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**TERM PREMIUM AND QUANTITATIVE EASING IN A FRACTIONALLY
COINTEGRATED YIELD CURVE**

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Abstract

The co-movement of US sovereign rates suggests a long-run common stochastic trend. Traditional cointegrated systems need to assume that interest rates are unit roots and thus imply non-stationary and non-mean-reverting dynamics. Based on recent econometric developments, we postulate and estimate a fractional cointegrated model (FCVAR) which allows for a mean-reverting stochastic trend. Our results point to the presence of such mean-reverting fractional cointegration among sovereign rates. The implied term premium is less volatile than the classic $I(0)$ stationary and $I(1)$ unit root models. Our analysis highlights the role of real factors (but not inflation) in shaping term premium dynamics. We further identify the dynamic effects of quantitative easing policies on our identified term premium. In contrast to the stationary-implied term premium, we find a significant term premium decline following these large-scale asset purchase programs.

JEL Classification: C2, C3, E4, G1

Keywords: U.S. yield curve; stochastic trend; fractional cointegration; term premium; quantitative easing

1 Introduction

Sovereign yield curve dynamics is of crucial concern to investors, bankers, policy makers and researchers. As such, it has attracted a great deal of attention in these domains and in the media. The joint co-movement of interest rates across maturities is a specific source of term structure attention. US sovereign rates track each other quite closely despite their different maturities (see Figure 1). Why is this the case? Many equilibrium models, such as those based on no-arbitrage, propose that common factors (level, slope and curvature) drive yield dynamics across all maturities. At the same time, researchers and policy makers have long pointed to long-rates embedding expectations of short-rates. Consequently, it is critical both to produce the accurate short-term forecasts and to capture the common dependence of rates across maturities. Thus, empirical models aim to improve both the characterization and estimation of joint bond yield dynamics. The correct exploitation of this cross-sectional term structure co-movement has important economic implications for both fiscal and monetary policy, term premium identification, predictability of future macro variables and banking management.

[Insert Figure 1: US Sovereign Interest Rates]

Figure 1 also reveals a long-run dependence across the term structure of interest rates. In the term structure literature, this long-run trend has been traditionally characterized via cointegration techniques (see Campbell and Shiller (1987) for a seminal study). In short, traditional cointegration requires that all interest rates are unit roots or $I(1)$ processes and that they cannot diverge from each other for long periods of time. While this methodology has certain advantages, such as exploiting this long-term relation across rates, the structure imposes an unappealing non-mean reversion in rates. As discussed by Campbell, Lo and MacKinlay (1997) and Diebold and Rudebusch (2013), this im-

plies that shocks to interest rates have permanent effects, despite the fact that sovereign interest rates, at least in most industrialized economies, do not exhibit such behavior.

Therefore, capturing this joint co-movement across maturities and at least allowing for mean-reversion dynamics should be on the agenda of any natural term structure model. This is what we explore and test in this paper, where we apply novel multivariate fractional cointegration techniques which allow for a flexible stochastic trend in the term structure of interest rates. This econometric model simultaneously identifies the order of the integration of rates (one, zero or a fractional number) and the potential presence of one (or several) cointegration relationships. Indeed, whether interest rates are cointegrated, fractionally cointegrated or not cointegrated is an empirical question which we tackle in this paper. To this end, we estimate a novel fractional cointegration vector auto-regressive (FCVAR) model (Johansen and Nielsen, (2012)) with US sovereign rates of different maturities.

We find that the U.S. term structure of interest rates exhibits a long-run mean-reverting stochastic trend. Our estimation results show that the order of integration of the interest rates is 0.756 with monthly data and statistically different from zero and one. Our results thus reject modeling sovereign rates in a unit-root cointegration framework. An implication of this result is that the common macro-finance shocks affecting the yield curve stochastic trend turn out to have transitory (rather than permanent) though long-lasting effects on the term structure. Our results also reject the joint modeling of interest rates in standard stationary vector auto-regressive systems, given that we estimate the order of integration to be well and significantly above zero and that we find that a stochastic trend captures the long-run dependence across the term structure of interest rates. We also show that the estimated long-run stochastic trend is consistent with monetary policy being the driving force of this joint low-frequency co-movement.

Our analysis yields a natural estimate of the term premium on long-term bonds, an important object of analysis for policy makers. Higher-term premiums reveal that investors require higher returns for long-term bonds, which may indicate a number of macro-financial or policy risks for the economy. The term premium associated with our fractional cointegrated system displays a marked degree of persistence and is clearly counter-cyclical. We analyze the sources of our term premium dynamics and show that they diverge with respect to term premiums implied by stationary $I(0)$ and unit-root $I(1)$ models. In particular, unemployment is key to understanding its counter-cyclical dynamics, whereas inflation plays no role in this respect. Moreover, we demonstrate that the term premium response to recent quantitative easing (QE) policies is significantly different from that of stationary models. In particular, we identify a dynamic decline of the term premium following these large expansions in asset purchases, whereas no significant effect is found in the stationary model. QE shocks are also shown to increase both economic activity and inflation. Our results thus pose relevant policy implications in the current debate on the effects of monetary stimulus withdrawal (see Yellen, 2017).

The paper proceeds as follows. Section 2 summarizes the fractional cointegration econometric framework and describes the economic implications of this modeling strategy for the term structure of interest rates. Section 3 discusses the data, empirical strategy and estimation procedure. Section 4 presents the empirical results of the paper. It shows the nature of the stochastic trend present in the US yield curve, the implied term premium and its economic sources (comparing to $I(0)$ and $I(1)$ alternatives), and the effects of unconventional monetary actions across macro-finance variables. Section 5 concludes.

2 Fractional Cointegration

In this section, we first briefly outline the multivariate fractional cointegration framework and lay out some of its general economic implications. Then, we go on to explain why fractional cointegration can be an appropriate modeling technique for the term structure of interest rates.

2.1 Econometric Setting

Our methodology to model term structure dynamics is based on the concept of long memory behavior. Given a covariance stationary process $x_t, t = 0, \pm 1, \dots$, a series has long memory if its spectral density function contains a pole or singularity at least at one frequency in the spectrum. Alternatively, it can be defined in the time domain by saying that x_t displays the property of long memory if the infinite sum of the auto-covariances is infinite. A typical model satisfying the above two properties is the fractionally integrated or $I(d)$ model, where d is a positive value and can be formulated as:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

with $x_t = 0$ for $t \leq 0$, where L represents the lag-operator, i.e. $Lx_t = x_{t-1}$, and u_t is an $I(0)$ or short-memory process defined in the frequency domain as a process with a spectral density function that is positive and bounded at all frequencies. Note that in this context, if $d > 0$, the spectral density function of x_t is unbounded at the smallest (zero) frequency, and the polynomial on the left hand side of equation (1) can be written for all real d as:

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = \left(1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 \dots \right), \quad (2)$$

and thus:

$$(1 - L)^d x_t = x_t - dx_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \frac{d(d-1)(d-2)}{6} x_{t-3} + \dots, \quad (3)$$

so that equation (1) can be expressed as:

$$x_t = dx_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \frac{d(d-1)(d-2)}{6} x_{t-3} + \dots + u_t. \quad (4)$$

Thus, the differencing parameter d plays a crucial role in describing the degree of dependence (persistence) in the data: The higher the value of d , the higher the level of dependence between observations. Three values of d are of particular interest. First, the case of $d = 0$ that implies short memory behaviour as opposed to the case of long memory with $d > 0$. Second, $d = 0.5$, since x_t becomes non-stationary as long as $d \geq 0.5$.¹ Finally, if $d < 1$ x_t is mean reverting with the effect of the shocks disappearing in the long run, contrary to what happens if $d \geq 1$ with shocks having permanent effects and lasting forever.

