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OPTIMAL LONG-RUN INFLATION AND THE INFORMAL ECONOMY[*]

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Abstract

This paper studies the optimal long-run rate of inflation in a two-sector model of the Lithuanian economy with informal production and price rigidity in the regular sector. The government issues no debt and is committed to follow a balanced budget rule. The informal sector is unregulated and untaxed and its existence limits the government's ability to collect revenues through fiscal policy. Such environment provides therefore the basis for quantifying the possible existence of a public finance motive for inflation. The main results can be summarized as follows: First, there is a strong heterogeneity in the optimal inflation rate which depends on the tax rate that is endogenously adjusted to keep the budget balanced. Inflation can be as high as 6.77% when the capital tax rate is endogenous, but when labor income taxes are adjusted optimal policy calls for a rate of deflation such that the nominal interest rate hits the zero lower bound. Second, the optimal inflation rate is a non-decreasing function of the size of the informal economy and, in most cases, there is a positive relationship between the two. Finally, substantial deviations from zero inflation are observed even in presence of a plausible degree of price rigidity.

Keywords: Optimal Inflation, Informal Economy, Endogenous Tax Changes.
J.E.L. Codes: E26, E52, H26.

1 Non-Technical Summary

The informal economy, which in the definition used in this paper encompasses all those transactions that are voluntarily kept hidden from the fiscal authorities to avoid the payment of taxes, is a widespread phenomenon in modern economies and a relevant issue for policymakers. Tax evasion limits, in fact, governments' ability to collect revenues in order to meet their obligations, thus forcing them to resort to debt issues, tax increases or, when allowed, to money printing. As taxpayers are the agents who suffer from tax hikes and debt financing the most, these measures provide a further incentive for evading taxes. It is then clear how easily governments can get caught in a vicious circle and why policymakers should be seriously concerned by the size of the informal economy.

This paper studies the optimal long-run level of inflation that should theoretically prevail in an economy where a share of production occurs in the informal sector. In such environment, the inflation target is in principle different from that arising in the absence of tax evasion. This claim directly follows from an intrinsic feature of the activities that take place in the informal sector in that they are, usually, entirely regulated by cash. Therefore, the policymaker has an incentive to set a positive rate of inflation as a mean to tax the informal activities that cannot be otherwise reached by standard taxation.

The model used in this analysis has two production sectors, formal and informal, and is calibrated to Lithuania, which features a sizable share of underground economy. The optimal level of inflation, defined as the one that maximizes the households' utility, is computed under the hypothesis that the government is committed to follow a balanced budget rule (i.e., it cannot issue debt to finance imbalances between revenues and expenditures). On the revenue side, the government budget constraint is composed of tax revenues (which can be lump-sum or levied on labor income, consumption and capital interests) and seigniorage revenues, that arise from money printing. It thus follows that changes in the rate of inflation (and, consequently, in seigniorage revenues) must be accompanied by changes in tax revenues. In this exercise, the optimal inflation target is computed by making one tax rate endogenous at a time.

The main results can be summarized as follows: First, there is a strong heterogeneity in the optimal inflation rate which depends on the tax rate that is endogenously adjusted to keep the budget balanced. Inflation can be as high as 6.77% when the capital tax rate is endogenous, but when labor income taxes are adjusted optimal policy calls for a rate of deflation such that the nominal interest rate hits the zero lower bound. This result is well rationalized throughout the paper. Second, the optimal inflation rate is a non-decreasing function of the size of the informal economy and, in most cases, there is a positive relationship between the two. As already stated, the existence of a cash-intensive underground sector provides an incentive to use inflation as a tax on informal activities. Finally, substantial deviations from zero inflation are observed even in presence of a plausible degree of price rigidity. The meaning of this last statement can be grasped by reckoning that price changes are costly for the firms: the interplay between prices and quantities demanded (which in turn depend on prices) affects, in fact, firms' profits and inflation clearly exerts influence on the firms' pricing behavior. This result contrasts most of the findings of the literature on

optimal monetary policy with price rigidity, in which price stability generally emerges as the optimal policy prescription.

2 Introduction

What is the optimal inflation target a Central Bank should aim for? The answer to this question is, in general, not trivial and unique and depends on the criterion defining optimality as well on the underlying features of the economy and on the consequences of monetary policy for the government budget.

In this regard, this paper studies the long-run rate of inflation that maximizes the agents' utility in an economic environment characterized by input and output imperfections arising from the existence of an untaxed informal sector. The reason why optimal monetary policy analysis is attractive in this framework stems from the fact that the informal economy relies heavily on cash as method of payment. Since inflation is actually a tax that penalizes cash and highly liquid investments, a policymaker might be tempted by using it as a tool to tax the otherwise untaxed underground economy. Moreover, there is evidence that countries where a large share of total activities occurs in the informal sector rely more on seigniorage revenues to meet their obligations and to avoid to excessively squeeze those who regularly pay taxes (see Mazhar and Méon, 2017).

The present work is not the first to study the optimal rate of inflation in a general equilibrium model with underground production. To the best of my knowledge, there are other few papers dealing with this issue. More precisely, Cavalcanti and Villamil (2003) and Koreshkova (2006) study the Ramsey optimal rate of inflation in a two-sector flexible prices economy where the income tax is the only source of revenue for the government. They both find that inflation is a positive function of the dimension of the underground economy and is particularly high for large values of the latter. Their contribution is important as they prove and motivate the non-optimality of the Friedman rule[1] in the presence of informality and tax evasion. Ahiabu (2006) derives the optimal inflation target in a model with bilateral trading and two markets, formal and informal, and proves that inflation is positive but can be large or small depending on the intrinsic characteristics of both markets. In models with costly tax collection, Arbex and Turdaliev (2011) and Arbex (2013) show that the Friedman rule ceases to be optimal but can be recovered under certain conditions.

This paper differs from these contributions along three dimensions: First, it builds a model that includes nominal frictions in the form of price adjustment costs, which makes it more suitable for carrying out policy analysis. Moreover, the degree of price rigidity greatly impacts the optimal rate of inflation. Second, the model features taxes on consumption, labor income and capital and envisages endogenous tax changes that follow the optimal choice of the inflation target, with the government honoring a balanced budget rule. Third, instead of following the Ramsey approach this analysis relies on static welfare maximization.[2,3]

[1]The Friedman rule is a well-known policy prescription claiming that in a monetary economy with flexible prices the optimal rate of inflation is negative and equal to minus the real rate of interest - such that the nominal interest rate is equal to zero. The Friedman rule has proved robust in a number of different frameworks (see Chari et al., 1996; Khan et al., 2003; Correia et al., 2008; Cunha, 2008).

[2]Research works that assess the optimal rate of inflation through a static measure of welfare include Cooley and Hansen (1991); De Gregorio (1993); Casares (2004); Amano et al. (2009); Fagan and Messina (2009); Antinolfi et al. (2016) and Brunnermeier et al. (2016).

[3]This approach leads to the calculation of the so called *golden rule* in the terminology of King and Wolman

The model presented in the next section is a two-sector model with informal production. The formal sector is characterized by a search and match mechanism *à la* Mortensen and Pissarides (1994). The informal sector is modeled assuming that the agents engaging in underground activities are informal self-employed workers as in Fernández and Meza (2015). A demand for money arises as a share of consumption purchases has to be paid cash, with the informal sector being cash-intensive compared to the regular one.[4] The model is calibrated to match as much as possible available data of the Lithuanian economy and is then used to compute the optimal inflation target under different assumptions regarding which tax rate is made endogenous following the implementation of the optimal policy.

Overall, the main results can be summarized as follows. First, there is a strong heterogeneity in the optimal inflation target in this economy which depends on the tax rate that is endogenously adjusted to satisfy the balanced budget rule. A change in the inflation target produces, in fact, two effects on the government budget constraint: a direct effect, in that it impacts seigniorage revenues, and an indirect effect as monetary policy affects the efficient allocation of resources which brings about changes in the tax base. In all the policy experiments considered in this paper, the optimal inflation target is the one which enables the policymaker to reduce the selected distortionary tax rate while keeping the government budget in balance. In the baseline calibration of the model, when the consumption tax rate is endogenous the optimal annual inflation target is 1.21%; when capital taxes are endogenous, optimal inflation is 6.77% and the tax rate is reduced by almost 6 p.p.; but when labor income taxes are adjusted, optimal policy calls for a rate of deflation such that the nominal interest rate hits the zero lower bound. When the government budget is brought back to equilibrium through lump-sum taxation, long-run inflation is low as it is not used to reduce distortionary taxes.

Second, the optimal rate of inflation is a non-decreasing function of the size of the informal economy. Precisely, it is increasing in the level of informality when lump-sum, consumption, or capital taxes are endogenous, though in the latter case the relationship is indirectly driven by the increase in the total amount of cash in the economy. With endogenous labor income taxes, the inflation target does not depend on the size of the informal sector unless there is limited use of cash in the formal sector.

Finally, nominal frictions do not prevent the policymaker from setting a rate of inflation different from zero because of the combined effect of the latter on the informal economy and on endogenous taxes. This statement holds true even in the presence of a significant degree

(1999), or the constrained long-run rate of inflation. Essentially, it is the inflation rate that maximizes the household's instantaneous utility under the constraint that the steady state conditions of the economy are imposed *ex-ante*. It is the rate of inflation that would arise if the policymaker can commit herself to follow such policy forever and is different from the unconstrained long-run rate of inflation, which can be computed using the Ramsey approach, where the steady state conditions are imposed *ex-post* on the first order conditions of the Ramsey problem. The choice of computing the constrained optimal rate of inflation in this model is motivated by the complexity of the problem when monetary policy is carried-out in the presence of endogenous distortionary tax rates. The Ramsey approach is certainly on the agenda for future research.

[4]Cash-intensity is a well-known feature of irregular activities. It is also the main intuition behind the currency-demand approach to estimate the size of the informal economy.

of price rigidity.

The paper is organized as follows. Section 3 presents the two-sector model with underground production. Section 4 describes the calibration strategy. Section 5 shows the results regarding optimal policy with endogenous taxes. It does so for the baseline calibration of the model and then extends the analysis to understand the role of the dimension of the informal economy, as well as that of cash and of price rigidity. Section 6 checks the robustness of the results to different modeling assumptions and to different values of some key parameters. Finally, Section 7 concludes and provide some comment on possible extensions for future research.

