Personal Bankruptcy, Bank Portfolio Choice and the Macroeconomy

By Eglė Jakučionytė
PERSONAL BANKRUPTCY, BANK PORTFOLIO CHOICE AND THE MACROECONOMY

Eglė Jakučionytė*

* Tinbergen Institute and the University of Amsterdam, the Netherlands. Address: Roetersstraat 11, 1018 WB Amsterdam. Email address: e.jakucionyte@uva.nl.

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Abstract

This paper explores the spillover effects from increasing personal bankruptcy protection. Innovatively, the paper shows that the spillover effects can be influenced by the bank portfolio choice. Since a low level of personal bankruptcy protection keeps an insolvent individual liable until her debt is repaid in full, lender’s returns on mortgages are less uncertain than returns on other assets ceteris paribus. Risk-averse banks would prefer mortgages over other types of assets such as corporate loans. Corporate lending and thus equilibrium output would fall. In contrary to the popular view that creditor protection smooths credit provision and makes the allocation of resources more efficient, I show that in some cases a low level of personal bankruptcy protection can lead to aggregate consumption losses. Also I show that macroprudential policies (LTV ratios) can successfully complement higher personal bankruptcy protection in ensuring even higher welfare.

Keywords: Personal bankruptcy, household debt, housing, general equilibrium, bank portfolio choice;

JEL codes: E44, G11, G21, K35, R21
1 Non-technical summary

Strict personal bankruptcy laws discipline borrowers to repay, however, they also deprive insolvent borrowers from more generous consumption insurance. Several studies have augmented this canonical trade-off of personal bankruptcy protection with general equilibrium considerations. This paper acknowledges the significance of financial intermediaries in allocating credit and explores the general equilibrium effects in the presence of banks. To capture the bank-dominant financial structure in Europe, I model banks as the only financial intermediaries and choose this model structure to explore the relationship between personal bankruptcy protection, credit allocation and the macroeconomy.

The paper focuses on the macroeconomic (spillover) effects of personal bankruptcy protection that arise due to changes in bank credit supply to financially constrained firms. To analyze relevant channels, I develop a small-scale general equilibrium model with mortgage default and corporate default. By assumption only banks collect deposits and allocate credit to household and firms. The resulting credit allocation affects corporate investment and output. Importantly, all households experience output gains or losses in the form of wage income or profits (if they are firm owners). Then changes in personal bankruptcy protection affect household consumption not only due to better consumption smoothing but also due to changes in equilibrium income, the spillover effect.

My main result is that a low level of personal bankruptcy protection can lead to aggregate consumption losses. A low level of personal bankruptcy protection allows banks to seize a share of insolvent individual’s earnings if collateral value is insufficient. It follows that a low level of personal bankruptcy protection makes bank returns on mortgages higher in expectation and also less uncertain than returns on other assets ceteris paribus. If banks can choose between mortgages and corporate loans to invest in, then under a low level of personal bankruptcy protection banks would prefer mortgage credit rather than corporate credit, leading to a reduction in finance for financially constrained firms and lower total output. Aggregate consumption would fall in response to lower wages and profits.

Ignoring the bank portfolio choice would omit the spillover effect on production and the benefits of higher personal bankruptcy protection would be underestimated considerably. I find that the optimal level of earnings exemptions in the considered setup is strictly larger than zero because improved consumption sharing and higher equilibrium income both contribute significantly
However, a starting level of personal bankruptcy protection matters. If a level of personal bankruptcy protection is sufficiently low, a marginal increase in the level of protection may cause a sufficiently small increase in borrowing costs such that household borrowers would not only demand but also afford more mortgage credit. If the resulting increase in mortgage credit is larger than the regulator prefers, such macroprudential tools as LTV ratios for mortgages in combination with a higher level of personal bankruptcy protection can be helpful in bringing demand for mortgages down while preserving insurance for insolvent individuals.