The natural generalization of the concept of fractional integration to the multivariate case is the idea of fractional cointegration. In this paper, we employ the Fractionally Cointegrated Vector AutoRegressive (FCVAR) model recently introduced by Johansen and Nielsen (2012). This method is used to determine the long-run equilibrium relation-

¹It is non-stationary in the sense that the variance of the partial sums increases in magnitude with d .

ship between series. Given two real numbers d, b , the components of the vector z_t are said to be cointegrated of order d, b , denoted $z_t \sim CI(d, b)$, if all the components of z_t are $I(d)$ and there exists a vector $\alpha \neq 0$ such that $s_t = \alpha' z_t \sim I(\lambda) = I(d - b), b > 0$.² The Fractionally Cointegrated Vector AutoRegressive (FCVAR) model introduced by Johansen (2008) and further expanded by Johansen and Nielsen (2010, 2012) is a generalization of Johansen (1995) Cointegrated Vector AutoRegressive (CVAR) model which allows for fractional processes of order d that cointegrate to order $d - b$. In order to introduce the FCVAR model, we refer first to the well-known, non-fractional, CVAR model. Let $Y_t, t = 1, \dots, T$ be a p -dimensional $I(1)$ time series vector. The CVAR model is:

$$\Delta Y_t = \alpha \beta' Y_{t-1} + \sum_{i=1}^k \Gamma_i \Delta Y_{t-i} + \varepsilon_t = \alpha \beta' L Y_t + \sum_{i=1}^k \Gamma_i \Delta L^i Y_t + \varepsilon_t, \quad (5)$$

where Δ refers to the first difference operator, i.e., $\Delta = (1 - L)$, α is the vector or matrix of adjustment parameters, β is the vector or matrix of cointegrating vectors and the sequence of matrices Γ_i governs the short-run $I(0)$ VAR dynamics. The simplest way to derive the FCVAR model is to replace the difference and lag operators Δ and L in (5) by their fractional counterparts, Δ^b and $L_b = 1 - \Delta^b$, respectively. We then obtain:

$$\Delta^b Y_t = \alpha \beta' L_b Y_t + \sum_{i=1}^k \Gamma_i \Delta^b L_b^i Y_t + \varepsilon_t, \quad (6)$$

which is applied to $Y_t = \Delta^{d-b}(X_t - \mu)$, where X_t is the $p \times 1$ vector of our series of interest and μ is a level parameter which accommodates a non-zero starting point for the first

² A more general definition of fractional cointegration permits different orders of integration for each individual series. See, e.g., Robinson and Marinucci (2001), Robinson and Hualde (2003) and others.

observation on the process. We therefore have that:

$$\Delta^d(X_t - \mu) = \alpha\beta' L_b \Delta^{d-b}(X_t - \mu) + \sum_{i=1}^k \Gamma_i \Delta^d L_b^i(X_t - \mu) + \varepsilon_t, \quad (7)$$

where ε_t is p -dimensional independent and identically distributed with mean zero and covariance matrix Ω . The parameters have the usual interpretations known from the CVAR model. In particular, α and β are $p \times r$ matrices, where $0 \leq r \leq p$. The columns of β are the cointegrating relationships in the system, that is, the long-run equilibria. The parameters Γ_i govern the short-run behavior of the variables (with k being the lag length of the VAR) and the coefficients in α represent the speed of adjustment towards equilibrium for each of the variables. Thus, the FCVAR model permits simultaneous modelling of the long-run equilibria, the adjustment responses to deviations from the equilibria and the short-run dynamics of the system. In Johansen and Nielsen (2012) and Nielsen and Popiel (2016) one can find estimation and inference explanations of the model, and the latter provides Matlab computer programs for the calculation of estimators and test statistics.

2.2 Fractionally Cointegrated Sovereign Rates

2.2.1 Why?

That the term structure of interest rates displays an empirical long-run trend is hard to argue against (see Figure 1 and the Introduction). While no-arbitrage across maturities in US bond markets seems a reasonable assumption, departures from it are surely small enough to ensure that no-arbitrage forces lie behind the relation of the different yield maturities. Indeed, most term structure models assume that a number of factors drive

the entire term structure of sovereign interest rates. This is true in models with absence of arbitrage (Dai and Singleton (2000), Ang and Piazzesi (2003) or, more recently, Monfort, Pegoraro, Renne and Roussellet, (2017)) and without this absence (see Nelson and Siegel (1987) or Abbritti, Gil-Alana, Lovcha and Moreno (2017)). These factors are typically characterized as the level, slope and curvature of the yield curve and shift all rates across maturities. Some studies have even tried to develop an underlying macroeconomic interpretation behind these factors, with expected inflation driving the level and economic growth shaping the slope. In this context, several authors have proposed the presence of long-run attractors (such as inflation target, the natural real rate or the natural level of economic activity) to which both factors and the term structure converge (Dewachter and Lyrio, (2006), Bekaert, Cho and Moreno (2010), Cieslak and Povala (2015) and Bauer and Rudebusch (2017)).

There are additional reasons why long-rates may be related to short-rates. The expectations hypothesis of interest rates, based on investors exploiting arbitrage opportunities, is a case in point. If the long-rate is the sum of current and expected short-term rates, then long-rates should naturally inherit dynamics of short-rates, often driven by monetary policy actions. While the expectations hypothesis tends to be statistically rejected in the data, most studies point to some economic truth in it (see, for instance, Campbell and Shiller (1987) or Bekaert, Hodrick and Marshall, (2001)). As a result, a general formulation of the relation between long-rates and short-rates typically includes an expectations hypothesis part and a time-varying term (risk) premium (the expectations hypothesis assumes that this term premium is constant). Given the subsequent empirical analysis, we characterize here the relation between the 10-year rate ($i_t^{(120)}$) and 1-year

rates ($i_t^{(12)}$) with monthly data:

$$i_t^{(120)} = \frac{1}{10} E_t \sum_{j=0}^9 i_{t+12j}^{(12)} + tp_t^{(120)}, \quad (8)$$

where $tp_t^{(120)}$ is the associated 10-year term premium and the remaining right-hand side of the equation constitutes the risk-neutral rate. The term premium can be seen as the compensation demanded by investors for bearing the interest rate risk associated with a 10-year (longer-term) security. As is well known, the short-rate is affected by macroeconomic variables through monetary policy setting. Take, for instance, credible changes in the Central Bank inflation target. As shown in several economic models (see Gürkaynak, Sack and Swanson (2005) or Bekaert, Cho and Moreno (2010)), this policy change (which directly influences the monetary policy (short-term) rate) can be priced by investors on bonds with different maturities and subsequently transmitted through the entire yield curve. This has significant economic policy implications, as monetary policy makers often resort to short-term interest rate policy to influence long-term expectations and the overall economy (see Bernanke, 2006, and the policies applied by the Federal Reserve after 2008, such as forward guidance).

At the same time, and from an empirical perspective, interest rates tend to display significant long-memory (fractional integration) dynamics, as found in Backus and Zin (1993), Gil-Alana and Moreno (2011) and Abbritti, Gil-Alana, Lovcha and Moreno (2016), among many others. There are several reasons for interest rates being fractionally integrated. Inspired by the work of Robinson (1978) and Granger (1980), Altissimo, Mojon and Zaffaroni (2009) show that aggregation of sub-indices can explain inflation persistence. If aggregation explains fractional integration in inflation, then interest rates can all inherit fractional integration due to standard inflation targeting strategies by

monetary policy makers.

Thus, if interest rates exhibit both stochastic long-run co-movement and fractional integration, it makes sense to feature them jointly in order to capture the correct dynamics. This is what we pursue in this paper. Of course, there are alternative techniques for modeling interest rates. One of them is regime switching (see Ang and Bekaert (2002) and Baele, Bekaert, Cho, Inghelbrecht and Moreno (2015), among others). Regime switching has the appealing feature of allowing shifts in meaningful key reduced-form or policy parameters, such as the reaction to inflation deviations from target or changes in interest rate inertia induced by financial stability purposes. These shifts influence the whole term structure, thus shaping joint yield dynamics. While the fractional cointegration approach does not model these parameter shifts, it can be consistent with regime-switching dynamics. Indeed, as discussed by Diebold and Inoue (2001), the dynamics of fractional integration and regime switching are easily confused because fractional integration can capture some of the embedded autocorrelations derived from regime-switching processes.