3 The Model

The model formally takes into account that, as in the real world, some of the economic transactions remain unobserved and untaxed as they occur in the so called *informal*, *shadow*, or *underground economy*. To this end, production in this environment is assumed to take place in two sectors: a formal sector with a labor market characterized by search frictions à la Mortensen and Pissarides (1994), and an informal sector where workers are self-employed as in Fernández and Meza (2015) and their activity remains concealed from fiscal authorities.

This economy is populated by a representative household who has preferences over a basket of formal and informal goods, whereas only labor supplied in the informal sector reduces her utility. This circumstance is a natural feature of search and match models of unemployment, but in this case is also supported by the social stigma attached to underground labor. A fraction of consumption purchases has to be regulated by cash through the inclusion of a cash-in-advance constraint, which motivates the household's demand for money. The household also undertakes investment decisions. The production sector consists of formal firms producing differentiated intermediate goods under monopolistic competition, of informal perfectly-competitive self-employed workers and of retailers who operate in perfect competition, buy the intermediate goods and aggregate them in order to produce a homogeneous final formal good.

I each period before the formal labor market opens, the household's members decide whether, and to what extent, to work in the informal sector: those who decide to do so are excluded from the job matching process in the regular sector. In order to be able to hire workers, formal firms are required to post vacancies and this activity is subject to a vacancy posting cost. Jobs are exogenously destroyed at a constant rate. Formal output is produced through a constant return to scale technology combining capital and labor. The informal sector, instead, produces via a decreasing return to scale technology with labor as the only input. In the intermediate formal sector, prices are sticky as in Rotemberg (1982). Finally, the government finances its expenditure by levying consumption, capital, labor income and lump-sum taxes and maintains a balanced budget rule over time.

This section provides a description of the model and of the primitives that regulate the economy.

3.1 The Representative Household

The representative household consists of a continuum of family members of mass 1 who constitute the labor force. The mass of formal employees, informal self-employed and unemployed is denoted by $N_{F,t}$, $N_{I,t}$ and U_t, respectively, and is such that:

$$1 = N_{F,t} + N_{I,t} + U_t. \tag{1}$$

In a generic period, after all the decisions have been taken and executed, a constant exogenous fraction ρ of people employed in the formal sector are separated from their jobs. At the beginning of each period, instead, before the formal labor market opens and production takes place, the household chooses the fraction of workers to be employed in the informal sector. The latter produce the informal consumption good and cannot participate as job seekers in the formal labor market.[5] Hence, the mass of family members who actively search for a job in a generic period t is defined by:

$$S_t = 1 - (1 - \rho)N_{F,t-1} - N_{I,t}. \tag{2}$$

Firms in the formal market post vacancies, v_t, at a constant per-vacancy cost. An attribute of the formal labor market is its tightness, θ_t, which is an indicator of the facility with which employers fill their vacant jobs and is defined by the following ratio:

$$\theta_t = \frac{v_t}{S_t}. \tag{3}$$

Jobs are created according to a standard matching technology

$$\mathcal{M}(S_t, v_t) = \chi S_t^{\xi} v_t^{1-\xi}, \tag{4}$$

which is strictly increasing and strictly concave in both its arguments and exhibits constant returns to scale. Using equations (3) and (4), the vacancy filling rate and the job finding rate can be defined, respectively, as follows:

$$q(\theta_t) = \frac{\mathcal{M}_t}{v_t} = \chi \theta_t^{-\xi}, \tag{5}$$

$$f(\theta_t) = \frac{\mathcal{M}_t}{S_t} = \chi \theta_t^{1-\xi}. \tag{6}$$

Assuming instantaneous hiring,[6] employment in the formal sector evolves according to

$$N_{F,t} = (1 - \rho)N_{F,t-1} + v_t q(\theta_t). \tag{7}$$

[5] This is a quite natural assumption since searching for a job is time consuming.

[6] This assumption is standard in most recent search models. It is also sensible given that the time period is a quarter.

The representative household's time t objective function is given by

$$U(\tilde{C}_t, N_{I,t}) = \ln \tilde{C}_t - \frac{N_{I,t}^{1+\varphi}}{1+\varphi}, \tag{8}$$

where $\tilde{C}_t$ represents aggregate consumption and is modeled as a CES aggregator of formal and informal consumption goods $C_{F,t}$ and $C_{I,t}$, respectively:

$$\tilde{C}_t = \left(\omega C_{F,t}^{\eta} + (1-\omega) C_{I,t}^{\eta}\right)^{\frac{1}{\eta}}. \tag{9}$$

Parameter ω is the weight attached to formal consumption, and η regulates the elasticity of substitution between the two goods. It can be noted that formal labor, $N_{F,t}$, does not appear in equation (8). This is a standard practice in labor markets characterized by search frictions, as workers bargain with firms over wages and supply labor inelastically at the equilibrium wage rate. The household, instead, derives disutility from working in the informal sector and the parameter φ represents the inverse of the Frisch elasticity of informal labor supply.[7]

The household's members regularly working in the formal sector earn the after-tax real wage $(1-\tau_t^w)w_{F,t}$ and those who remain unemployed are entitled to real unemployment benefits b. Consumption risks are perfectly shared among the family members and all the decisions are collectively taken by the household.[8] In this regard, she decides how much to consume and invest, supplies labor in the informal sector, buys one-period nominal bonds B_{t+1} at a price equal to the inverse of the nominal interest rate, $E_t Q_{t,t+1} = R_t^{-1}$, receives nominal profits D_t from the ownership of intermediate formal firms and pays lump-sum taxes T_t^{ls}. The household accumulates capital, K_t, in order to rent it out to formal firms in exchange of the net return $(1-\tau_t^k)r_t^k$, according to following law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{10}$$

where δ is the capital depreciation rate. Moreover, it is assumed that a constant fraction of total consumption expenditure in both sectors requires the household to hold cash, which gives rise to a demand for money

$$M_t \geq \nu_F(1+\tau_t^c)P_t C_{F,t} + \nu_I P_{I,t} C_{I,t}, \tag{11}$$

where τ_t^c is the consumption tax rate, P_t is the price of the formal good and $P_{I,t}$ that of the informal good. Parameters $\nu_F, \nu_I \in [0,1]$ respectively measure the cash intensity in the formal and informal sector, that is the fraction of total purchases of the consumption good paid in cash. In principle, $\nu_I > \nu_F$ as the transactions that takes place in the informal sector are generally regulated by cash. The informal self-employed family members directly

[7]Indeed, the parameter φ is not exactly the inverse of the Frish elasticity, as the latter involves the existence of a wage rate in the informal sector, which is absent in this economy due to the modeling assumptions. More precisely, it is the inverse of the elasticity of informal labor supply to a change in the marginal revenue of self-employed underground activities.

[8]See Andolfatto (1996) and Merz (1995).

produce the informal output using a decreasing return to scale technology[9] with parameter α_I and with labor as the sole production input

$$Y_{I,t} = N_{I,t}^{\alpha_I}. \tag{12}$$

The household's budget constraint in nominal terms is therefore

$$\begin{aligned} P_t C_{F,t}(1+\tau_t^c) + P_{I,t}C_{I,t} + E_t Q_{t,t+1} B_{t+1} + M_t - M_{t-1} + P_t I_t \\ = P_t w_{F,t} N_{F,t}(1-\tau_t^w) + P_t r_t^k K_t (1-\tau_t^k) + P_{I,t} Y_{I,t} + B_t + P_t b U_t + D_t + T_t^{ls}. \end{aligned} \tag{13}$$

The household takes decisions on $C_{F,t}, C_{I,t}, K_{t+1}, N_{I,t}, B_{t+1}$ and M_t, taking as given the tax rates τ_t^c, τ_t^w and τ_t^k, the wage rate $w_{F,t}$, the nominal interest rate R_t, the return on capital r_t^k, both prices P_t and $P_{I,t}$, the unemployment benefit b and the probability of being hired in the formal sector $f(\theta_t)$, in order to maximize

$$E_0 \sum_{t=0}^{\infty} U(\tilde{C}_t, N_{I,t})$$

subject to the constraints (1), (2), (7) and (9)−(13). After some substitutions, the household's behavior is entirely described by the following equations:

$$\frac{\omega}{\tilde{C}_t}\left(\frac{\tilde{C}_t}{C_{F,t}}\right)^{1-\eta} = (1+\tau_t^c)\lambda_t\Big(1+\nu_F(1-R_t^{-1})\Big), \tag{14}$$

$$\frac{(1-\omega)}{\tilde{C}_t}\left(\frac{\tilde{C}_t}{C_{I,t}}\right)^{1-\eta} = p_{I,t}\lambda_t\Big(1+\nu_I(1-R_t^{-1})\Big), \tag{15}$$

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[(1-\tau_{t+1}^k)r_{t+1}^k + 1 - \delta\right], \tag{16}$$

$$\alpha_I p_{I,t} N_{I,t}^{\alpha_I - 1} = \frac{N_{I,t}^{\varphi}}{\lambda_t} + f(\theta_t)(1-\tau_t^w)w_{F,t} + (1-f(\theta_t))b, \tag{17}$$

$$\frac{1}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}, \tag{18}$$

where π_{t+1} is the change in the formal sector price level between dates t and $t+1$.[10] The first two equations represents the demand for the formal and informal goods, respectively, when cash payment are considered, Equation (16) regulates investment decisions, whereas the last equation is the standard Euler condition. Equation (17) is the condition that regulates the supply of informal labor and, therefore, the participation in the formal labor market.

[9]This assumption is common in the literature of the shadow economy. As noted by Koreshkova (2006), it is equivalent to the assumption of the existence of convex costs of concealing the informal business.

[10]The price of the formal good acts as numeraire in this economy.