Finally, the regulator who aims at reducing housing demand by fine-tuning personal bankruptcy regulations in this setup would achieve only small changes to equilibrium housing. Households value housing for its utility value and consumption smoothing opportunities, so they reduce housing purchases gradually.
2 Introduction

Mortgage credit is usually perceived as a safer type of investment from the perspective of a bank: for a loan of the same maturity mortgage borrowers pay less than businesses and during the last recession banks cut new household loans by less than loans extended to firms (ECB (2013)). In most cases a mortgage is a relatively standard and better collateralized type of credit compared to a corporate loan, however, mortgage credit supply is unavoidably shaped by present institutions such as bankruptcy laws as well. One of the key but often overlooked differences between the corporate and household borrowers stems from the bankruptcy law. Particularly in emerging Europe, the corporate bankruptcy law is rather developed but in many cases individual insolvency rules did not exist a few years ago. If an individual became insolvent, her debt would not be cancelled but she would remain liable until the debt would be repaid in full. Thus, a low level of personal bankruptcy protection increases the returns to the lender in bad states making his total returns on mortgages vary less. Lender’s returns on mortgages are less uncertain than returns on other types of assets ceteris paribus. In that case, credit price would adjust to reflect that risk-averse lenders would be incentivized to give mortgage credit at the expense of other loans, including corporate credit. I show that the resulting credit allocation would have far-reaching implications for the macroeconomy and welfare.

Before the crisis personal debt discharge was to some extent available in the Czech Republic and Poland only. In other countries bankruptcy institutions were asymmetric: the development of the personal bankruptcy institution lagged behind the corporate bankruptcy one. The different treatment of different types of insolvent borrowers and thus stronger lenders’ incentives to issue mortgage credit can partially explain the neck-breaking speed of mortgage credit expansion in emerging Europe before the crisis. In most of the countries mortgage credit increased from a virtual zero to the heights of corporate credit. High income growth and speedy financial development can only partially explain credit expansion, because bank capital inflows to emerging Europe contributed to the credit boom as well (Bakker and Gulde (2010); Bakker and Klingen (2012)) and parent bank funding was central to the boom in the Baltic countries (Bakker and Klingen (2012)). Moreover, the empirical study by Everaert et al. (2015) shows that credit supply rather than demand drove credit growth in many of emerging countries in Europe. I could add

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1 The recent crisis generated pressure to create or improve the legal framework for personal bankruptcy, as evidenced by the considerable number of governments in emerging Europe that enacted bankruptcy laws in 2008-2013. Besides the introduction of the personal bankruptcy laws in Latvia (2008, 2010) and Lithuania (2013), such laws were enacted in Poland (2009) and Slovenia (2008) (CESifo, 2015). Hungary and Romania followed in 2015.
the deterioration of credit standards (e.g. LTV and LTI went up) to the evidence in favor of the supply-side effects as well. Thus although credit inflows facilitated catching-up in the region, excessive lending and lenders’ preferences were a part of the story too. While there could be several reasons behind the lender’s incentives to invest heavily in the real estate directly and indirectly though real estate loans, the potential role of the sharp difference between personal bankruptcy and corporate bankruptcy procedures in the lenders’ choices should not be underestimated as well. I take the expansion in mortgage credit in emerging Europe as an inspiration for my further analysis.

An empirical investigation of the credit allocation channel is challenging for various reasons, most importantly, the lack of variation personal bankruptcy protection across Eastern Europe. Therefore, I choose to present the mechanism of how the differences in bankruptcy institutions can affect the credit allocation. I develop a coherent theoretical framework with a bank portfolio choice between mortgage credit and corporate credit and use it to understand the potential outcomes for the macroeconomy as well as policy implications. The model structure is such that the differential of mortgage default risk and default risk of businesses as well as the associated expected returns determine the supply side of the credit equilibrium. I assume that the lender’s choice between loans depends on these differentials: the lender prices in differences in risk and expected returns. Changing the bankruptcy law for one type of borrowers can increase the difference in the risk and the expected return affecting the lender’s choice between loans and thus the credit allocation. The model allows me to illustrate how a decrease (increase) in the level of personal bankruptcy protection makes risk averse lenders increase (decrease) the supply of mortgage credit at the expense of loans to the corporate sector. Credit reallocation is triggered by the change in expected payoffs and uncertainty associated with returns on household loans: a decrease (increase) in the level of personal bankruptcy protection increases (decreases) expected returns on mortgages and decreases (increases) the associated risk for lenders. Credit reallocation triggers a general equilibrium effect: the supply of corporate loans determines the amount of finance available for credit-constrained firms and in turn the capital stock in the economy. It follows that the optimal level of earnings exemptions is strictly larger than zero: it offers consumption sharing and higher equilibrium income. In contrast to the popular view that creditor protection smooths credit provision and makes the allocation of resources more efficient, I show that in some cases undeveloped personal bankruptcy institutions can distort credit allocation and create output losses from the social planner’s perspective.
Importantly for a regulator, the effects are non-linear. For low levels of personal bankruptcy protection, a marginal increase in the level of protection may create only a small increase in borrowing costs, leading to a higher equilibrium mortgage credit and lower welfare (the opposite of the previously described effect).