2.2.2 What Differences Does It Make?

Long-run equilibrium relations stem from many economics and finance models (e.g., growth theory and asset pricing, among other areas). This is one of the reasons why cointegration has attracted so much attention in empirical studies. But we are left with the question: What are the new advantages that fractional cointegration brings to the modeling of long-run relations, such as the yield curve? We cite here three relevant novel features: First, letting the data choose the order of integration offers an important advantage, namely, avoiding the risk of over/under-differencing the variables (see Cochrane and Piazzesi (2008) on this point). The fractional integration setting lets the data choose what kind of long-run relationship there is among sovereign interest rates (sometimes al-

lowing for weaker or stronger, more realistic, autocorrelation in the model variables). Second, and related to the first point, by assuming I(1) cointegration or an I(0) VAR model, we may be mis-specifying the model estimates, parameters, test restrictions and implied dynamics, such as the term premium. This is an especially important point today, when scholars and policy makers alike strive to understand the effects of quantitative easing policies (and subsequent tapering) on term premium dynamics (see D’Amico, English, López-Salido and Nelson (2012) and Yellen (2017), among others). Indeed, we demonstrate below that both term premium interpretation and policy evaluation can crucially differ, depending on term premium identification.

Third, by allowing for a stochastic trend of order lower than unity, we allow economic shocks to have temporary mean-reverting effects on the variables of interest. This introduces a higher degree of flexibility in modeling both theoretical and applied macro dynamics, where the stochastic long-run term structure trend can be formed by shocks having transitory effects on interest rates. In particular, this has implications for macro-finance models elaborating on the relations between shocks and propagation. Importantly, neither theoretical nor applied modelers have to assume potentially I(1) behavior for sovereign interest rates, given the widely accepted common trends or variables (such as the standard term structure factors) affecting the yield curve.

3 Data and Estimation

In our empirical work, we employ monthly series corresponding to the U.S. Treasury Yield Curve. The data was obtained online from the work of Gürkaynak, Sack and Wright (2007). Their yield curve estimates are updated periodically and provide a benchmark US sovereign zero-coupon yield curve. Our baseline specification includes four series,

namely, the one-, three-, five- and ten-year sovereign rates. In this way, our data vector X_t includes information about the short, medium and long end of the yield curve. By including different parts of the term structure, our model captures key macro-finance information, including future economic and financial expectations. Our dataset covers observations from August 1971 up to April 2018. Figure 1 shows the dynamics of the four interest rates for our sample period.

In terms of estimation, we proceed as follows: We first assume that a sample of length $T + N$ is available on X_t , where N denotes the number of observations used for conditioning. As shown in Johansen and Nielsen (2016), model (7) can be estimated by conditional maximum likelihood, conditional on N initial values, by maximizing the following function:

$$\log L_T(\lambda) = \frac{T}{2} (\log(2\pi) + 1) - \frac{T}{2} \log \det \left\{ T^{-1} \sum_{t=N+1}^{T+N} \varepsilon_t(\lambda) \varepsilon_t(\lambda)' \right\}. \quad (9)$$

For model (7) the residuals are:

$$\varepsilon_t(\lambda) = \Delta^d (X_t - \mu) - \alpha \beta' \Delta^{d-b} L_b (X_t - \mu) - \sum_{i=1}^k \Gamma_i \Delta^d L_b^i (X_t - \mu), \quad (10)$$

with $\lambda = (d, b, \mu, \alpha, \beta, \Gamma_i)'$. It is shown in Johansen and Nielsen (2012) and Dolatabadi, Nielsen and Wu (2016) that, for fixed (d, b) , the estimation of model (6) is carried out as in Johansen (1995). In this way, the parameters $(\mu, \alpha, \beta, \Gamma_i)'$ can be concentrated out of the likelihood function. Then, we only need to optimize the profile likelihood function over the two fractional parameters, d and b . Through our analysis, we demonstrate the results implied by estimation which allow for different estimates of d and b . We also comment below on the estimates implied by a model where d is forced to be equal to b . As explained by Johansen and Nielsen (2018), the likelihood ratio test of the usual

CVAR is asymptotically $\chi^2(2)$ and the likelihood ratio test of the hypothesis that $d = b$ in the fractional model is asymptotically $\chi^2(1)$. Hence, these tests are easy to implement and can be calculated using Nielsen and Popiel's (2016) software package.

4 Empirical Results

In this section, we present and discuss the empirical results of the paper. We first show the empirical estimates of the FCVAR model and relate the implied model rates to monetary policy management. We then extract the FCVAR-term premium, compare it with alternative I(0) and I(1) counterparts and provide an interpretation of its underlying economic sources. Finally, we examine the dynamic effects of the recent quantitative easing policies on term premium dynamics.

4.1 Long-Run Trend and Monetary Policy Role

The dataset in Gürkaynak, Sack and Wright (2007) provides daily data of sovereign rates from maturities 1-year to 30 years. To capture some relevant maturities at the short, medium and long end of the yield curve, we work with the 1-year ($i_t^{(12)}$), 3-year ($i_t^{(36)}$), 5-year ($i_t^{(60)}$) and 10-year ($i_t^{(120)}$) US sovereign rates. We work with the monthly frequency, as results can then be related to key macro variables, such as unemployment, consumer inflation and industrial production. We use end-of-the-month interest rate observations over each month to construct the monthly dataset, which spans the August 1971-March 2017 sample period.

When we run the FCVAR system with the four interest rates, we obtain the following estimated model:

$$\Delta^{0.756}(X_t - \mu) = \alpha\beta' L_{1.184} \Delta^{0.756-1.184}(X_t - \mu) + \sum_{i=1}^k \Gamma_i \Delta^{0.756} L_{1.184}^i (X_t - \mu) + \varepsilon_t. \quad (11)$$

Results are based on a VAR(1) for short-run dynamics ($k = 1$), as selected by the Hannan-Quinn criterion. Table 1 reports the cointegrating rank test and identifies a single stochastic trend for interest rates. In turn, the alternative of not having a cointegrating relationship is clearly rejected. Hence the FCVAR model is validated. This stochastic trend is of fractional nature, as the estimated common order of integration of the four interest rates is 0.756, with a 95% confidence interval including the set (0.688, 0.824). This value turns out to be statistically higher than 0.5 and different from 0 and 1. Table 2 also shows the results of an LR test and reveals that the CVAR is rejected in favor of the FCVAR. The parameter b is estimated to be 1.184 (with standard deviation 0.087). This implies that the error terms display anti-persistence, being therefore stationary and with shocks reverting more often than those expected from a random series. In turn, the level parameter is μ estimated at [5.264, 5.771, 5.993, 6.188]’.

[Insert Tables 1 and 2: Results of the Cointegrating Rank and LR Tests]

The estimated long-run fractional cointegration vector is:

$$\hat{\beta}' = [1, -2.598, 2.347, -0.760]’,$$

where the elements of this vector are associated with the 1-, 3-, 5- and 10-year bond rates, respectively. Thus, loadings on the medium end of the yield curve are more than twice higher than those in the short and long ends. In turn, the corresponding speed of adjustment vector is estimated at:

$$\hat{\alpha}' = [0.016, \quad 0.042, \quad 0.053, \quad 0.064]'$$

As a result, the implied speed of adjustment with respect to deviations from the long-run relationship is fastest (and statistically significant, given that its standard deviation is 0.021) for the 10-year rate. In contrast, the 1-year rate adjustment to deviations from this fractional cointegration is very sticky, almost null (and statistically insignificant, given that its standard deviation is 0.020). The short-rate thus tends to be less driven by the long-run relation among rates and more influenced by its own short-run dynamics, at least at high frequencies. So, shocks affecting specifically the medium and long end of the yield curve –and which generate deviations from the long-run relationship– are transmitted to the short-rate very slowly, while specific shocks affecting the short and medium ends of the yield curve –and, again, to the extent that they generate deviations from the long-run relationship– are transmitted to the 10-year rate relatively fast.