Consider the opportunity cost of working in the informal sector, that is the right hand side (RHS) of (17). It is given by the labor disutility borne by the informal self-employed plus a weighted average of the after tax formal wage and the unemployment benefit, where the weight is represented by the probability of finding a job if participating in the search and match process.[11] The self-employed family member supplies labor in the informal sector until the marginal revenue of this activity, left hand side (LHS) of (17), equals the opportunity cost. Whenever conditions in the formal labor market improve (i.e., the agent observes an increase in $f(\theta_t), w_{F,t}, b$ or a decrease in τ_t^w), informal labor supply is reduced and participation in the formal labor market is increased. In this sense, labor force participation in the two sectors is endogenous.[12]

3.2 Formal Retailers and Intermediate Goods Producers

Formal retailers operate under perfect competition. They buy the differentiated intermediate goods and aggregate them into a final good using the the following CES technology:

$$Y_{F,t} = \left(\int_0^1 Y_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Profit maximization gives the demand for the j^{th} intermediate good:

$$Y_{F,t}(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_{F,t}, \tag{19}$$

where the associated price index is given by

$$P_t = \left(\int_0^1 P_t(j) dj \right)^{\frac{1}{1-\epsilon}}.$$

As in the standard Mortensen and Pissarides (1994) framework, any j-th $\in [0,1]$ intermediate firm that is not matched with a worker at time t might post a vacancy and pay a cost of κ units of the final consumption good. If a firm and a worker match, production occurs in the same period by means of a constant returns to scale Cobb-Douglas technology

$$Y_{F,t}(j) = N_{F,t}(j)^{\alpha_F} K_t(j)^{1-\alpha_F} \tag{20}$$

with parameter α_F. Intermediate formal firms operates under monopolistic competition and face convex adjustment costs when changing prices. Firm j nominal profits reads as follows:

$$D_{F,t}(j) = P_t(j)Y_{F,t}(j) - P_t w_{F,t}(j) N_{F,t}(j) - P_t r_t^k K_t(j) - P_t \kappa v_t(j) - \frac{\phi_F}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 P_t Y_{F,t}(j),$$

[11] This result is obtained under the assumption that those working as informal self-employed do not receive the unemployment benefit despite they do not appear among the employed. The reason is that by filing for the benefit the informal worker gets exposed to fiscal authorities.

[12] Although with due distinctions, this feature is similar to the economy described by Campolmi and Gnocchi (2016), where the outside option is given by home production.

where ϕ_F is a parameter regulating the degree of price rigidity. The firm j's problem is to choose $N_{F,t}(j), v_t(j)$, $K_t(j)$ and $P_t(j)$ in order to maximize the present value of the future stream of profits

$$E_t \sum_{n=0}^{\infty} \frac{\lambda_{t+n}}{\lambda_t} D_{F,t+n}(j)$$

subject to the constraints (19) and (20). The solution to this problem is represented by the following three equations that respectively describe the vacancy posting condition, the demand for capital and the so called New-Keynesian Phillips curve, where the j index has been eliminated as convex adjustment costs result in a symmetric behavior among firms:

$$\frac{\kappa}{q(\theta_t)} = \alpha_F mc_{F,t} \frac{Y_{F,t}}{N_{F,t}} - w_{F,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\frac{\kappa}{q(\theta_{t+1})}, \tag{21}$$

$$r_t^k = (1-\alpha_F) mc_t \frac{Y_{F,t}}{K_t}, \tag{22}$$

$$\epsilon - 1 - \epsilon mc_{F,t} + \phi_F(\pi_t - 1)\pi_t - \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\phi_F(\pi_{t+1}-1)\pi_{t+1}\frac{Y_{F,t+1}}{Y_{F,t}} = 0. \tag{23}$$

3.3 Wage Determination

In the formal sector, firms and workers negotiate the wage rate in order to share the surplus of a match. The worker's value of being employed is given by

$$V_t^E = w_{F,t}(1-\tau_t^w) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\Big\{(1-\rho)V_{t+1}^E + \rho f(\theta_{t+1})V_{t+1}^E + \rho(1-f(\theta_{t+1}))V_{t+1}^U\Big\}. \tag{24}$$

Equation (24) shows that the worker's value of a match depends in the current period on the after-tax wage paid by the firm. In the next period, the worker retains his job with probability $1-\rho$, whereas he will lose it with probability ρ. If separation occurs, the worker might find another job in the same period with probability $f(\theta_{t+1})$ or stay unemployed with probability $1-f(\theta_{t+1})$. The value of being unemployed is, in turn, given by

$$V_t^U = b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\Big\{f(\theta_{t+1})V_{t+1}^E + (1-f(\theta_{t+1}))V_{t+1}^U\Big\}. \tag{25}$$

According to (25), the unemployed worker receives the unemployed benefit b and, in the following period, he will be matched with a formal firm with probability $f(\theta_{t+1})$ or will remain unemployed with probability $1-f(\theta_{t+1})$.[13]

Similarly, the firm's value of a match is simply

$$V_t^F = \alpha_F mc_{F,t}\frac{Y_{F,t}}{N_{F,t}} - w_{F,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left\{(1-\rho)V_{t+1}^F\right\}. \tag{26}$$

[13]Note that the possibility of supplying labor in the informal sector does not affect the worker's value functions. This happens as the latter describe the condition of a worker who is participating in the formal labor market, whereas informal-self employed workers are non-participants.

As a standard practice in search models of unemployment, wage is determined by means of Nash bargaining over the match surplus. The surplus sharing rule is given by

$$\frac{\psi}{1-\psi}(1-\tau_t^w)V_t^F = V_t^E - V_t^U,$$

where $\frac{\psi}{1-\psi}$ is the relative bargaining power of workers. The resulting equilibrium wage is then found to be

$$w_{F,t} = \psi\alpha_F mc_t\frac{Y_{F,t}}{N_{F,t}} + \frac{(1-\psi)}{(1-\tau_t^w)}b - \psi\beta\frac{\lambda_{t+1}}{\lambda_t}(1-\rho)\frac{\kappa}{q(\theta_{t+1})}\left\{(1-\theta_{t+1}q_{t+1})\frac{(1-\tau_{t+1}^w)}{(1-\tau_t^w)} - 1\right\}. \tag{27}$$

3.4 Government Sector and Aggregation

The government issues money and collects consumption, labor income and lump-sum taxes in order to finance its exogenous purchases and to pay benefits to the unemployed workers. In doing so, it maintains the following period-by-period balanced budget rule:

$$\bar{G} + bU_t = \tau_t^w N_{F,t} w_{F,t} + \tau_t^c C_{F,t} + \tau_t^k r_t^k K_t + m_t - \frac{m_{t-1}}{\Pi_t} + T_t^{ls}. \tag{28}$$

Assuming that public expenditure is entirely in formal goods, there are two separate aggregate resource constraints for the formal and informal sector, respectively:

$$Y_{F,t} = C_{F,t} + \bar{G} + I_t + \kappa v_t + \frac{\phi}{2}(\Pi_t - 1)^2 Y_{F,t}, \tag{29}$$

$$Y_{I,t} = C_{I,t}. \tag{30}$$

4 Calibration

This section describes the model calibration. The objective is to match as much as possible data of the Lithuanian economy, although some of the parameter values are set according to the standard literature. The time period is a quarter.

A first set of parameters and steady state ratios is related to the labor market. The unemployment rate U_t is set according to the 2015 figure provided by Eurostat and is equal to 9% of total labor force. The share of informal to total workers equals 12.08% and is drawn from the estimate provided by Pocius (2015). The job finding probability, $f(\theta_t)$, is computed with the procedure suggested by Shimer (2012) using the data in the Eurostat Labor Force Statistics (LFS) and is equal to 0.1423; the job separation rate is implicitly computed by combining equations (2) and (6) and is equal to 0.0187, in line with the data in the LFS that show a rate between 0.01 and 0.02 over the year 2015. The vacancy filling rate, $q(\theta_t)$, is set to 0.6, which delivers a number of vacancies v equal to 0.0249. Given these numbers, market tightness θ is 0.2372. As for the wage bargaining process, it is well known from

Petrongolo and Pissarides (2001) that ψ has to lie in the interval $[0.3, 0.5]$ and, therefore, the intermediate value 0.4 is chosen. In order to satisfy the Hosios condition, the elasticity of matches ξ in the matching function is set to 0.6, which delivers χ equal to 0.2530.

A second set of parameters concerns the household's behavior. The inverse of the Frisch elasticity of labor supply in the informal sector, φ is set to 1. In equation (9), the parameter η governs the elasticity of substitution between formal and informal goods. Given the absence of data on informal consumption, the value of this parameter is set to 0.875 (implying an elasticity of substitution equal to 8) in accordance with previous works by Restrepo-Echavarria (2014) and Fernández and Meza (2015). The formal good share of consumption, ω, is implicitly derived and equals 0.5644. The subjective discount factor β equals 0.99, implying an annual nominal interest rate of about 4% in the absence of trend inflation (i.e., $\pi = 1$). Two relevant parameters in this model are ν_F and ν_I, which determine the share of consumption purchases in both sectors paid in cash, therefore affecting the quantity of money circulating in the economy and, consequently, the conduct of optimal monetary policy. In the informal sector, it assumed that all the transactions require cash. Hence, ν_I is set equal to 1. In order to calibrate cash-intensity in the formal sector, ν_F, information provided in the Payments Market Review by the Central Bank of Lithuania (2016) is used. According to this study, 33% of the funds that are connected to a credit/debit card is used for payments at the point of sale or via the internet (the remaining 67% is withdrawn at the ATM). The strategy is then as follows: if the household has €1, €0.33 is spent for consumption purchases that not require the use of cash (i.e., $(1-\nu_F)(1+\tau^c)C_F + (1-\nu_I)p_I C_I$), whereas €0.67 is withdrawn and spent in cash-regulated transactions (i.e., $\nu_F(1+\tau^c)C_F + \nu_I p_I C_I$). Given this information, the parameter ν_F is treated as an unknown variable and computed along with other unknowns as the solution to a system of equations. The result is $\nu_F = 0.6229$. The parameter α_I in the informal technology is set equal to 0.8.

As concerns the formal production sector of the economy, the elasticity of substitution among intermediate good varieties, ϵ, is set to 7.5 in order to obtain a price mark-up of about 15%, as suggested by Thum and Canton (2015). The parameter in the formal technology, α_F, is set equal to 0.65. The degree of price rigidity, ϕ_F, is set to 50 as in Faia (2008) and the vacancy posting cost, κ, to 0.05.