I touch upon the role of macroprudential policy in the presence of personal bankruptcy protection as well. The two policies prove to be complements in terms of welfare: I show that the combination of higher level of personal bankruptcy protection and lower LTV ratios allows to crowd-in capital even more while preserving consumption insurance to insolvent borrowers.

Since the lender is risk averse, his portfolio choice depends on the risk associated with returns. The risk aversion assumption enables richer mechanics of the credit allocation problem and strengthens the supply effect in response to personal bankruptcy protection. This extension is important in the light of the usual ex-ante trade-off related to personal bankruptcy protection. The literature (e.g. Athreya (2002)) explains the trade-off as the competing demand and supply effects. For instance, a low level of personal bankruptcy protection discourages demand for mortgage credit: borrower's consumption is less insured against bad states of nature. The positive supply effect would manifest in looser borrowing conditions. Based on the empirical literature\(^2\) I argue that the supply effect can dominate the demand effect. Thus I can extend my model with risk aversion on the lender side to strengthen the supply effect.

Since mortgage credit is the largest component of household credit, personal bankruptcy institutions would have an effect on the housing market as well. Therefore, besides exploring the effect of personal bankruptcy institutions on the bank portfolio choice, I study the corresponding macroeconomic implications for housing demand.

I contribute to the personal bankruptcy literature by highlighting the role of the bank portfolio choice in the presence of mortgage return uncertainty. So far, several studies, which abstract from housing, show that a decrease in the level of personal bankruptcy protection can be followed by an increase in capital stock Li and Sarte (2006) or reduction in savings Chatterjee and Gordon (2012). Studies with equilibrium mortgage default are limited to a one-asset framework on the bank balance sheet: the bank can invest in mortgages only, which implies that there are no portfolio choice considerations and direct implications of the personal bankruptcy policy beyond the

\(^2\)Gropp et al. (1997) use the variation in personal bankruptcy exemptions across US states and find that states with generous exemptions are associated with higher interest rates and poorer access to credit for borrowers with few assets. Pence (2006) finds that lenders in states with borrower friendly foreclosure laws tend to give smaller mortgages. Lin and White (2001) confirm positive relationships debtor-friendly bankruptcy rules and the difficulty of getting mortgage and unsecured loans by using the cross-state variation in bankruptcy exemption levels.
housing market and savings market. For instance, Jeske et al. (2013), Corbae and Quintin (2015), Chatterjee and Eyigungor (2015), and Garriga and Schlagenhauf (2009) attempt to evaluate the effects of government housing market policy on the housing market. Most importantly, these studies abstract from the productive asset, capital, and the feedback effect of personal bankruptcy regulation into production. I extend this framework by introducing capital and considering the bank portfolio choice explicitly which yields strong macroeconomic effects.

Section 2 presents the model. Section 3 discusses the results. Robustness checks and a brief discussion about future extensions follow. Section 6 concludes.

### 3 Model

The main goal of my model is to illustrate the mechanism through which personal bankruptcy protection impacts the credit allocation in the economy and thus I abstract from many details a macroeconomic model would feature otherwise. The model assumes only a few actors in the economy (see Figure 1). I assume that there are only two households. By assumption they differ in their discount factors which allows for the positive supply and positive demand for savings. Importantly, intermediation of savings requires a special technology that is available to the banker only. The bank collects deposits and lends to one of the households and the financially constrained production firm. The banker is a part of the saving household and thus transfers profits or losses to his household at the end of every period.

![Figure 1: Model economy.](image-url)
The model economy exists for three periods. It is a closed economy with production and housing. One household (out of two) takes a mortgage in period 1 and can default in period 2, if she finds default welfare-increasing. The main reason of household default in this model is the fall of the collateral (housing) value below the debt value. Potential default makes bank returns uncertain and, since banks are risk-averse, they do incorporate the variance of returns into their lending decision.

Bank returns on corporate loans can fall below the contracted amount as well, if productivity unexpectedly takes a low value in period 2. Then, under chosen parametrization, the firm finds it optimal to default and let the bank seize profits. Thus there are two sources of uncertainty in the model: the housing value shock $\zeta$ and the productivity shock $z$ where $\zeta \in \{\zeta^L, \zeta^H\}$ and $z \in \{z^L, z^H\}$ with commonly known probabilities. The shocks are independent and occur in period 2 only. To draw a difference between the productive asset and the unproductive asset, I assume that the productivity shock generates on average higher returns than the housing shock ($E\zeta = 1, Ez = 1.1$) to reflect higher fundamental productivity of capital, but variance of both shocks is set equal.