To understand the impact of monetary policy in our fractionally cointegrated yield curve, Figure 2 plots the monetary policy rate (Federal Funds Rate, FFR), together with the 1-, 3-, 5- and 10-year bond rates implied by the long-run equilibrium relation vector β' . Figure 2 shows that the FFR is very similar to the 1-year and 3-year rates implied by the long-run relation. So our estimated fractional cointegration relation captures the fact that monetary policy is the driving factor behind the short-end of the yield curve at both high and low frequencies (see Moreno (2004)). When comparing the FFR with the medium and long ends of the yield curve (5 and 10-year rates), some differences arise. In particular, while the long-run trend of both rates is similar, the strong counter-cyclical dynamics of the FFR are not replicated by the implied 10-year rate, which exhibits a less volatile pattern and more nuanced changes. In the next subsection, we turn to further scrutinize the dynamics of the 10-year rate by breaking it down into the risk-neutral rate and the term premium.

[Insert Figure 2: Federal Funds Rate and FCVAR-implied Rates]

Finally, we note in this subsection that when we estimate the FCVAR imposing that $d = b$, we also obtain a unique fractional cointegration relation with $d=0.765$ –very similar to our benchmark 0.756– and a standard deviation of 0.050. Table 3 shows the results of a likelihood ratio testing the benchmark model ($d \neq b$) versus the restricted model ($d = b$). The table shows a statistical rejection of the restricted model. From an economic perspective, subsequent results turn out quite similar under both specifications, although the term premium implied by our more flexible benchmark (a topic to which we now turn) is even more stable.

[Insert Table 3: LR Test, $d = b$ v/s $d \neq b$]

4.2 Term Premium Analysis

Once we have determined that sovereign rates are fractionally cointegrated and assessed their relation with monetary policy management, we can examine the implied term premium, which is obtained as the difference between the 10-year rate and the average of current and expected future 1-year rates. Following our previously introduced relation between long-rates and expected short-rates (see equation (8)) and based on the estimates of our FCVAR model (equation (11)), we can identify the implied term premium associated with the 10-year bond rate ($tp_t^{(120)}$). This is plotted in Figure 3. As the figure shows, the implied term premium is markedly counter-cyclical and no clear trend emerges. While the term premium is positive during most of the sample period, it also displays low negative values at the end of the 70s and beginning of the 80s (reaching values around -0.5%). During the recent 2008 financial crisis, the term premium also

increased to values close to 3%, but it has declined since then, with term premium levels ranging between 0% and 1% by the end of the sample.

[Insert Figure 3: Term Premium Implied by the FCVAR System]

Table 4 shows the mean and standard deviation of the term premiums and risk-neutral rates implied by the I(0)-VAR, I(1)-CVAR and FCVAR models, respectively. The I(0)-VAR model generates the least variable risk neutral rate, due to the fast mean reversion of forward-looking expectations. The opposite is the case for the CVAR model, where expectations are the most volatile. The FCVAR model clearly delivers the most stable term premium in terms of standard deviation (one-third lower than its CVAR and I(0)-VAR counterparts). Its mean is also the lowest, 20 and 30 basis points lower than the CVAR and I(0)-VAR models, respectively. Table 5 shows the correlation of the term premiums and risk-neutral rates with four macro variables: Federal Funds Rate, unemployment, industrial production growth and the term premium itself. While the FCVAR and the CVAR term premiums display a negative correlation with the Federal Funds Rate, this correlation is positive for the I(0)-VAR. Also, the risk-neutral rates implied by the FCVAR and the CVAR have a negative correlation with their respective term premiums, whereas the opposite is the case for the I(0)-VAR. All term premiums have a positive correlation with unemployment, a theme we revisit below.

[Insert Tables 4 and 5: Term Premium Descriptive Statistics]

The top graph in Figure 4 plots the term premiums implied by the three models. It shows how the I(0)-implied term premium was substantially higher during the early 80s and has become increasingly negative since 2015. The bottom graph in Figure 4 shows the differences between the term premium implied by both the I(0) and CVAR models,

respectively, and that implied by our FCVAR. The differences are quite sizable during some periods. The $I(0)$ -implied term premium is higher than the FCVAR-term premium from 1976 to 1995 (reaching nearly 3% in the early 80s). This gap exhibits a downward trend, revealing the downward trend in the $I(0)$ implied term premium during the first part of the sample. The downward trend in the $I(0)$ -implied term premium thus reveals the challenge that $I(0)$ models face when describing the true counter-cyclical nature of the term premium (see Bauer, Rudebusch and Wu (2012) small-sample analysis of $I(0)$ -type models). In contrast to the $I(0)$ -implied model, the CVAR-implied term premium is lower than the FCVAR-implied one during most of the first 15 years of the sample. This difference reaches its maximum value in the last years of the 70s and the first years of the 80s (nearly -3%), when sovereign rates were especially volatile due to monetary policy tightening in an era of high inflation rates.

By the last years of the sample –when policy rates close to the zero lower bound–, we see important remaining differences with different signs, depending on the model at hand: around 0.5% higher in the CVAR and almost 2% lower in the $I(0)$ VAR. The first column of Figure 5 examines the patterns in the 1-year rate expectations for the three models during the post-2006 period for three alternative horizons (1-year, 5-year and 10-year). The differences are striking. While the $I(0)$ model produces long-run (10-year) expectations above 4% (close to the full sample average) –implying a negative term premium by the end of the sample (see top graph in the second column of Figure 5)–, the opposite is the case for the $I(1)$ -CVAR, where implied expectations are very close to zero (in fact, they are negative for almost three years!). The FCVAR-implied long-run expectations are between 1 and 2%, showing a realistic slow mean reversion in the context of a slow economic recovery. In sum, our FCVAR-identified term premium is less volatile than its $I(0)$ and CVAR counterparts. Our analysis demonstrates that this is due to the

fact that the I(0) model assumes too little volatility for the risk-neutral rate, whereas the CVAR assumes too much volatility.

[Insert Figures 4 and 5: Term Premium and Risk-Neutral Differences:

FCVAR v/s CVAR and VAR]

Theoretical and empirical research identifies two main reasons for an increase in term premiums: an increase in inflation uncertainty (see, e.g., Wright, 2011) and an increase in economic risk (see e.g. Bauer, Rudebush and Wu, 2012). Thus, a correct identification of the term premium is crucial for grasping the economic forces behind term premium dynamics as well as for determining the appropriate policy response. In fact, these two risk factors call for opposite monetary policy responses: Central banks should increase interest rates if increasing risk premiums reflect inflation uncertainty, while they should reduce them when a spike in term premiums reflects economic and financial risk (see related comments in Bernanke, 2006). It is thus vital to identify the main forces that drive term premium increases.

To shed some light on this issue, we follow, e.g., Backus and Wright (2007), Gagnon, Raskin, Remache and Sack (2011), and Wright (2011), who introduce the following ordinary least squares regression model to explain historical time variation in the term premium:

$$tp_t = \alpha + \beta x_t + \eta_t, \tag{12}$$

where tp_t is a measure of the term premium –I(0)-VAR, FCVAR or CVAR–, x_t denotes a vector of regressors and η_t is the error term. In practice, we will consider two models. In the first model, which is very similar to those in Backus and Wright (2007), Wright (2011) and Bauer, Rudebusch and Wu (2012), we regress the term premium on measures of inflation uncertainty and real economic activity. Specifically, we measure inflation

uncertainty with the long-run inflation disagreement series measured by the Michigan Survey of Consumers, which captures the interquartile range of five-to-ten-year-ahead inflation expectations. Business cycle uncertainty is captured with the unemployment rate and an NBER recession dummy.

We compare in Table 6 the results obtained with the I(0)-VAR, the CVAR and the FCVAR term premium. The dimension of the full sample, which starts in April 1990 and ends in April 2018, is constrained by the availability of the long-run inflation disagreement series. We find that the correct identification of the persistence of the term premium has a strong influence on its interpretation. All three term premiums react positively to the inflation dynamic, but while the stationary I(0) term premium is strongly positively related to inflation uncertainty, the opposite is the case for the unit root-I(1) term premium, as the conditional correlation with inflation uncertainty is negative. In contrast, the results of the FCVAR model show no evidence of correlation with our measure of inflation uncertainty. Given that such results can be perceived as controversial, we welcome further discussion on this issue, which we aim to address in future work.