Finally, the central government collect taxes and issues money in order to finance its expenditures. Lithuania has a flat 15% income tax rate, therefore $\tau^w = 0.15$. Also, Lithuanian tax code treats capital gains as ordinary taxable income, therefore $\tau^k = 0.15$. The consumption tax, τ^c is set to 0.21, equal to the country's VAT.[14] Lump-sum taxes are endogenously determined and equal -0.0367 in the baseline calibration - that is, they represent net transfers to the household sector. The unemployment benefit is also endogenous and equal to about 84% of the equilibrium wage in the formal sector. Public expenditure is chosen so as to match the 2015 ratio $G/Y_F = 0.175$. Capital depreciation, δ, equals 0.03 and implies a ratio $I/Y_F = 0.193$ that perfectly matches what observed in the data for 2015. The model parametrization is summarized in Table 1.

[14]Consumption tax and VAT are not equivalent concepts. Although this lack of exactness, it is common to set the two equal in macroeconomic models.

5 Optimal Long-Run Inflation, Taxes and the Informal Economy

The choice of the inflation target is a delicate task for central bankers. Inflation targeting was first introduced in 1990 in New Zealand and has since then been adopted by several countries. The introduction of such a policy has brought benefits in the form of increased Central Banks' credibility and a more stable economic environment. In macroeconomic models, the theoretical value of the inflation target is generally established through a welfare criterion and depends largely on the fundamentals of the economy under analysis.

Differently from standard analysis of this type, the problem described here is made more interesting by the fact that the economy is characterized by the existence of an informal sector where transactions remain untaxed and are entirely regulated by cash. Therefore, monetary policy affects - and is affected by - the dimension of the unobserved economy. Moreover, monetary policy is also linked to the existence of endogenous taxes that can be set accordingly by the policymaker. In fact, the government's commitment to follow a balanced budget rule such (28) implies that, under the assumption of fixed government purchases, changes in the level of inflation must be followed by a passive endogenous adjustment in the other sources of revenue (i.e., in lump-sum or distortionary taxes). Hence, the model allows to study different monetary and fiscal policy mixes and their interaction with the degree of informality in the economy.

The section is divided in three parts: First, given the baseline calibration of the model, the welfare-maximizing steady state inflation is computed under different hypotheses concerning the fiscal policy instrument that is adjusted in response to the change in resources available to the government; second, it will be shown how changes in the size of the informal sector affect the results obtained in the previous part; finally, the section is concluded with a discussion on the role of cash and of the degree of price rigidity in the determination of the optimal inflation target.

5.1 Optimal Long-Run Inflation with Endogenous Taxes

Consider a policymaker who must decide on the optimal long-run rate of inflation in the economy. This type of problem is common in monetary policy analysis an its solution is largely affected by the underlying characteristics of the economy, as well on the assumption regarding the conduct of fiscal policy.

In the model presented in Section 3, the most important feature that is likely to affect the conduct of monetary policy is the existence of an informal, untaxed, cash-intensive production sector.[15] Inflation can be deemed, in fact, as a tax that applies to all the economic agents who thus experience a decrease in the real value of their money holdings. However, the fact that all the transactions taking place in the informal sector involve the sole use of

[15]To be precise, the degree of price rigidity is another important factor. Even though the focus of this study is on the relationship between the informal economy and monetary policy, the last part of this section will isolate and uncover the role of price rigidity.

cash should provide an incentive to set a larger inflation target than the one observed in an economy without informal production. The policymaker might find attractive to use the instrument of inflation for two reasons: first, it is a way to tax those activities that otherwise would not be subject to any form of taxation; and second, it might reduce the total amount of transactions occurring underground.

Another important aspect in the determination of the inflation target is the relationship between monetary and fiscal policy. Such a relationship can be easily grasped by taking a look at the steady state version of the government budget constraint (28):

$$\bar{G} + bU = \tau^w N_F w_F + \tau^c C_F + \tau^k r^k K + \frac{\pi^\star - 1}{\pi^\star} m + T^{ls}. \tag{31}$$

The LHS of (31) represents the overall government expenditure, which is composed of a fixed amount of public consumption, $\bar{G}$, plus the benefits payed to the unemployed workers. The RHS of (31) contains all the sources of government revenues, which depend both on monetary and fiscal policy. Fiscal rule (31) also implies that the government is committed to maintain its budget balanced without resorting to debt issues. Changes in the long-run inflation rate produce two types of effects in identity (31): a direct effect in the measure of changes in seigniorage revenues for every value taken by m, and an indirect effect in the measure that changes in the rate of inflation affect the tax base and the level of unemployment. It thus follows that changes in the long-run rate of inflation require endogenous changes in one or more tax rates in order to maintain the equality in (31). In the terminology proposed by Leeper (1991) monetary police is *active*, whereas fiscal policy is *passive*.[16]

The discussion proposed above therefore motivates the following exercise: starting from the model equilibrium with zero inflation, the optimal inflation target is computed under four different hypotheses concerning the tax rate made endogenous. These are:

- P_1: endogenous lump-sum taxes T^{ls};
- P_2: endogenous consumption tax rate τ^c;
- P_3: endogenous capital tax rate τ^k;
- P_4: endogenous labor income tax rate τ^w.

Formally, the problem consists in

$$\max_{\pi^\star} U(\tilde{C}, N_I)$$

$$\text{s.t. the steady state version of equations}$$
$$\left\{(1)-(7), (9)-(12), (14)-(23), (27)-(30)\right\}.$$

[16]This statement deserves attention since, as it will become clear in a while, optimal monetary policy will provide different results depending on the choice of which tax rate to be made endogenous. However, fiscal policy can be deemed passive in that it responds to a well defined balanced budget rule.

Moreover, the policymaker faces an additional constraint when solving the problem stated above, that is the zero lower bound (ZLB) on the nominal interest rate. Since the steady state value of the latter is simply given by the ratio $\pi^\star/\beta$, it follows that the additional constraint is

$$\pi^\star \geq \beta.^{17}$$

Table 2 summarizes the optimal inflation target and the associated tax rates implied by policies P_1–P_4. A fist glance at this table immediately reveals a strong heterogeneity in the optimal policy prescriptions that essentially depends on which tax instrument is endogenously adjusted along with the chosen inflation rate. Notably and differently from policies P_1–P_3, the optimal inflation target is negative in P_4 at a level for which the nominal interest rate hits the zero lower bound. In the other three cases, the inflation target is positive and ranges from a value slightly above zero with endogenous lump-sum taxes to more than 6.5% when the capital tax rate is endogenous.

Understanding this huge difference in results requires some preliminary considerations. First of all, it is extremely important to identify the channels through which the rate of inflation affects the economy in this model. As consumption purchases require the use of cash, changes in the inflation level alter the real value of money holdings, bringing about changes in the consumers' purchasing power. In this regard both the formal and the informal sectors are affected by changes in $\pi^\star$, with higher inflation levels associated to a lower purchasing power. However, the cash-intensive nature of the transactions occurring in the informal sector implies that the latter is hit more strongly by increases in $\pi^\star$ than its formal counterpart, meaning that the informal good consumption becomes relatively more costly. Inflation plays also an important role by affecting the government budget constraint. Isolating its effect from the influence on the other variables of the economy (i.e., a partial equilibrium analysis), higher inflation generates larger seigniorage revenues that increase the availability of resources in the RHS of (31), which could be used, at least in principle, to reduce taxes. Finally, inflation - better, deviations from price stability - represents a cost in terms of output due to the Rotemberg's pricing assumption. In fact, the steady state formal sector aggregate resource constraint

$$Y_F \left[1 - \frac{\phi}{2}(\pi - 1)^2\right] = C_F + \bar{G} + I + \kappa v$$

clearly shows how nominal rigidities create a wedge between output and formal consumption, public expenditure, investment and vacancy posting cost because part of the output goes in the price adjustment cost.

A second set of considerations concerns the nature and the objective of these policy experiments. The policymaker aims, in fact, at maximizing consumers' welfare by choosing the optimal rate of inflation and observing the constraint (31). A common trait of policies P_1–P_4, as shown in Table 2, is that the optimal inflation target is always set at a value that is compatible with a reduction of the corresponding tax rate. Because of the purely static nature of the problem, every chosen value of the policy instrument $\pi^\star$ involves a

[17] Precisely, $\pi^\star/\beta$ is steady state value of the gross nominal interest rate. Hence, a value lower than unity implies a negative net rate of interest.

simultaneous adjustment of all the other endogenous variables in the model, including the change in the tax rates that satisfies (31). In this context, policies P_1–P_4 are equivalent to another set of policies in which the very same tax rates are used as instruments by the policymaker and the inflation rate is endogenously determined by the equilibrium set of equations describing the model economy. Changes in monetary and fiscal policies must be thus deemed as contemporaneous and there is no causal effect of one on the other.[18]

With these considerations in mind, the results presented in Table 2 can now be discussed taking also advantage of the data reported in Table 3, which shows the new equilibrium values of the selected variables that result from the implementation of Policies P_1–P_4, in percentage deviation from their initial steady state. Start from P_1, in which the observance of the fiscal rule (31) is guaranteed through the adjustment of lump-sum taxes. In this scenario, the annualized optimal rate of inflation is small and equal to 0.67%. As inflation mainly negatively affects the cash-intensive informal sector, a $\pi^\star > 0$ induces the households to substitute formal consumption for informal consumption and generates, therefore, a shift of resources towards the regular economy. Agents are better off as they prefer formal good consumption and dislike irregular work. However, since the resources freed by larger seigniorage revenues can be used to reduce lump-sum, non distortionary, taxes only, gains from a positive $\pi^\star$ are limited and small compared to its costs and the resulting optimal inflation target is small too.

The outcomes of Policies P_2 and P_3 can be easily discussed following the same line of reasoning used in the previous analysis of P_1. In both P_2 and P_3 the optimal inflation target is positive and larger than the one obtained under P_1, despite the magnitude of $\pi^\star$ dramatically differs between the two policies. In fact, the annual inflation target equals 1.21% when the related proceeds are used to reduce consumption taxes (i.e., Policy P_2) and is as high as 6.77% in P_3. This more than 5 p.p. difference in results arises as capital taxes are in general highly distortionary and the resulting optimal inflation rate enables a reduction of τ^k of almost the same amount of the increase in $\pi^\star$ (see Table 2). This result appears to be consistent with the literature on optimal capital taxation, which advocate the long-run optimality of non-positive capital taxes in macroeconomic models with homogeneous agents.[19] Since capital accumulation is a powerful driver of economic growth, the costs of high inflation are overcome by the large cut in τ^k. Moreover, in this model capital is a production input in the formal sector only and the combined effect of higher inflation and

[18]This last consideration will prove very useful in discussing the results displayed in Tables 2 and 3, especially those concerning Policy P_4.

