8 suggests that in 2005 there were: i) fewer sectors producing “general purpose” products used as inputs in many other industries, or ii) fewer sectors producing inputs to other sectors that produce “general purpose” inputs.

Fig. 10 and 11 present the empirical CCDFs of the first-order and second-order degrees on a log-log scales that captures the first-order and second-order heavy-tailed distributions in 2005, the same as in 2010. The tails of the first-order and second-order distributions are well approximated by a power law distribution for both data sets.

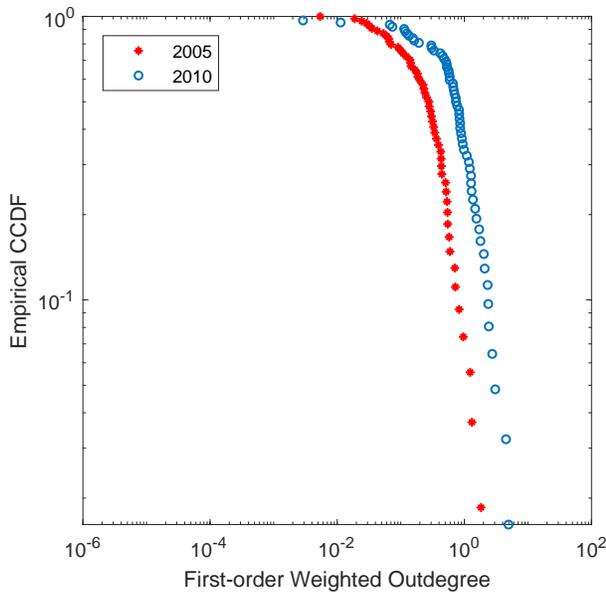


**Figure 8** – First-order weighted out-degree for Lithuanian industries

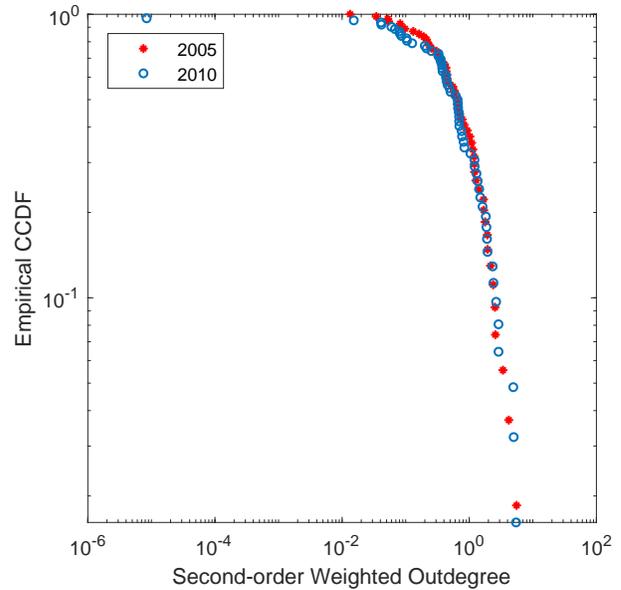


**Figure 9** – Second-order weighted out-degree for Lithuanian industries

Note: Figures present the nonparametric estimates of empirical densities of the weighted first-order (left figure) and second-order (right figure) out-degrees in 2005 and 2010. Both of them are skewed with right tails.



**Figure 10** – CCDF of the first-order degree



**Figure 11** – CCDF of the second-order degree

Note: Figures present the empirical CCDFs of the first-order (left figure) and second-order (right figure) degrees on a log-log scales in 2005 and 2010. The tails of both distributions are well approximated by a power law distribution.

**Table 2 – Estimation of  $\beta$  and  $\zeta$** 

year	$\beta$	$\zeta$	cut-off value	n
2005	1.71 (0.42)	1.41 (0.35)	32	54
2010	1.70 (0.40)	1.28 (0.30)	37	62

Note: Table presents OLS estimates of the first-order and second-order degrees ( $\beta$  and  $\zeta$  accordingly) with standard errors in the brackets. The cut-off value presents the number of sectors used in estimation of the shape parameters and  $n$  devotes the total number of sectors in the economy.

In Table 2, we present the OLS estimates of the first-order and second-order degree parameters,  $\beta$  and  $\zeta$  respectively, with corresponding standard errors (in the brackets), while  $n$  devotes the total number of sectors and the cut-off value presents the number of sectors used in estimating the shape parameters. The results of 2005 data set are in line with the data set for 2010:  $\zeta$  is smaller than  $\beta$  (1.41 and 1.71 accordingly). When it comes to the first-order degree, the aggregate volatility decays at the rate smaller than  $n^{(1.71-1)/1.71} = n^{0.42}$ . Regarding the second-order degree, the aggregate volatility decays at the rate smaller than  $n^{(1.41-1)/1.41} = n^{0.29}$ , that is, again, much lower than  $\sqrt{n}$  – the rate suggested by the standard diversification argument.