[Insert Table 6: Term Premium Sources Regressions, Simple Model]

This first specification is admittedly very simple, probably too simple to derive definitive conclusions on the relationship between term premiums and macroeconomic uncertainty. To test for robustness of these findings to additional controls, we follow Gagnon, Raskin, Remache and Sack (2011) and expand the set of variables considered by Wright (2011) and Bauer, Rudebush and Wu (2012). In particular, we control for three additional variables:

- Core PCE inflation (year-on-year), which proxies for the long-run level of inflation and is also correlated to inflation uncertainty.

- Six-month realized volatility of the ten-year Treasury yield, which proxies for interest rate uncertainty.
- The Economic Policy Uncertainty index by Baker, Bloom and Davis (2016), which proxies for policy-related economic risk.

The results for the I(0), I(1), I(d) term premiums, displayed in Table 7, confirm the main message of the baseline regression: A correct term premium identification is crucial for economic analysis. According to the I(0)-VAR model, the term premium is strongly related to both inflation and inflation uncertainty. According to the I(1)-CVAR model, term premiums are positively related to unemployment rates and recession dummies, and negatively related to long-run inflation disagreement. The FCVAR model strikes a balance between the two and allows us to conclude that the term premium does not grow in line with inflation and inflation uncertainty.

[Insert Table 7: Term Premium Sources Regressions, Augmented Model]

4.3 Quantitative Easing and the Term Premium

In this final subsection, we provide a further illustration of the importance of correct term premium identification for economic analysis and policy. The illustration is based on a topic that has garnered a great deal of attention in recent years: The effects of large assets purchase programs (also called “quantitative easing”) on macro-finance variables, such as the term premium. To analyze this issue, we compute impulse responses to large shocks to the Central Banks balance sheet by means of local projections (LPs). The local projections methodology, developed by Jordà (2005), consists of running sequential predictive regressions of the endogenous variables on a structural shock and a set of

controls for different prediction horizons. Specifically, we estimate local projections of the following form:

$$y_{t+h} = \alpha_h + \beta_{0(h)}\epsilon_t + \sum_{i=1}^p \gamma_{i(h)}w_{i,t} + u_{(h),t+h}, \quad (13)$$

where y_{t+h} is the projection of the endogenous variable at the horizon h , ϵ_t is the shock of interest and $w_{i,t}$ is a vector of control variables. We consider four endogenous variables: The term premium, the risk neutral rate, core inflation and the year-on-year industrial production growth. The vector of control variables include a constant, the term premium, industrial production, inflation, the Gilchrist and Zakrajšek (2012) credit spread and the nominal Federal Funds Rate. We also control for the lags 1 to 6, 9 and 12 of each of these variables and of the endogenous variable of interest. The model is estimated by simple OLS, and confidence intervals can be computed using Newey-West corrected standard errors.

As discussed in Jordà (2005), LPs present several advantages with respect to a standard VAR. LPs do not require the specification and the estimation of an unknown data generating process and are therefore more robust to specification, are less affected by the curse of dimensionality and can easily accommodate non-linearities. He also shows that, even though LPs estimates of impulse responses are less efficient than VAR-based estimates, when the VAR is correctly specified and it is the true model, these efficiency losses are usually not large.

We identify a “quantitative easing” (QE) shock with two different strategies:

- First, we identify the shock as the residual of the regression of the first difference of the St. Louis Adjusted Monetary Base, Δm_t , on a set of controls: $\epsilon_t = \Delta m_t - E(\Delta m_t | w_{1,t}, \dots, w_{p,t})$. The set of controls includes industrial production

growth, the inflation rate, the Gilchrist and Zakrajsek (2012) credit spread, the term premium and the short-term interest rate. We also control for 12 lags of each of these variables and for 12 lags of Δm_t .

- Second, we identify the QE shock using the actual timing of the FED announcements. Specifically, we build a shock variable defined as the product of a dummy D taking value 1 when a new round of quantitative easing is implemented, and the growth rate in monetary base, Δm_t , which proxies for the importance of the policy change: $\epsilon_t = D \cdot \Delta m_t$. D is a dummy that takes value 1 in the month of the QE announcement and Δm_t is the monthly growth rate of the monetary base. In particular, the variable D takes value 1 in the following dates:

1. December 2008 (QE1): The FOMC approves the purchase of agency mortgage-based securities (MBS) and agency debt for up to 600 billion dollars.
2. March 2009: The FOMC expands its asset purchase program to a total of 1.25 trillion in purchases of agency MBS, 200 billion in government-sponsored enterprises (GSE) obligations, and up to 300 billion of longer-term securities.
3. November 2011 (QE2). The FOMC announces the intention of purchasing 600 billion of longer-term securities. Since, in our data, the actual monetary base started to increase the following month (in December 2012), the QE dummy takes a value 1 in December. Note, however, that this timing assumption does not affect the results.
4. September 2012 (QE3): The FOMC announces an open-ended commitment to purchase 40 billion agency MBS per month. Since, in our data, the actual monetary base started to increase the following month, the QE dummy takes a value 1 in October 2012. Again, this timing assumption does not affect the results.

The dummy variable is multiplied by the log change of monetary base, so that the resulting coefficients in the impulse response functions correspond to the actual percent change in the term premium when the change in QE episodes is 1%. Figure 6 shows the impulse responses of the macro-finance variables (including the FCVAR implied term premium and risk-neutral rate) to the QE shock following the second identification strategy. Results from the first strategy are quite similar, and we do not report them for brevity. The figure shows a protracted decline of the term premium together with an increase in the risk-neutral rate. This increase in the risk-neutral rate seems to be driven by the expansionary effects in inflation and economic activity which are shown in the two bottom panels of the Figure. Thus, the QE shocks had a clear effective expansionary effect in the real economy together with a considerable dynamic reduction in the risk aversion of investors toward long-term bonds. Because of this expected increase in real activity and inflation, investors (in a zero-lower bound environment) may have priced a future increase in the future monetary policy (short-term) interest rate. While this increase may have only materialized at a later date (see Bauer and Rudebusch (2016)), the risk neutral rate clearly reflects these expectations. For these empirical applications, we have not discriminated across different QE periods of time, which is something we will take into account in future research.

As a final illustration of the differences among the term premiums identified through the alternative modelling strategies, Figure 7 compares the impulse responses of three term premiums and associated risk-neutral rates (I(0)-VAR, I(1)-CVAR and FCVAR) to the same QE shock. Qualitatively, the responses of the FCVAR and the I(1)-VAR are similar (decline of the term premium and increases in the risk-neutral rate), but the responses of the I(1)-VAR are quantitatively larger. This is in contrast to the I(0)-VAR model, where the term premium response to the QE shock is basically null and not even

statistically significant. Thus, policy analysis based on purely I(0) VAR models can be especially misleading when focusing on the monetary policy effects on term premium dynamics. This is particularly relevant now, at this point in time, when policy makers are measuring the effects of future tapering of the monetary stimulus on term premium dynamics. While the current withdrawal of monetary stimulus is asymmetric in timing and size with respect to the QE injections (see Yellen (2017)), our results support the view that term premiums will tend to increase over time.

[Insert Figures 6 and 7: Term Premium and Risk-Neutral Responses to QE Shock]

5 Conclusions

This paper presents a natural model of the yield curve, capturing both the joint co-movement of U.S. sovereign rates and the existence of a common long-run trend present in the data. Our estimates of the flexible FCVAR model confirmed the existence of this trend and characterized it as a fractionally integrated mean-reverting process. Our analysis also rejects some of the standard stationary and unit-root alternatives to joint modelling of interest rates. The estimates implied by our general FCVAR model are thus able to capture both the low-frequency movements in bond yields and the mean reversion commonly assumed in many financial models.