[19]In the standard Ramsey model without frictions and rigidities, the optimal capital tax is zero. See Judd (1985, 1999); Atkeson et al. (1999) and, more recently, Teles et al. (2016). In a medium-scale model of the U.S. economy with imperfect competition and nominal and real rigidities, Schmitt-Grohé and Uribe (2005) show that optimal capital taxation is characterized by a large and volatile subsidy. Positive taxes on capital can be optimal under particular features of the economy. For instance, Aiyagari (1995) finds that the optimal capital tax rate is positive in a model with heterogeneous agents, incomplete markets and borrowing constraints - i.e., in the *Bewley* class of models. When search-and matching frictions are taken into account, Arsenau and Chugh (2006) prove the optimality of positive capital taxes provided that employed and unemployed workers display welfare heterogeneity. When this assumption is relaxed, optimal capital taxes are, again, zero.

lower capital taxes represents a strong incentive to divert resources away from the informal economy.

The results associated with Policy P_4, in which labor income taxes adjust along with the optimal inflation rate, are indeed more controversial and cannot be explained by the very same mechanism underlying Policies P_1–P_3. In order to understand why a reduction in the labor income tax rate is associated with a negative rate of inflation, it is important to remind that the change in τ^w and that in $\pi^\star$ occur contemporaneously and that the very same outcome could be reached with τ^w rather than $\pi^\star$ as policy instrument. Interpreting the results in P_4 would, therefore, require answering the following question: Why a reduction in the labor income tax rate is associated with a negative rate of inflation? Differently from Policies P_1–P_3, in P_4 the policymaker directly intervenes on the formal labor market by reducing τ^w and, consequently, lowering the equilibrium wage in the formal sector, w_F (see the last column of Table 3). Cheaper labor increases regular workers hiring and the rate of unemployment, U, considerably falls from 9% to about 8.2%. Such a reduction in U, which does not occur in any of the other policies as unemployment remains relatively stable in P_1–P_3, has a beneficial effect on the expenditure side of the government budget constraint (31) as it reduces the amount of benefits to be payed to the unemployed. On the revenue side of (31), the increase in formal sector employment broaden the tax base and, therefore, tax revenues: as expenditures reduce and revenues increase, maintaining the budget balanced requires setting a negative rate of inflation. Having imposed that the chosen rate of inflation cannot drive the nominal interest rate below zero, optimal policy calls for a rate of deflation such that the ZLB is binding. Interestingly, negative inflation also favors the informal sector and the new equilibrium under P_4 is characterized by an increase of informal employment and consumption.[20] Reducing the informal economy is not, in fact, a direct objective of the policymaker, who only seeks to maximize the agents' utility: the beneficial effect of a smaller underground sector comes indirectly from a lower disutility of labor in (8) and from a preference for formal goods consumption in (9), but the nature of the solution to the maximization problem depends on the interaction between monetary and fiscal policies.

To sum up, the choice of the inflation target is intimately related to the hypothesis concerning the availability of different tax instruments. Nonetheless, all the policies share the common trait that monetary policy is flanked by a reduction in taxation. When the policymaker have no access to distortionary taxes, the optimal policy imposes an equilibrium that is close to the initial steady state of the economy. When consumption or capital taxes are available, the inflation rate is positive and seigniorage revenues are channeled into the reduction of τ^c or τ^k. In the new equilibrium, the informal sector shrinks. Finally, when the adjustment involves labor income taxes, the welfare maximizing inflation target is negative at the value imposed by the ZLB on the nominal interest rate. All the policies are welfare improving, with largest increase in welfare arising in P_3, followed by P_4 and P_2.[21] However,

[20] Although the informal sector is positively affected by P_4, the share of informal workers and the dimension of the shadow economy - defined as the ratio $\frac{p_I Y_I}{p_I Y_I + Y_F}$ (not reported in Table 3) - remain almost unchanged due to the concomitant increase of the formal sector.

[21] The reader should be cautioned that a proper welfare evaluation among policies is not meaningful because the magnitudes of the interventions are not comparable.

this static measure of welfare should be interpreted cautiously in P_4 as the long-run benefits of disinflation are typically economy-depressing in the short-run and depend largely on the timing of policy implementation.[22]

5.2 Optimal Inflation and the Size of the Informal Economy

In a model featuring informal employment, it is essential to understand how the results obtained in the previous section depend on the existence and the dimension of the shadow economy. What would the optimal policy be in the absence of the informal sector? And what in the presence of a large share of underground economy? In order to answer these questions, Figure 1 plots the optimal inflation target, expressed in annual terms, under policies P_1–P_4 for different dimensions of the informal economy. In particular, on the horizontal axis the share of informal employment ranges from 0% to 50% of total employment.[23]

A first striking result depicted in Figure 1 is that one of the three policies, namely P_4, is independent of the size of the informal sector. This result is motivated by the fact that the policymaker always aims at setting $\pi^\star$ so as to reduce the labor income tax rate τ^w. As previously discussed, a reduction in formal labor market distortions lowers unemployment and broadens the tax base, thus leaving a negative inflation rate as the only possible solution in order to satisfy Equation (31). Interestingly, this optimal behavior is not affected by the size of the shadow economy.

When the change in government revenues triggered by a change in the inflation target affects the marginal tax rates τ^c and τ^k, or lump sum taxes, the size of the informal sector has important implications for the optimal conduct of monetary policy. As shown in columns one to three of Table 3, in fact, a positive inflation rate shifts resources from the informal sector to the formal one, thus broadening the tax base and increasing total tax revenues for given tax rates. Moreover, a positive rate of inflation generates seigniorage revenues that are proportional to the amount of money in the economy, which is in turn a positive function of the size of the informal sector. Given the combination of these two effects, there is an incentive to raise the inflation target as the share of informal workers increases. This type of behavior enables the policymaker to reduce the size of the informal sector and to collect increasing revenues from seigniorage with which to finance tax reductions in the attempt of mitigating the welfare loss induced by widespread informality.

This mechanism explains the increasing patterns depicted in Figure 1, where, in the extreme scenario with half of the employed workers operating in the informal sector, the optimal inflation target is as large as 10% when the capital tax rate is endogenous; reaches about 5% when the endogenous change occurs in the consumption tax rate; and is above 2% when distortionary taxes are not available. Also, when the share of informal workers equals zero optimal inflation is essentially zero in P_1 and P_2: this is a standard result in models with price rigidities in which deviations from price stability are inefficient as they generate

[22] See Ascari and Ropele (2013), among others.

[23] Since the informal technology uses labor as the only production input, changes in the share of informal to total workers strictly mirror changes in the size of the informal economy.

costs that absorb a share of the produced output. Conversely, zero inflation is not optimal even in the absence of the informal sector in P_3, that is the policymaker uses the inflation tax to raise resources in order to reduce the capital tax rate. Again, this result is driven by the fact that reductions in τ^k stimulate investments and provide a strong boost to economic activity.

5.3 The Role of Cash and of Price Rigidity

This section explores the role of cash and of the degree of price rigidity in determining the results obtained in Section 5.1. Consider the latter first. Figure 2 shows the optimal inflation target for different values of ϕ_F, the parameter regulating the degree of price rigidity in the economy. Not surprisingly, the inflation target is a negative function of ϕ_F and approaches zero as the latter goes to infinity. From this point of view, Figure 2 does not add anything new to monetary policy theory. However, it should be noted that two policies, namely P_3 and P_4, still exhibit a rate of inflation substantially different from zero even with large values of ϕ_F. A rate of inflation different from zero can be thus well tolerated even in presence of a large degree of price rigidity if the policymaker is able to reduce the distortions generated by capital and labor income taxes. Secondly, extremely large rate of inflation cannot be observed unless ϕ_F is set very low, which explains the extremely high values of the optimal inflation target observed in the literature of macroeconomic models with underground economy and flexible prices.[24]

A more important concern is whether the optimal inflation targets discussed under policies P_1–P_4 are motivated by the total amount of cash in the economy rather than by the distinction between money demanded for purchases of formal or informal goods. Since all the informal good consumption must be paid in cash, increases in the dimension of the informal sector bring about increases in the total amount of money in the economy as well, which change the incentive to use inflation to generate revenues (see Equation 31). Hence, what observed in Figure 1 might be caused by an increase in M rather than by a larger dimension of the informal sector. In order to dispel any doubt, Figures 3 and 4 plot the optimal inflation target for different degrees of cash intensity in the informal and formal sectors, respectively. Importantly, the analysis is carried out isolating the cash demand in each sector, that is in Figure 3 the amount of cash required for buying the formal good is set equal to zero (i.e., $\nu_F = 0$), whereas Figure 4 is derived under the - admittedly unusual - hypothesis that informal consumption does not require the use of cash (i.e., $\nu_I = 0$).

Observed together, Figures 3 and 4 immediately show that the positive inflation targets observed in policies P_1 and P_2 are entirely motivated by the existence of a cash-intensive informal sector. With $\nu_F = 0$, the optimal inflation target is a positive function of ν_I in Figure 3 and for $\nu_I = 1$, as in the baseline calibration of the model, the value of $\pi^\star$ is consistent with that reported in Table 2. Figure 4 shows indeed that Policy P_2 does not depend on the amount of cash used in formal sector transactions, whereas P_1 is slightly negatively affected by the value of ν_F.

[24]See Cavalcanti and Villamil (2003) and Koreshkova (2006).