The timing of events ensures that bankruptcy institutions can determine lender’s choices ex-ante and the borrower’s default decision ex-post. The last period features exclusion from credit markets as an additional default penalty.

- **Period 1**
  1. The saver and the borrower receive an endowment.
  2. The firm is set up and plans its production in the next period.
  3. The bank receives deposits from the saver and faces demand for mortgage credit and corporate credit.
  4. Demand for credit is satisfied at the price that ensures a zero profit for the bank.
  5. Households consume.

- **Period 2**
  1. Uncertainty in the housing market and the production sector is resolved.
  2. Households supply labor at the predetermined wage inelastically.
  3. The borrower and the firm choose whether to default and the lender transfers realized returns to the saver.
4. The borrower may borrow for the next period (if did not default this period) and the saver decides how much to save through deposits.

5. The old firm transfers profits/losses to the saver and exits.

6. There is a new firm that plans its production in the next period.

7. The lender satisfies new demand for mortgage credit and corporate credit at the price that ensures a zero profit for the bank.

8. Households consume.

• Period 3

1. Households supply labor at the predetermined wage inelastically.

2. The borrower and the firm repay their loans and the bank transfers realized returns to the saver.

3. Households consume their entire income and die.

I solve model with backwards induction using terminal values indicated in the model description. The solution algorithm and the baseline set of parameters are given in the appendix.

3.1 Households

Households are of two types. The household with a higher discount factor $\beta_S$ is the saver as opposed to the borrowing household ($\beta_B < \beta_S$).

3.2 Saver

In the beginning of period 1 the saver receives an endowment $y_1/2$. Labor supply is inelastic, so the household chooses only consumption $c_1^S$ and how much to save by putting deposits $d_1$ such that her value function is given by

$$V_1^S = \max_{\{c_1^S, d_1\}} \left\{ \frac{(c_1^S)^{1-\sigma} - 1}{1-\sigma} + \beta_S V_2^S (d_1; \zeta, z) \right\}$$

s.t. $c_1^S + d_1 \leq \frac{y_1}{2}$

In period 2 the decision set for the saver is the same as before, however, instead of endowment, the saver receives labor income $w_2 n^S$, gross returns on deposits $(1 + r_1) d_1$ and a lump-sum profit transfer from firms and the bank $\pi_2$. The saver owns all firms and the bank in the economy, thus,
all profits/losses get attributed to him at the end of the period. The value function in period 2 can be expressed as

\[
V^S_2 = \max_{\{c_2^S, d_2\}} \left(\frac{(c_2^S)^{1-\sigma} - 1}{1 - \sigma}\right) + \beta_2 V^S_3 (d_2)
\]

s.t. \(c_2^S + d_2 \leq w_2 n^S + (1 + r_1) d_1 + \pi_2\)

In the last period the household consumes all income and does not save anymore:

\[
V^S_3 = \max_{\{c_3^S\}} \left(\frac{(c_3^S)^{1-\sigma} - 1}{1 - \sigma}\right)
\]

s.t. \(c_3^S \leq w_3 n^S + (1 + r_2) d_2 + \pi_3\)

\[d_3 = 0\]

The assumption \(d_3 = 0\) will be used for backward induction.

### 3.3 Borrower

The housing value shock is the direct source of uncertainty that affects household’s wealth and the collateral value and can trigger household default. The setup allows for the household to default if her financial benefits from personal bankruptcy are high enough: the collateral value falls below the debt value and the costs associated with default are not high enough to make up for the difference between the collateral value and the debt value.