As an important outcome of our exercise, this term structure model permits the identification of a credible term premium which can be readily used by both scholars and policy makers. We also shed light on the sources of the term premium, which are mainly real, i.e. while economic growth lowers the term premium, economic slack and recessions increase the risk priced by investors in long-term bonds. In a final exercise, we investigate the role of quantitative easing policies on term premium dynamics. In contrast to the

$I(0)$ stationary-implied term premium, our estimates of the FCVAR imply a significant term premium decline following the quantitative easing episodes.

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Table 1: Cointegrating Rank Test, Sovereign Yields

| Rank | Log-Likelihood | LR statistic |
|----------|-----------------|---------------|
| 0 | 1343.603 | 60.193 |
| 1 | 1751.443 | 25.127 |
| 2 | 1371.076 | 5.248 |
| 3 | 1372.619 | 2.162 |
| 4 | 1373.700 | — |

This table shows the results of the cointegrating rank test for the FCVAR model. In bold, the selected cointegration rank.

Table 2: LR Test, Sovereign Yields, CVAR v/s FCVAR

| | |
|------------------------|----------|
| Unrestricted log-like: | 1361.136 |
| Restricted log-like: | 1345.663 |
| LR statistic: | 30.947 |
| p-value: | 0.000 |

This table shows the results of the Likelihood Ratio (LR) Test, testing the likelihood of the FCVAR model vis à vis the I(1) CVAR model.

Table 3: LR Test, Sovereign Yields, $d = b$ v/s $d \neq b$

| | |
|------------------------|----------|
| Unrestricted log-like: | 1361.136 |
| Restricted log-like: | 1357.200 |
| LR statistic: | 7.873 |
| p-value: | 0.014 |

This table shows the results of the Likelihood Ratio (LR) Test, testing the likelihood of the FCVAR model with d different from b and the FCVAR model with the restricted model where $d = b$.

Table 4: Term Premium, Risk Neutral Rate: Descriptive Statistics

| | Variable | Mean | St.dev. |
|-----------|-------------------|--------|---------|
| I(0)-VAR | Risk neutral rate | 4.9527 | 2.1520 |
| | Term Premium | 1.4666 | 1.3163 |
| I(1)-CVAR | Risk neutral rate | 5.0422 | 3.7325 |
| | Term Premium | 1.3771 | 1.2590 |
| FCVAR | Risk neutral rate | 5.2297 | 3.1312 |
| | Term Premium | 1.1897 | 0.8628 |

This table shows the first and second moments of the term premium and risk neutral rates implied by the three alternative term structure models.

Table 5: Term Premium, Risk Neutral Rate: Descriptive Statistics

| | Variable | Corr w/FFR | Corr w/tp | Corr w/unempl | Corr w/ ΔY |
|-----------|-------------------|------------|-----------|---------------|--------------------|
| I(0)-VAR | Risk neutral rate | 0.96 | 0.54 | 0.001 | 0.21 |
| | Term Premium | 0.56 | 1 | 0.59 | -0.08 |
| I(1)-CVAR | Risk neutral rate | 0.98 | -0.66 | 0.009 | 0.15 |
| | Term Premium | -0.68 | 1 | 0.35 | -0.15 |
| FCVAR | Risk neutral rate | 0.98 | -0.22 | 0.11 | 0.11 |
| | Term Premium | -0.29 | 1 | 0.50 | 0.02 |

This table shows the correlations of the three alternative term premiums and risk neutral rates with several macro-finance variables: Federal Funds Rate, term premium, unemployment and industrial production growth, respectively.

Table 6: Term Premium Drivers: Simple Model

| 1990m4-2017m12 | TP I(0) | TP I(1) | TP FCVAR |
|---------------------------------|--------------------|--------------------|-------------------|
| Constant | -4.05*** (0.75) | -0.30 (0.50) | -0.62 (0.42) |
| Long-run Inflation Disagreement | 1.26*** (0.23) | -0.37*** (0.18) | -0.09 (0.14) |
| Unemployment Rate | 0.20*** (0.05) | 0.51*** (0.05) | 0.35*** (0.05) |
| Recession dummy | 0.30 (0.24) | 0.54*** (0.21) | -0.10 (0.14) |
| Adj. R2 | 0.41 | 0.52 | 0.38 |

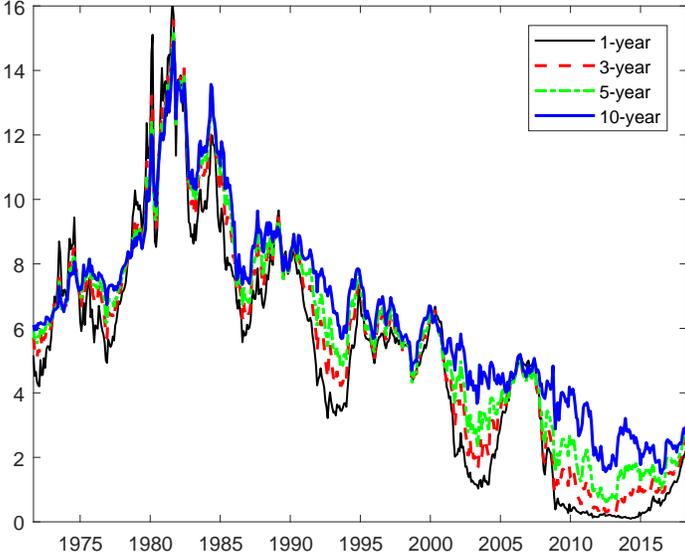
This table shows the result of the simple OLS regressions of the alternative term premium on macro-finance variables. Newey-West-corrected standard errors appear in parentheses.

Table 7: Term Premium Drivers: Augmented Model

| 1990m4-2017m12 | TP I(0) | TP I(1) | TP FCVAR |
|---------------------------------|--------------------|-------------------|-------------------|
| Constant | -3.54*** (0.65) | -0.26 (0.51) | -0.14 (0.44) |
| Core inflation | 0.50*** (0.13) | 0.02 (0.13) | 0.13 (0.11) |
| Long-run Inflation Disagreement | 0.63*** (0.22) | -0.38** (0.18) | -0.34** (0.17) |
| Unemployment Rate | 0.32*** (0.05) | 0.53*** (0.06) | 0.46*** (0.05) |
| Recession dummy | 0.45** (0.24) | 0.58*** (0.24) | 0.13 (0.18) |
| Int. rate volatility | 1.77*** (0.48) | 0.68 (0.56) | 0.57 (0.56) |
| Policy uncertainty | -0.76** (0.29) | -0.25 (0.28) | -0.75 (0.28) |
| Adj. R2 | 0.53 | 0.53 | 0.42 |

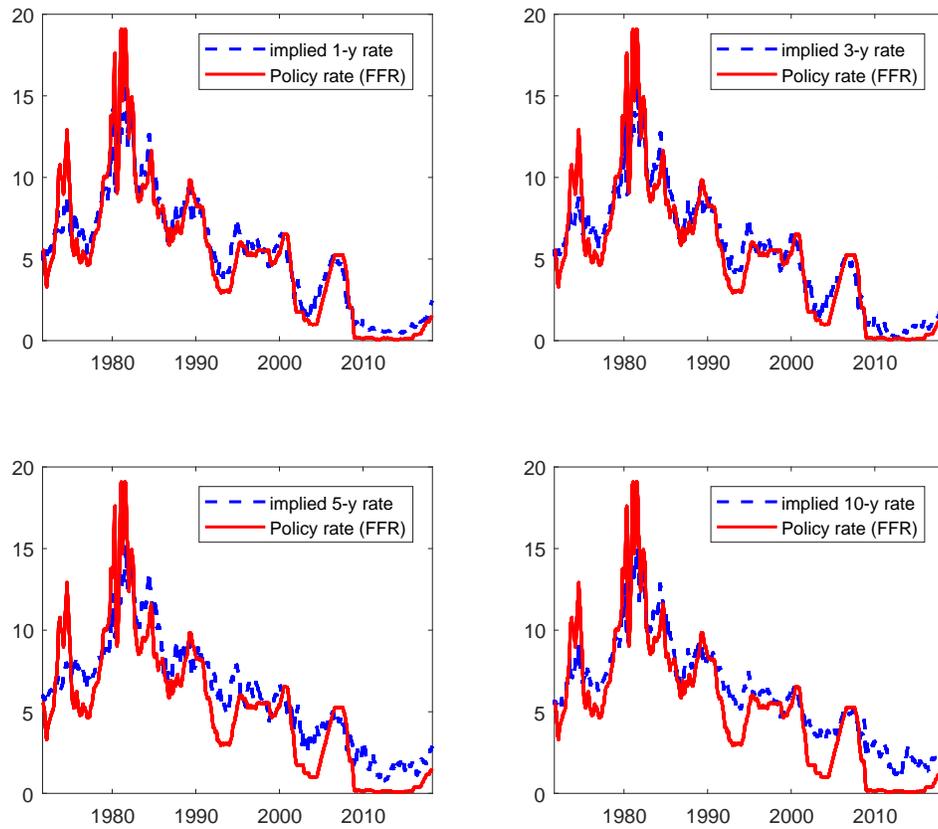
This table shows the result of the augmented OLS regressions of the alternative term premium on macro-finance variables. Newey-West-corrected standard errors appear in parentheses.