On the contrary, the situation is completely different in policies P_3 and P_4. When seigniorage revenues can be channeled into the reduction of the capital tax rate, that is Policy P_3, the policymaker does not make any distinction between using inflation to tax cash-holdings in either sectors. The larger the amount of cash in the economy, the higher the inflation target as the larger will be the reduction in τ^k. Again, the possibility to stimulate investments and increase production dominates the welfare cost of higher inflation. When the labor income tax rate is endogenous (i.e., Policy P_4), the optimal inflation target is found to be a decreasing function of the total amount of cash in the economy, which adds important insights to what previously observed. Figure 1 showed in fact that, independently of the dimension of the informal economy, Policy P_4 would call for a rate of deflation such that the ZLB on the nominal interest rate is binding. Figures 3 and 4 finally allow to shed light on the optimal behavior suggested by P_4. When decreasing $\pi^\star$ in order to lower τ^w, the policymaker has to strike a balance among the benefits of lower income taxes, the absorption of resources caused by deviating from zero inflation, and the increase in informal self-employment. Under the hypothesis that cash is not used at all in formal sector transactions, in fact, the optimal rate of deflation is smaller than that observed in the baseline calibration of the model. Intuitively, when $\nu_F = 0$ a negative inflation target represents a relative advantage to the informal sector against the formal economy, that is why $\pi^\star$ is smaller in Figure 3 compared to Figure 4. However, the decreasing pattern of $\pi^\star$ in Figure 3 is still observed because the combined effects of deflation and of lower labor income taxes on aggregate consumption of the cash-constrained household dominate the relative increase in the share of self-employed informal workers.

6 Robustness Analysis

This section tests the robustness of the results to two different modeling assumptions. First, price rigidity is also introduced in the informal sector. In this version of the model informal labor does not take the form of self-employment, but is rather given by the intersection of demand and supply as in standard frameworks. Second, the assumption of decreasing returns to scale in the informal sector is relaxed and all the simulations are carried out with constant returns to scale in the informal sector. Finally, sensitivity analysis on two key parameters, namely the inverse of the elasticity of informal labor supply, φ, and the elasticity of substitution between formal and informal consumption goods, η, is performed.

6.1 Informal Monopolistic Firms with Price Rigidity

The two most peculiar features of the economy presented in this paper are the choice of modeling all the households working in the informal sector as self-employed workers and the fact that price rigidity affects the formal sector only. One might therefore wonder whether the results reported in Tables 2 and 3 are influenced by such assumptions.

In order to resolve this concern, Policies P_1–P_4 are implemented in a model with monopolistically competitive informal firms which face convex costs when changing prices. As

in the model with informal self-employed workers, search frictions occur only in the regular sector and the timing of intra-period actions remains unchanged. Households and firms respectively take decisions on the supply and the demand of informal labor depending on the observed wage rate in the informal sector and taking into account the opportunity cost of informality, which consists in the inability of searching for a regular job in the same period. In this version of the model, the household's budget constraint becomes

$$\begin{aligned} P_{I,t}C_{I,t} + P_tC_{F,t}(1+\tau_t^c) + E_tQ_{t,t+1}B_{t+1} - B_t + M_t - M_{t-1} + P_tI_t \\ = P_tw_{F,t}N_{F,t}(1-\tau_t^w) + P_tr_t^kK_t(1-\tau_t^k) + P_tw_{I,t}N_{I,t} + P_tbU_t + D_{F,t} + D_{I,t} + T_t^{ls}, \end{aligned}$$

and the first order condition with respect to $N_{I,t}$ is, accordingly:

$$w_{I,t} = \frac{N_{I,t}^{\varphi}}{\lambda_t} + f(\theta_t)(1-\tau_t^w)w_{F,t} + (1-f(\theta_t))b. \tag{32}$$

As regards the production sector, monopolistic competition occurs in the intermediate goods sector, whereas the final producer operates in perfect competition. Informal firm i profits are

$$D_{I,t}(i) = P_{I,t}(i)Y_{I,t}(i) - P_tw_{I,t}(i)N_{I,t}(i) - \frac{\phi_I}{2}\left(\frac{P_{I,t}(i)}{P_{I,t-1}(i)} - 1\right)^2 P_tY_{I,t}(i),$$

where ϕ_I is the degree of price rigidity in the informal sector.[25] Profit maximization with respect to $N_{I,t}$ and $P_{I,t}$ results in the following two equations describing the informal labor demand and the price setting equation, respectively:[26]

$$w_{I,t} = \alpha_I mc_{I,t}N_{I,t}^{\alpha-1}, \tag{33}$$

$$p_{I,t}(1-\epsilon) - \phi_I(\pi_{I,t}-1)\pi_{I,t} + \epsilon mc_{I,t} + \beta\frac{\lambda_{t+1}}{\lambda_t}\phi_I(\pi_{I,t+1}-1)\pi_{I,t+1}\frac{Y_{I,t+1}}{Y_{I,t}} = 0. \tag{34}$$

The most important difference with respect to the baseline version of the model with flexible prices in the informal sector is the existence of a further channel through which the rate of inflation negatively affects the shadow economy. In addition to raising the cost of consuming the good produced in the informal cash-intensive sector, inflation directly reduces informal consumption due to price rigidity, as shown in the modified version of Equation (30):

$$Y_{I,t}\left[1 - \frac{\phi_I}{2}\left(\pi_{I,t}-1\right)^2\right] = C_{I,t}. \tag{35}$$

[25] This parameter is set equal to half of the parameter regulating price rigidity in the formal sector, $\phi_I = \phi_F/2$.

[26] Note that in this modified model, the inflation rate and the price level in the informal sector cannot be uniquely determined by the set of equilibrium conditions as the price level in the formal sector plays the role of numeraire. One possible solution to this form of indeterminacy would require to detail a process linking the inflation rates in the two sectors. Since the aim of this paper is to present an analysis of inflation in the long-run, the assumption here is that the steady state levels of inflation in the two sectors will eventually converge to a common value.

Tables 4 and 5 report the results of the policy experiments P_1–P_4 in this different version of the model. Comparing Table 4 with Table 2, it is clear that the inclusion of price stickiness in the informal sector significantly affects the optimal level of the rate of inflation. In Policies P_1–P_3, the latter is in fact higher than what observed in the baseline version of the model. In particular, the most striking difference between the two models is observed in Policy P_2: when price rigidity in the informal sector is considered, the policymaker has an incentive to set a higher inflation rate in order to undertake a quite large cut in the consumption tax τ^c, as the informal economy is much more sensitive to inflation and the latter triggers a sort of self-reinforcing positive effect on the budget balance. In fact, not only higher inflation generates larger revenues *per se*, but shifts also resources towards formal consumption, which in turn allows to reduce τ^c, which further stimulates C_F (and so on). The combined effect of high inflation and lower taxes on formal goods consumption has a strong negative influence on the consumption of the informal good, which falls by 11.5%. Because of this large drop in informal consumption, the aggregate consumption level, $\tilde{C}$, little increases despite the 1.7% jump in formal consumption. The gain in welfare is therefore mostly due to the reduction in the number of family members employed in the informal sector rather than to the mild increase in $\bar{C}$. Interestingly, the optimal inflation target in P_4 is invariant to this alternative model specification, but the outcome on our variables of interest is quite different as shown in Table 5. In particular, the increase in formal consumption is lower and the increase in informal consumption - as well as in the share of informal workers - is much larger than what observed in Table 3. The reason for this differences lies on the fact that, in the model with informal intermediate producers, the increase in the demand for the informal good translates into the increase of the demand for informal workers, which in turn results in a higher equilibrium wage in the underground sector.

6.2 Constant Returns to Scale in the Informal Sector

Another assumption that might potentially affects the optimal rate of inflation is the existence of decreasing returns to scale in the informal technology. This section explores this possibility by substituting Equation 12 with a linear technology

$$Y_{I,t} = N_{I,t}. \tag{36}$$

Tables 6 and 7 show the results of Policies P_1–P_4 when the informal sector technology exhibits constant returns to scale. Again, the optimal inflation level is higher compared to the baseline model in P_1–P_3, whereas that resulting from P_4 is still equal to minus the real rate of interest. With constant returns to scale, informal consumption is more sensitive to changes in the after-tax real wage in the formal sector due to the equality between N_I and C_I, which makes inflation more distortionary to the informal economy.[27] In fact, the household partly benefits from a greater reduction in informal employment, but has also to compensate for the lower level of C_I by increasing consumption of the formal goods. For this reason, the policymaker exploits the stronger effects of inflation by setting it at a higher

[27]Note that the elasticity of the real formal sector wage to inflation does not change with α_I.

level. For the same reason, although under Policy P_4 inflation and the labor income tax in Table 6 are exactly equal to those observed in Table 2, the last column of Table 7 differs from that of Table 3, as this policy brings about a reduction of the formal sector wage.

All in all, the following two conclusions can be drawn from robustness analysis, with the exception of Policy P_4: first, both the experiment of replacing informal self-employed workers with informal monopolistic firms facing price rigidity and that of assuming constant returns to scale in the informal technology have a remarkable impact on the optimal long-run rate of inflation; and second, the policymaker shows a greater propensity to use the instrument of inflation the more effectively the latter shifts resources from the informal to the formal sector. However, although the optimal level of inflation and the effects on the economic variables vary with the two different assumptions considered above, the following feature of optimal monetary show robustness to departures from the baseline model: when the potential seigniorage revenues can be channeled into the reduction of the capital tax rate (i.e., Policy P_3), the inflation target is the highest and the households accrue the largest welfare gain. As concerns Policy P_4, the optimal rate of inflation is instead always equal to the value that sets the nominal interest rate to zero, but the effects on the economy differ depending on the specification of the model.

6.3 Sensitivity Analysis

This section concludes the analysis on the robustness of the model results by performing sensitivity analysis on two key parameters that might in principle influence the results displayed in Table 2. The first is the parameter regulating the elasticity of informal labor supply, φ, in the utility function of the representative household. Compared to the baseline calibration of the model where $\varphi = 1$, Table 8 reports the results for the optimal policy rates when informal labor supply is more sensitive to changes in the marginal revenue from underground activity ($\varphi = 0.8$), whereas the results with a lower elasticity are shown in Table 9 ($\varphi = 1.2$).[28] As can be seen, the optimal rate of inflation is only slightly affected by changes in the elasticity of informal labor supply and Policies P_1 and P_4 are invariant to changes in φ. In P_2 and P_3, the optimal inflation target is somewhat higher in Table 9 as the workers are less prone to abandon informal employment and the policymaker is therefore induced to give them a greater incentive.

Another parameter that deserves attention is that regulating the elasticity of substitution between formal and informal goods in the CES consumption function (9). This parameter, η, is set to 0.875 in the baseline calibration of the model. Table 10 shows the optimal rates under the assumption of perfect substitutability between formal and informal goods (i.e., $\eta = 1$). As perfect substitutability between C_F and C_I implies that the agents switch from one good to another very easily in response to a change in the relative cost of one good, the inflation rate is a powerful tool to be exploited in the hand of the policymaker via the cash-in-advance constraint (11). For this reason, $\pi^\star$ is larger in Table 10 compared to Table 2.