In the beginning of period 1 the borrowing household chooses consumption \(c_1^B\), housing stock \(h_1\) and how much to borrow from the bank \((m_1)\) such that her value function is maximized:

\[
V_1^B = \max_{\{c_1^B, h_1, m_1\}} \left(\frac{(c_1^B)^{1-\sigma} h_1^{\sigma_h}}{1 - \sigma}\right) + \beta_B V_2^B (h_1, m_1, \zeta; z)
\]

subject to the budget constraint:

\[c_1^B + h_1 \leq \frac{y_1}{2} + m_1\]

Moreover, I assume that the household borrows the maximum allowed amount given by \(\rho(1 - \delta)\tilde{\zeta} h_1 / (1 + r_1^m)\). \(\rho (0 < \rho < 1)\) is an exogenous LTV ratio, \(\tilde{\zeta}\) denotes an expected value of the housing shock \(\zeta\) and \(r_1^m\) stands for the mortgage net interest rate. The household pledges her housing \(h_1\) which can be seized in case of default in period 2. The housing value shock \(\zeta\) can
take two values only: \( \zeta \in \{ \zeta_L, \zeta_H \} \). The probability of a bad state \( \zeta_L \) occurring is denoted by \( p_L \) and is commonly known. The value \( \zeta_L \) would reduce the value of the housing asset possibly triggering household default (though not necessarily). It is important to note that housing value is exogenous for the sake of simplicity.

I proxy a higher level of personal bankruptcy protection with higher earnings exemptions. Earnings exemptions matter in case of insolvency because, if the value of housing assets is insufficient to repay the mortgage, the bank can claim non-exempt earnings. Then higher earnings exemptions mean that the borrower can keep more of her earnings in case of default.

I denote a fraction of earnings that the household can keep in case of default by \( \kappa \). Then a fraction \( 1 - \kappa \) of insolvent household’s earnings is seized by the creditor together with housing assets in exchange for forgiving mortgage debt. A positive parameter \( \kappa \) implies personal bankruptcy protection. The setup loosely corresponds to no-recourse laws in the US: if the value of the house is insufficient to repay the mortgage, the bank can seize some but not all of other assets (including cash) to cover the losses. \( \kappa = 0 \) approximately corresponds to recourse laws in some states in the US.

Further I formalize maximization problems for different decisions of the borrower. If the borrower does not default, the value function for period 2 is the following:

\[
V_{BP}^2 = \max_{\{c_{B2}, h_2\}} \left( \left( \frac{c_{B2}}{2} \right)^{1-\sigma} \left( \frac{h_2}{2} \right)^{\sigma} \right)^{1-\sigma} \left( 1 - \frac{1}{\sigma} \right) + \beta_B V_{BP}^3 (h_2, m_2)
\]

\[
s.t. \quad c_{B2} + h_2 \leq w_2 n_B + \zeta (1 - \delta^h) h_1 - (1 + r^m_1) m_1 + m_2
\]

To ensure that the household with negative home equity does not borrow, I impose

\[
m_2 = \begin{cases} 
\rho (1 - \delta) h_2 / (1 + r^m_2) & \text{or} & 0, \quad \text{if} \quad \zeta (1 - \delta^h) h_1 - (1 + r^m_1) m_1 > 0 \\
0, \quad \text{if} \quad \zeta (1 - \delta^h) h_1 - (1 + r^m_1) m_1 \leq 0 
\end{cases}
\]

In my model the insolvent household is excluded from the credit market, thus \( m_2 = 0 \).\(^3\) Then the value of the borrower after default is given by

\[
V_{BD}^2 = \max_{\{c_{B2}, h_2\}} \left( \left( \frac{c_{B2}}{2} \right)^{1-\sigma} \left( \frac{h_2}{2} \right)^{\sigma} \right)^{1-\sigma} \left( 1 - \frac{1}{\sigma} \right) + \beta_B V_{BD}^3 (h_2, m_2)
\]

\(^3\)See Fisher et al. (2004) for empirical evidence that low credit scores restrict post-bankruptcy credit.
\[ s.t. \quad c^B_2 + h_2 \leq \kappa w_2 n^B \]

The household's choice whether to default or not is determined by the maximum expected value:

\[ V^B_2 = \max \left\{ V^{BP}_2, V^{BD}_2 \right\} \]

In the last period the borrower consumes everything which is left after repaying the last mortgage. Thus, the value attributed to the borrower in the final period depends on her decision in period 2. If the household paid back the mortgage, she can borrow again in period 2 and has to repay in period 3:

\[ V^{BP}_3 = \max \left\{ \left( c^B_3 \right)^{1-\sigma} \right\} \]

\[ s.t. \quad c^B_3 \leq w_3 n^B + (1 - \delta^h) h_2 - (1 + r^m_2) m_2 \]

\[ m_3 = 0, h_3 = 0 \]

Or, if the household defaulted in period 2,

\[ V^{BD}_3 = \max \left\{ \left( c^B_3 \right)^{1-\sigma} \right\} \]

\[ s.t. \quad c^B_3 \leq w_3 n^B + (1 - \delta^h) h_2 \]

\[ m_3 = 0, h_3 = 0 \]

I use the last condition as the terminal point for the backwards induction solution.