Figure 1: US Sovereign Rates



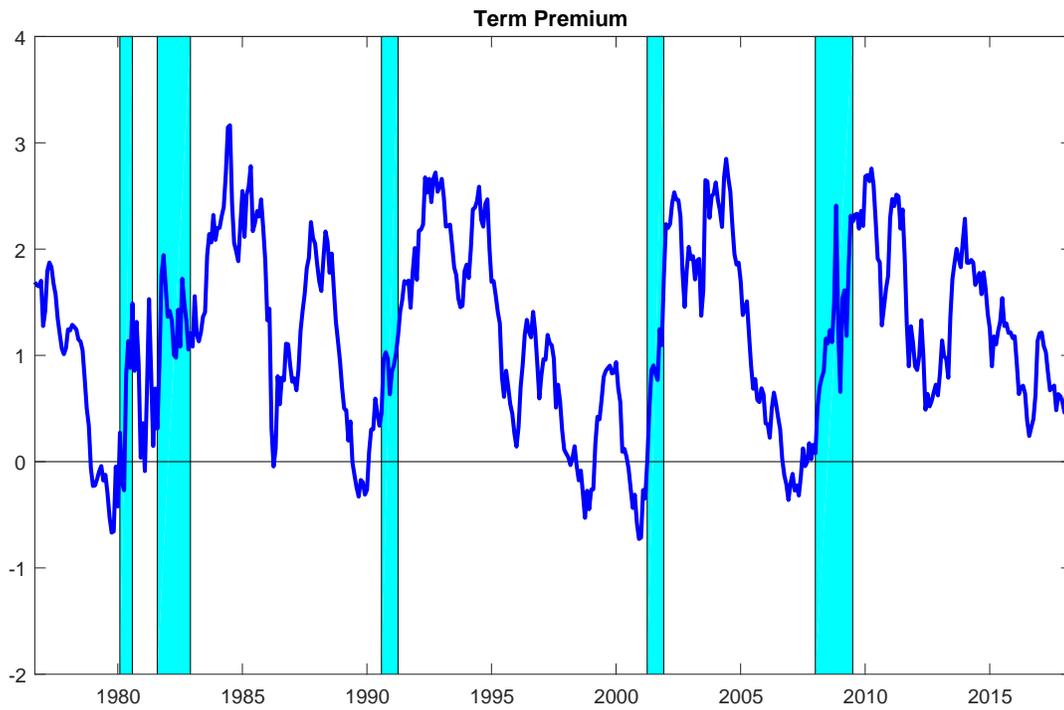
This figure plots the historical monthly series of zero-coupon US sovereign rates (1-year, 3-year, 5-year and 10-year).

Figure 2: Federal Funds Rate and FCVAR-implied Rates



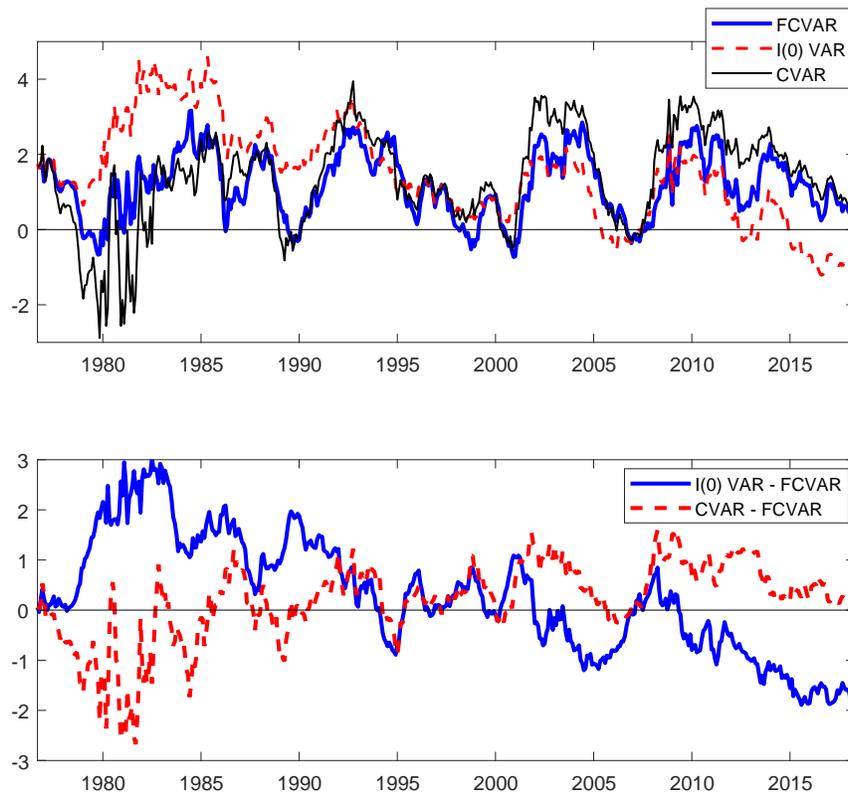
This figure plots the historical monthly series of the Federal Funds Rate (FFR) together with the FCVAR-long-run-implied US sovereign rates (1-year, 3-year, 5-year and 10-year).

Figure 3: Term Premium FCVAR



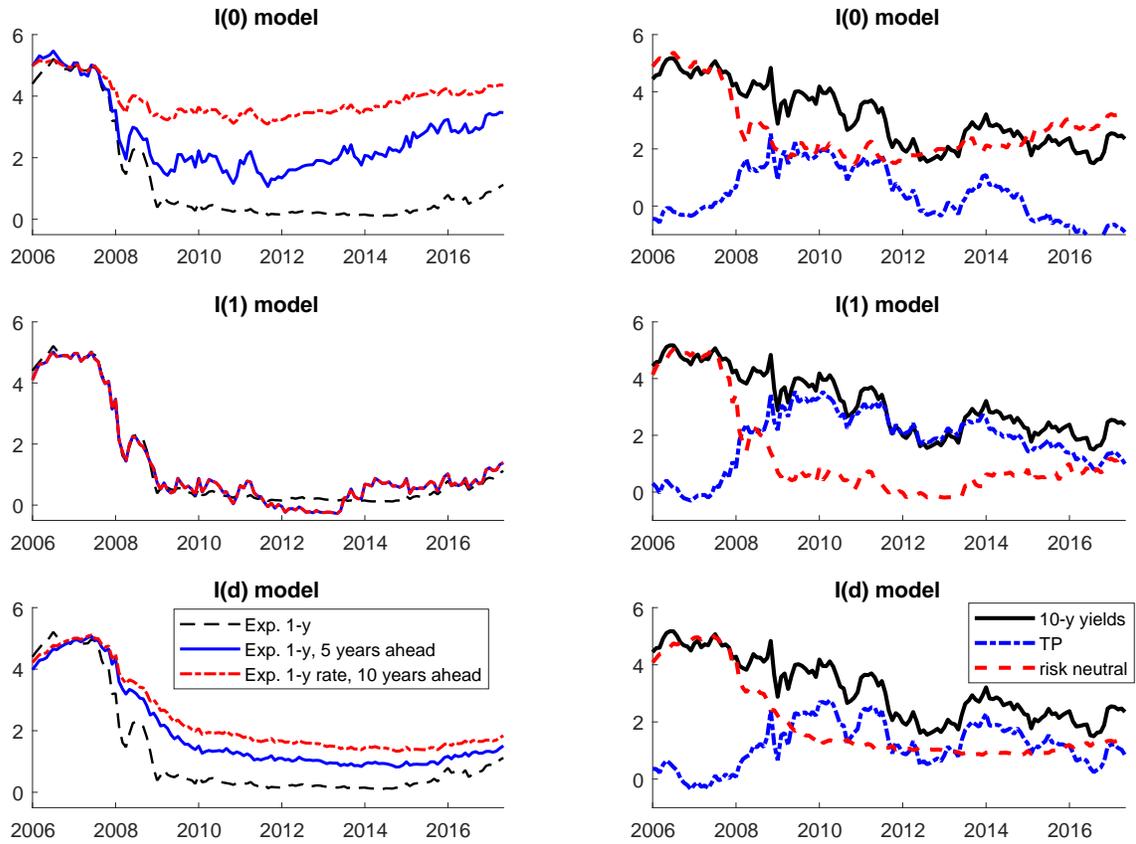
This figure plots the monthly term premium implied by the FCVAR. Shaded areas reflect NBER recession periods.