[28] As φ is the inverse of the elasticity of informal labor supply, the latter is higher when the former is lower (and vice-versa).

Following the same line of reasoning, it is clear that the optimal inflation target is, instead, lower in Table 11.

7 Conclusion

This paper offers an analysis of the optimal long run rate of inflation in a two-sector general equilibrium model with underground production calibrated to Lithuania. The analysis is carried out in an environment in which public expenditure is exogenously fixed and the government is committed to maintain a balanced budget. Under these restrictive assumptions on the conduct of fiscal policy, changes in the inflation level require some endogenous tax adjustment for the government budget constraint to be satisfied. Based on this premise, the optimal inflation target is computed under different hypotheses regarding the tax instrument that is endogenously adjusted. Precisely, four policies are considered: in three of them, the change in the inflation rate is coupled with an adjustment in distortionary taxes (namely, consumption, capital and labor income taxes), whereas one involves an adjustment in lump-sum taxes.

The first interesting result obtained is that monetary policy displays heterogeneity in the four policies. In the baseline version of the model, the optimal inflation target reaches an outstanding annual value of 6.77% when seigniorage revenues are used to decrease the capital tax rate. As explained in the paper, the economic stimulus provided by a large reduction in the latter dominates the cost of high inflation. When the budget balance is maintained through changes in consumption and lump-sum taxes, optimal long-run inflation is positive and equals 1.21% and 0.67%, respectively. In the first case, inflation has a dual role: it shifts resources from the cash-intensive informal sector to the formal one and allows to reduce the consumption tax rate. The low level observed in the second case, instead, is clearly due to the fact that the change does not involve distortionary taxes. The optimal policy prescription is completely different with endogenous labor income taxes as it calls for a negative inflation rate such that the ZLB on the nominal interest rate is binding. This result has been rationalized throughout the paper.

Secondly, the optimal rate of inflation is largely influenced by the size of the informal economy. In most cases, it is a positive function of the informal sector dimension. However, in policies where the capital or the labor income tax rates are endogenous, this result has to be interpreted with caution as it is in general affected, and partly driven, by the total amount of cash rather than by the underground economy itself.

Finally it is important to notice that, in this model, a relatively high rate of inflation can be optimal even in presence of price rigidity.

This analysis has, indeed, a limitation that is particularly important when optimal policy is studied in the hypothesis of endogenous income taxes. As already discussed in the Introduction, the optimal policy derived in this paper is the one the policymaker would aim for if she can stick to it forever. In this sense, the analysis is purely static. That is why the policymaker does not care about setting a rate of deflation such that the ZLB is binding. In a different approach where the steady state of the economy is imposed *ex-post* rather

than *ex-ante* - i.e., the so called Ramsey or unconstrained approach to monetary policy - the presence of the ZLB usually calls for a higher rate of inflation, because when near to the ZLB conventional monetary policy is ineffective (see Adam and Billi, 2004, 2007). Nonetheless, this issue suggests that further research in this direction is needed, but the results obtained in this paper offer important insights on the relationship between monetary policy and the underground economy.

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Tables

Table 1: Parametrization

Calibrated		
Parameter	**Interpretation**	**Value**
ψ	Formal workers bargaining power	0.4
ξ	Matching function elasticity	0.6
φ	Inverse of Frish elasticity in the informal sector	1
η	Elasticity of substitution formal/informal goods	0.875
β	Subjective discount factor	0.99
ν_I	Cash intensity in the informal sector	1
α_F	Formal technology parameter	0.65
α_I	Informal technology parameter	0.8
ϵ	Elasticity of substitution among intermediate goods	7.5
ϕ_F	Price rigidity	50
κ	Vacancy posting cost	0.05
τ^c	Consumption tax rate	0.21
τ^w	Labor income tax rate	0.15
τ^k	Capital tax rate	0.15
δ	Capital depreciation rate	0.03
Implied		
ρ	Job separation rate in the formal sector	0.0187
χ	Scale parameter in the matching function	0.2530
ω	Formal-good share of consumption	0.5644
ν_F	Cash intensity in the formal sector	0.6229
b	Unemployment benefit as a share of formal wage	0.84

Table 2: Inflation and tax rates implied by policies P_1–P_4

	$\pi^{\star}_{annual}$	τ^c	τ^h	τ^k	T^{ls}
Initial steady state	0.00%	21.00%	15.00%	15.00%	-0.08
P_1 : endogenous lump-sum taxes	0.67%	21.00%	15.00%	15.00%	-0.09
P_2 : endogenous consumption tax rate	1.21%	20.68%	15.00%	15.00%	-0.08
P_3 : endogenous capital tax rate	6.77%	21.00%	15.00%	9.24%	-0.08
P_4 : endogenous labor income tax rate	-3.94%	21.00%	14.89%	15.00%	-0.08

Table 3: Steady state effects of policies P_1–P_4, % variation from initial values

Variable	P_1	P_2	P_3	P_4
Y_F	0.0328	0.1213	4.9511	0.7871
Y_I	-0.1616	-0.6929	-6.8716	0.9789
C_F	0.0274	0.1119	2.9769	0.6193
C_I	-0.1616	-0.6929	-6.8716	0.9789
$\tilde{C}$	0.0067	0.0238	1.8809	0.6586
I	0.0456	0.1447	12.2055	0.7103
w_F	0.0197	0.0360	3.8004	-0.1372
U	0.0014	0.0082	-0.0570	-0.7975
Share of Informal Workers	-0.0242	-0.1035	-1.0355	0.0418
Welfare	0.0091	0.0342	1.9428	0.6395

Table 4: Inflation and tax rates implied by policies P_1–P_4: Alternative model with informal monopolistic firms with price rigidity.

	$\pi^{\star}_{annual}$	τ^c	τ^h	τ^k	T^{ls}
Initial steady state	0.00%	21.00%	15.00%	15.00%	-0.08
P_1 : endogenous lump-sum taxes	1.77%	21.00%	15.00%	15.00%	-0.09
P_2 : endogenous consumption tax rate	4.77%	19.41%	15.00%	15.00%	-0.08
P_3 : endogenous capital tax rate	6.93%	21.00%	15.00%	9.85%	-0.08
P_4 : endogenous labor income tax rate	-3.94%	21.00%	14.88%	15.00%	-0.08

Table 5: Steady state effects of policies P_1–P_4, % variation from initial values: Alternative model with informal monopolistic firms with price rigidity.

Variable	P_1	P_2	P_3	P_4
Y_F	0.2001	1.8110	3.9105	0.5008
Y_I	-1.1621	-11.5319	-2.7869	3.6126
C_F	0.1695	1.7335	1.9763	0.2543
C_I	-1.1621	-11.5319	-2.7869	3.6126
$\tilde{C}$	0.0524	0.5658	1.5338	0.5326
I	0.2341	1.9039	10.3520	0.4243
w_F	0.0523	0.1404	3.4299	-0.1400
U	0.0140	0.1521	-0.0977	-0.9323
Share of Informal Workers	-0.1734	-1.6981	-0.4319	0.4199
Welfare	0.0697	0.7210	1.5512	0.4740

Table 6: Inflation and tax rates implied by policies P_1–P_4: Constant returns to scale in the informal sector

	$\pi^{\star}_{annual}$	τ^c	τ^h	τ^k	T^{ls}
Initial steady state	0.00%	21.00%	15.00%	15.00%	-0.08
P_1 : endogenous lump-sum taxes	0.94%	21.00%	15.00%	15.00%	-0.09
P_2 : endogenous consumption tax rate	1.93%	20.47%	15.00%	15.00%	-0.08
P_3 : endogenous capital tax rate	7.42%	21.00%	15.00%	7.94%	-0.08
P_4 : endogenous labor income tax rate	-3.94%	21.00%	14.89%	15.00%	-0.08

Table 7: Steady state effects of policies P_1–P_4, % variation from initial values: Constant returns to scale in the informal sector

Variable	P_1	P_2	P_3	P_4
Y_F	0.0684	0.3083	6.8366	0.7069
Y_I	-0.4635	-2.3112	-16.4968	1.9948
C_F	0.0604	0.2922	4.5936	0.5173
C_I	-0.4635	-2.3112	-16.4968	1.9948
$\tilde{C}$	0.0136	0.0595	2.6657	0.6491
I	0.0865	0.3454	15.8680	0.6302
w_F	0.0278	0.0569	4.6170	-0.1375
U	0.0040	0.0234	-0.0016	-0.8179
Share of Informal Workers	-0.0555	-0.2762	-1.9930	0.1312
Welfare	0.0192	0.0870	2.7745	0.6207

Table 8: Inflation and tax rates implied by policies P_1–P_4: Higher elasticity of informal labor supply ($\varphi = 0.8$)

	$\pi^{\star}_{annual}$	τ^c	τ^h	τ^k	T^{ls}
Initial steady state	0.00%	21.00%	15.00%	15.00%	-0.08
P_1 : endogenous lump-sum taxes	0.67%	21.00%	15.00%	15.00%	-0.09
P_2 : endogenous consumption tax rate	1.18%	20.69%	15.00%	15.00%	-0.08
P_3 : endogenous capital tax rate	6.08%	21.00%	15.00%	9.34%	-0.08
P_4 : endogenous labor income tax rate	-3.94%	21.00%	14.89%	15.00%	-0.08

Table 9: Inflation and tax rates implied by policies P_1–P_4: Lower elasticity of informal labor supply ($\varphi = 1.2$)

	$\pi^{\star}_{annual}$	τ^c	τ^h	τ^k	T^{ls}
Initial steady state	0.00%	21.00%	15.00%	15.00%	-0.08
P_1 : endogenous lump-sum taxes	0.67%	21.00%	15.00%	15.00%	-0.09
P_2 : endogenous consumption tax rate	1.29%	20.66%	15.00%	15.00%	-0.08
P_3 : endogenous capital tax rate	6.89%	21.00%	15.00%	9.09%	-0.08
P_4 : endogenous labor income tax rate	-3.94%	21.00%	14.89%	15.00%	-0.08

Table 10: Inflation and tax rates implied by policies P_1–P_4: Perfect substitutability of informal vs. formal consumption ($\eta = 1$)