### 3.4 Firms

Firms in the model economy are owned by the saver. There are 2 firms (can be interpreted as 2 generations of firms) which enter sequentially as showed in Figure 2 and live for two periods in total. They buy capital \( k \) and hire labor \( n \) in the first period of their lives but produce in the second period of their lives only. It follows that both capital and wage is predetermined (labor supply is inelastic). I assume that firms need to finance capital expenditure by taking a loan from the bank. If the productivity shock \( z_t \) takes a low value, the firm defaults in the second period.\(^4\)

\(^4\)The predetermined working capital and the ability to default makes the firm subject to debt overhang. This is one of the few inefficiencies in the model next to incomplete markets. However, since the firm’s default probability is predetermined (coincides with the probability of the bad productivity shock), changes in the model structure will not reduce/increase debt.
Both firms use a perfectly competitive technology:

\[ y_t = z_t k_{t-1}^{\alpha_t} n_{t-1}^{1-\alpha_t}, \quad \forall t = \{2, 3\} \]

Every firm solves the following maximization problem in the first period of their lives:

\[
\max_{k_t, n_t} E_t \left\{ y_{t+1} + (1 - \delta) k_t - w_t n_t - \left(1 + r_t^l\right) l_t, \quad 0 \right\}
\]

\[ s.t. \quad l_t = k_t \]

$l_t$ denotes the loan that the firm takes from the bank to finance capital. If the firm does not default in the second period, it transfers profits to the patient household and exits.

### 3.5 Banking sector

A representative competitive bank is operated by the risk averse manager and owned by the saving household. I assume that in the beginning of every period the realized profits/losses from the previous period are transferred to the saving household which allows me to abstract from the dynamic properties of the bank problem and focus on the credit allocation in the current period only. The uncertainty of returns associated with different assets on their balance sheet play a role in this problem, because the assumed risk-aversion affects the bank manager decision how much to lend to the borrowing household and how much to the firm, i.e. his portfolio choice.
Both types of investment are risky: the mortgage is subject to default, whereas firms enjoy limited liability. Thus interest rates are set not only to make the bank break even in expectation, but also to account for the relative risk the bank manager is taking given the return on the mortgage and the return on the corporate loan. Thus, the bank would ask a higher interest rate on mortgages (or corporate loans) to compensate for otherwise lower expected returns compared to the net deposit rate but also to compensate for the uncertainty. Further I solve the portfolio allocation problem of the bank manager and obtain equilibrium conditions. If the personal bankruptcy law becomes more borrower-friendly, the bank would adjust the mortgage interest rate to account for the change in the risk that the bank manager has to bear and the change in the expected return.

In period 1 the bank manager expects a net return \( \tilde{r}_m \) on the mortgage and a return \( \tilde{r}_l \) on the loan, where the indicator \( I(\text{repaid}) \) denotes the borrower’s (either household’s or firm’s) decision to repay debt. The expected return on the mortgage reflects that in case of default the bank will seize the non-exempt earnings together with housing:

\[
E_1 (1 + \tilde{r}_m) m_1 = E_1 \{ I(\text{repaid}) (1 + r_m) m_1 \} + E_1 \left\{ (1 - I(\text{repaid})) \left( \zeta (1 - \delta^h) h_1 + (1 - \kappa) w_2 n^B \right) \right\}
\]

The expected return on the corporate loan reflects that in case of default the bank will seize the firm’s profits (including revenue for selling depreciated capital):

\[
E_1 \left( 1 + \tilde{r}_l \right) l_1 = \left\{ I(\text{repaid}) \left( 1 + r_l \right) l_1 \right\} + E_1 \left\{ (1 - I(\text{repaid})) (y_2 + (1 - \delta) k_1 - w_2 n) \right\}
\]

Denote a share of total funds allocated to finance mortgages as \( \gamma_m \), where \( \gamma_m \in (0, 1) \). Then the total expected net return is given by:

\[
E_1 \tilde{r}_2 = \gamma_m E_1 \tilde{r}_2^m + (1 - \gamma_m) E_1 \tilde{r}_2^l
\]  

(2)