It is worth mentioning that data set for 2005 captures a slightly weaker network effect as compared with 2010 results. Due to the first-order degree, aggregate volatility decays at rates smaller than  $n^{0.42}$  and  $n^{0.41}$  accordingly. Regarding the second-order degree, aggregate volatility decays at rates smaller than  $n^{0.29}$  and  $n^{0.22}$ . These results are in line with the theory which indicates that more aggregated data captures lower network effects. As mentioned before, the 2005 data set is more aggregated as compared with the data set for 2010 ( $n = 54$  and  $n = 62$  accordingly).

## 4 Conclusion

The current study investigates the importance of inter-sectoral linkages of intermediate products as conduits of sectoral shocks at the aggregate level in Lithuania. Specifically, we investigate whether the sectorial shock affects aggregate level via the network of intermediate products as argued by [Acemoglu et al. \(2012\)](#). To do so, we construct a domestic sector-by-sector direct requirements table using the Lithuanian interindustry transactions table, in the first step. Then, we applied the [Acemoglu et al. \(2012\)](#)'s network-based methodol-

ogy and [Gabaix and Ibragimov \(2011\)](#)'s modified log rank-log size regression to analyse the inter-sectoral linkages.

The results show that the first-order and second-order empirical distributions are skewed to the right, indicating that the network of intermediate products in Lithuania is unbalanced with some dominant sectors in the economy. The direct and indirect inter-sectoral linkages imply that aggregate volatility decays at the rate lower than  $\sqrt{n}$  as implied by the standard diversification argument. Consequently, this paper provides evidence that the inter-sectoral network in Lithuania, a small and open economy, is a potential propagation mechanism for idiosyncratic shocks, further supporting the findings of [Acemoglu et al. \(2012\)](#).

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# A Appendix: List of Lithuanian Sectors in 2010

Here is presented the list of 62 sectors in Lithuania in 2010:

1. Products of agriculture, hunting and related services
2. Products of forestry, logging and related services
3. Fish and other fishing products; aquaculture products; support services to fishing
4. Mining and quarrying
5. Food products, beverages and tobacco products
6. Textiles, wearing apparel and leather products
7. Wood and of products of wood and cork, except furniture; articles of straw and plaiting materials
8. Paper and paper products
9. Printing and recording services
10. Coke and refined petroleum products; Chemicals and chemical products
11. Basic pharmaceutical products and pharmaceutical preparations
12. Rubber and plastics products
13. Other non-metallic mineral products
14. Basic metals
15. Fabricated metal products, except machinery and equipment
16. Computer, electronic and optical products
17. Electrical equipment
18. Machinery and equipment n.e.c.
19. Motor vehicles, trailers and semi-trailers
20. Other transport equipment
21. Furniture; other manufactured goods
22. Repair and installation services of machinery and equipment
23. Electricity, gas, steam and air-conditioning
24. Natural water; water treatment and supply services
25. Sewerage; waste collection, treatment and disposal activities; materials recovery remediation activities and other waste management services
26. Constructions and construction works
27. Wholesale and retail trade and repair services of motor vehicles and motorcycles
28. Wholesale trade services, except of motor vehicles and motorcycles
29. Retail trade services, except of motor vehicles and motorcycles
30. Land transport services and transport services via pipelines
31. Water transport services
32. Air transport services
33. Warehousing and support services for transportation
34. Postal and courier services
35. Accommodation and food services
36. Publishing services
37. Motion picture, video and television programme production services, sound recording and music publishing; programming and broadcasting services
38. Telecommunications services
39. Computer programming, consultancy and related services; information services
40. Financial services, except insurance and pension funding

41. Insurance, reinsurance and pension funding services, except compulsory social security
42. Services auxiliary to financial services and insurance services
43. Real estate activities excluding imputed rents
44. Imputed rents of owner-occupied dwellings
45. Legal and accounting services; services of head offices; management consulting services
46. Architectural and engineering services; technical testing and analysis services
47. Scientific research and development services
48. Advertising and market research services
49. Other professional, scientific and technical services; veterinary services
50. Rental and leasing services
51. Employment services
52. Travel agency, tour operator and other reservation services and related services
53. Security and investigation services; services to buildings and landscape; office administrative, office support and other business support services
54. Public administration and defence services; compulsory social security services
55. Education services
56. Human health services
57. Social work services
58. Creative, arts and entertainment services; library, archive, museum and other cultural services; gambling and betting services
59. Sporting services and amusement and recreation services
60. Services furnished by membership organisations
61. Repair services of computers and personal and household goods
62. Other personal services