	$\pi^{\star}_{annual}$	τ^c	τ^h	τ^k	T^{ls}
Initial steady state	0.00%	21.00%	15.00%	15.00%	-0.08
P_1 : endogenous lump-sum taxes	0.78%	21.00%	15.00%	15.00%	-0.09
P_2 : endogenous consumption tax rate	1.57%	20.58%	15.00%	15.00%	-0.08
P_3 : endogenous capital tax rate	7.09%	21.00%	15.00%	8.42%	-0.08
P_4 : endogenous labor income tax rate	-3.94%	21.00%	14.89%	15.00%	-0.08

Table 11: Inflation and tax rates implied by policies P_1–P_4: Imperfect substitutability of informal vs. formal consumption ($\eta = 0.5$)

	$\pi^{\star}_{annual}$	τ^c	τ^h	τ^k	T^{ls}
Initial steady state	0.00%	21.00%	15.00%	15.00%	-0.08
P_1 : endogenous lump-sum taxes	0.47%	21.00%	15.00%	15.00%	-0.09
P_2 : endogenous consumption tax rate	0.78%	20.81%	15.00%	15.00%	-0.08
P_3 : endogenous capital tax rate	6.32%	21.00%	15.00%	10.17%	-0.08
P_4 : endogenous labor income tax rate	-3.94%	21.00%	14.89%	15.00%	-0.08

Figures

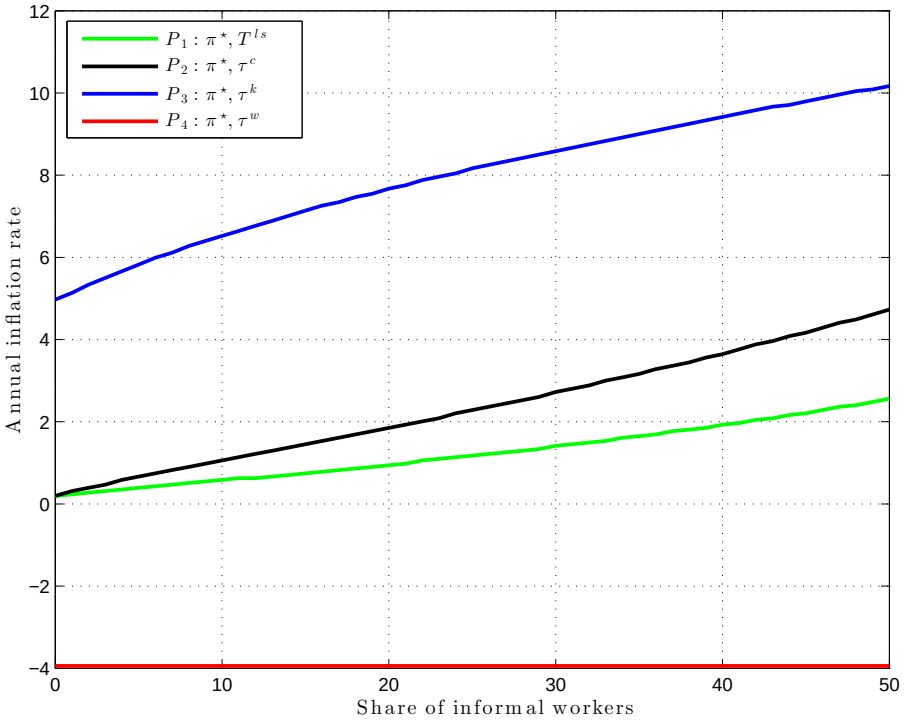

Figure 1: Optimal inflation target in policies P_1–P_4 for different shares of informal workers

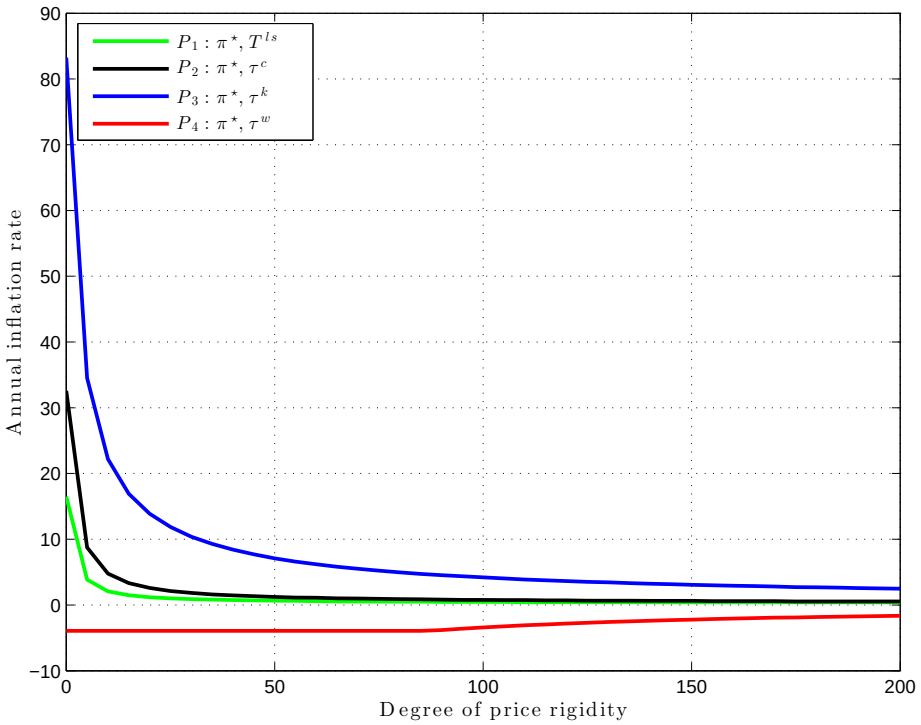

Figure 2: Optimal inflation target in policies P_1–P_4 for different degrees of price rigidity

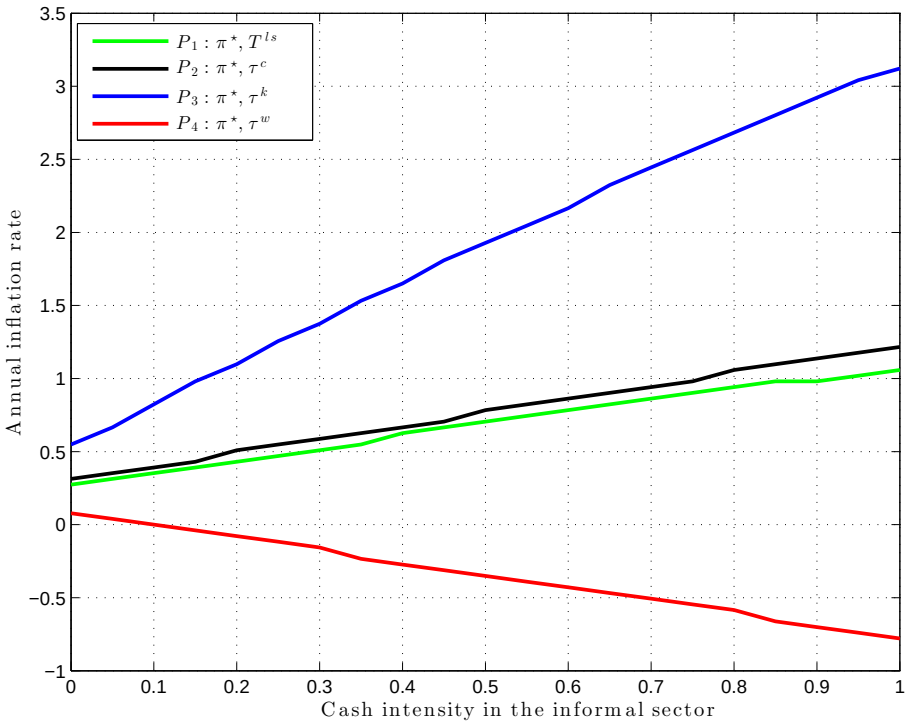

Figure 3: Optimal inflation target in policies P_1–P_4 for different degrees of cash intensity in the informal sector (setting cash intensity in the formal sector to zero)

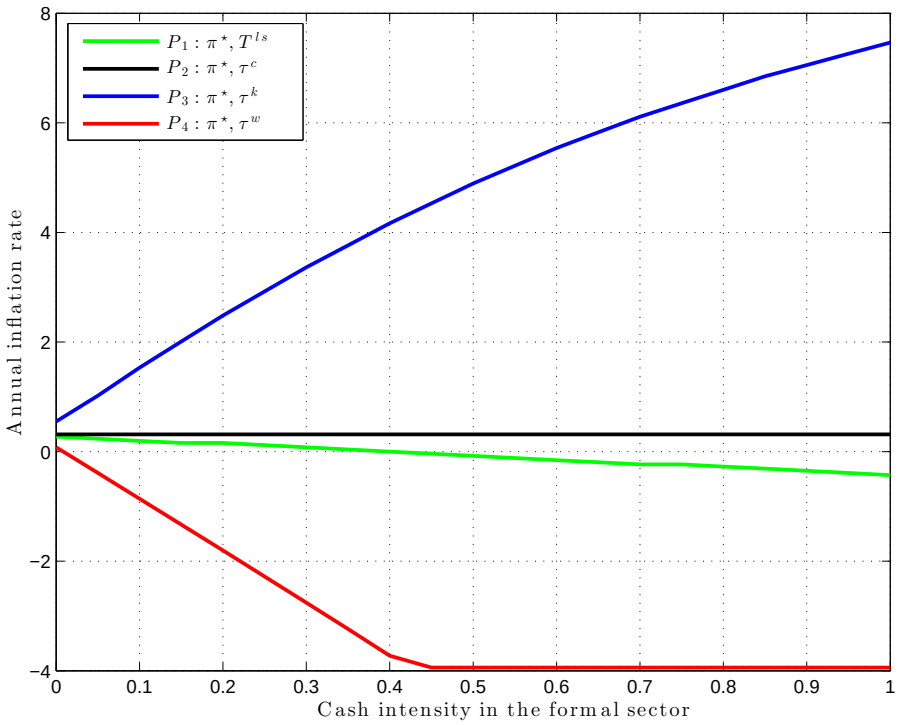

Figure 4: Optimal inflation target in policies P_1–P_4 for different degrees of cash intensity in the formal sector (setting cash intensity in the informal sector to zero)