Figure 4: Differences in Term Premium: I(0) and CVAR v/s FCVAR



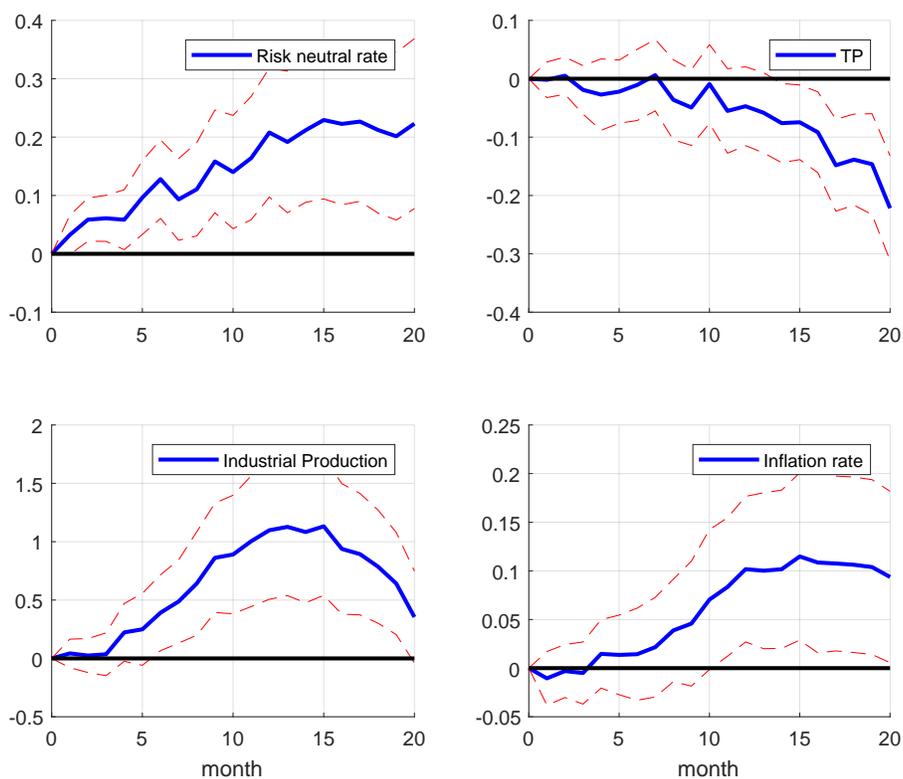
This figure plots the three monthly term premiums (CVAR, FCVAR and I(0)-VAR), as well as the differences between the FCVAR-implied term premium and the other two.

Figure 5: Differences in Long-Term Expectations, Term Premium and Risk-Neutral Rates: I(0) and CVAR v/s FCVAR



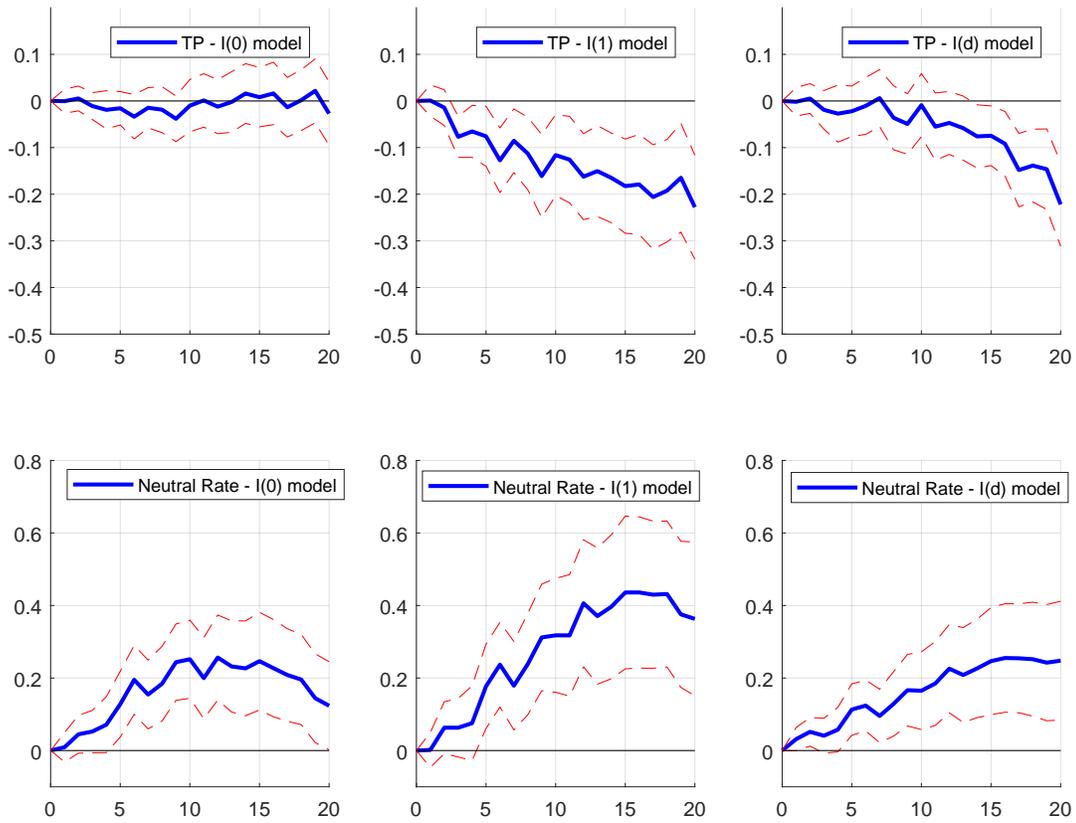
The graphs in the left column show the long-term expectations of the 1-year rate implied by the three models (I(0)-VAR, I(1)-CVAR and FCVAR (I(d))) for different horizons (1-year, 5-years and 10-years ahead), whereas those in the right column plots the implied term premium and risk-neutral rates across the three models together with the 10-year yields.

Figure 6: Macro-Finance Impulse Responses to QE Shock, FCVAR Model



This figure plots the impulse response function of the term premium, the risk-neutral rate, industrial production growth and inflation to a quantitative easing (QE) shock. The responses are identified via local-projection analysis and the term premium and risk-neutral rate through the FCVAR model with sovereign rates.

Figure 7: Term Premium and Risk-Neutral Rate Responses to QE Shock: I(0)-VAR, I(1) CVAR and FCVAR



This figure compares the impulse response function of the term premium and the risk-neutral rate to a quantitative easing (QE) shock (I(0)-VAR, I(1) CVAR and FCVAR (I(d) models)). The responses are identified via local-projection analysis.