To determine the optimal asset allocation, the bank manager maximizes expected returns minus the cost of funds subject to the associated risk where parameter \( A \) reflects the degree of bank manager’s risk aversion:

\[
\max_{\gamma_m} E_1 (\tilde{r}_2) - A \frac{\text{var}(\tilde{r}_2)}{2}
\]
The expression can be expanded into

\[
\max_{\gamma_m} \gamma_m E_1 \hat{r}_m^m + (1 - \gamma_m) E_1 \hat{r}_2^l - A \frac{\text{var} (\gamma_m \hat{r}_m^m + (1 - \gamma_m) \hat{r}_2^l)}{2}
\]

Due to my previous assumption on \( \gamma_m \in (0, 1) \), the return maximization problem already takes into account the balance sheet constraint of the bank: \( m_1 + l_1 = f_1 \).\(^5\)

\[
\Rightarrow \max_{\gamma_m} \gamma_m E_1 \hat{r}_2^m + (1 - \gamma_m) E_1 \hat{r}_2^l

- \frac{A}{2} \left( \gamma_m \text{var} (\hat{r}_m^m) + 2 \gamma_m (1 - \gamma_m) \text{cov} (\hat{r}_2^m, \hat{r}_2^l) + (1 - \gamma_m)^2 \text{var} (\hat{r}_2^l) \right)
\]

It follows that the optimal mortgage share in the bank’s asset portfolio is given by

\[
\gamma_m = \frac{E_1 \hat{r}_2^m - E_1 \hat{r}_2^l}{A \cdot \text{var} (\hat{r}_2^m - \hat{r}_2^l)} + \frac{\text{var} (\hat{r}_2^l) - \text{cov} (\hat{r}_2^m, \hat{r}_2^l)}{\text{var} (\hat{r}_2^m - \hat{r}_2^l)} \quad \text{(3)}
\]

Note that the stochastic properties of the housing shock and the productivity shock (more precisely, independence of the two shocks) imply that \( \text{cov} (\hat{r}_2^m, \hat{r}_2^l) = 0 \).

Next to the equilibrium condition presented in equation (3), I impose an additional condition to reflect a perfect competition environment for the bank: \( E_1 \hat{r}_2 = (1 + \tau) r_1 \). This condition can also be interpreted as a zero-profit condition of a perfectly competitive bank facing proportional lending costs \( \tau \) for every loan unit where \( 0 \leq \tau < 1 \). The bank passes through the costs to borrowers.\(^6\)

By assumption in period 2 the bank does not face credit risk, so he lends up to the point where the marginal expected net return on assets equals the net deposit rate \( r_2 \). Thus, the mortgage interest rate \( r_m^m \) and the firm loan interest rate \( r_2^l \) is set such that the bank breaks even in expectation and uncertainty considerations do not play a role in setting the loan rates anymore.\(^7\)

\(^5\)\( m_1 + l_1 = f_1 \Rightarrow m_1/f_1 + l_1/f_1 = 1 \Rightarrow \gamma_m + l_1/f_1 = 1 \Rightarrow l_1/f_1 = 1 - \gamma_m \). Thus the share of corporate loans in total assets has to be equal to \( 1 - \gamma_m \) which is what I assume from the beginning.

\(^6\)In concluding this section it is noteworthy to mention that the last condition does not occur in the conventional Markowitz portfolio problem (1964) due to exogenous asset returns while in my problem it is crucial to allow for endogenous asset returns and thus the additional condition is missing to pin down interest rates for the both assets in the bank’s portfolio.

\(^7\)I could assume proportional loan costs \( \tau \) again for the sake of model consistency, however, it would not affect the bank problem qualitatively: the bank manager solves a static problem and does not take into account the change in the proportional funding costs in the future, if there is any.
3.6 Other producers

Perfectly competitive housing producers receive an endowment $h_1$ in period 1 and can invest to increase the housing stock with the following technology:

$$h_t = i_t^h + (1 - \delta^h)h_{t-1}$$

Perfectly competitive capital producers receive an endowment $k_1$ in period 1 and can invest to increase the capital stock with the following technology:

$$k_t = i_t^k + (1 - \delta^k)k_{t-1}$$

3.7 Market clearing

In equilibrium total savings in the economy have to equal total demand for loans:

$$m_t + l_t = d_t, \quad \forall t = \{1, 2\}$$

The rest two conditions ensure market clearing in the labor market:

$$n_t = n, \quad \forall t = \{2, 3\}$$

$$n = n^B + n^S$$

4 Results

4.1 Effect on the credit allocation

An increase in the level of personal bankruptcy protection has a negative effect on the bank returns. First, it increases a fraction of an insolvent individual's earnings that is protected from seizure by the bank and the expected bank losses go up. Second, risk aversion on the bank side would result in an even higher contraction of credit than it takes to break even in expectation because the return would have to compensate for increased return uncertainty as well. In total, this would create a negative supply effect of the increase in the level of personal bankruptcy protection. The demand effect of higher personal bankruptcy protection is positive because the
individual is insured from bad states better thus would like to borrow more. The equilibrium outcomes in the credit market will depend on which of two effects, the supply effect and the demand effect, will dominate. I will keep referring to them throughout this section. Later I will show that, although a higher level of personal bankruptcy protection improves risk sharing considerably, at a sufficiently high level of earnings exemptions the negative supply effect is strong enough to offset the increased demand for credit and equilibrium mortgage credit decreases. Next to the first order effects of the change in personal bankruptcy protection, the bank portfolio choice will allow for secondary effects (the spillover effects) on other loans on the bank balance sheet. I will refer to this as the portfolio effect.

Evidence for the varying dominance of the supply effect in the mortgage market is presented in this paragraph. I proxy a higher level of personal bankruptcy protection with higher earnings exemptions $\kappa$: higher $\kappa$ means that, if the housing value is insufficient to cover the mortgage, the insolvent borrower can keep more of her earnings. As I will show later, this results in higher demand for mortgage credit, thus the positive demand effect, but firstly, I establish how the supply effect manifests in this setup. Higher $\kappa$ directly translates into an additional expected loss to the bank when the default happens. Figure 3\(^8\) shows how bank returns on mortgages in bad states decrease in the level of earnings exemptions $\kappa$. In good states bank returns rise to compensate for higher risk. At the same time, bank returns on mortgages become more uncertain, see Figure 4. The bank responds to higher earnings exemptions $\kappa$ by raising the interest rate spread for mortgages. The spread is defined as the difference between the mortgage rate and the risk-free rate ($r_m - r$) and exhibited in Figure 5. The spread accounts for both the default risk and the uncertainty risk that the bank has to bear.

The increase in the level of personal bankruptcy protection insures the borrower against bad states and depending on the starting level of personal bankruptcy protection the demand effect can be strong enough to dominate the supply effect. In Figure 6 I observe that for $\kappa < 0.5$ mortgage credit does not respond to changes in personal bankruptcy protection. The reason is that for these values of personal bankruptcy protection the borrower repays the mortgage even in bad states (see Table 1) and thus changes in earnings exemptions do not matter. Mortgage credit increases sharply if $\kappa \in (0.5, 0.6)$, suggesting the dominance of the demand effect, because in this region consumption insurance in bad states becomes sufficient to choose default in bad states and

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\(^8\)In Figure 3 and in the subsequent plots I plot equilibrium outcomes given a value of $\kappa$ where $0 \leq \kappa \leq 1$ and where a different value of $\kappa$ corresponds to a different economy. This allows me to show the changes in variables of interest as the level of personal bankruptcy protection ($\kappa$) varies.
stimulates overall mortgage credit demand. However, losses for the bank are sufficiently small, so borrowing costs increase marginally and the borrower can indeed afford more mortgage credit. When the exemption level increases further ($\kappa \geq 0.6$), equilibrium mortgage credit gradually decreases suggesting the dominance of the supply effect. Then, even though higher earnings exemptions lead to better risk sharing for the risk averse household by reducing variation of the borrower’s utility as depicted in Figure 7, the decline in bank returns in bad states is sharper and thus the declining supply starts dominating the equilibrium outcomes. The decline in equilibrium credit is in line with empirical evidence provided in Gropp et al. (1997), Pence (2006) and Lin and White (2001).

![Figure 3: Returns on mortgages in different states.](image1.png)  
![Figure 4: Variance of returns on mortgages and loans.](image2.png)  

![Figure 5: Mortgage interest rate spread in period 1.](image3.png)  
![Figure 6: Mortgages given in period 1.](image4.png)

The increasing generosity of personal bankruptcy rules affects not only the mortgage market.
equilibrium but the credit allocation as well. When mortgage default becomes sufficiently more borrower-friendly ($\kappa$ increases beyond 0.6), the bank not only decreases lending to households but shifts to corporate lending instead. Higher earnings exemptions generate a shift from mortgage credit to financing capital purchases: the portfolio effect decreases the interest rate spread for corporate loans $r_1 - r_1$ as exhibited in Figure 8 and firms’ borrowing increases (Figure 9). When less tight lending conditions for firms makes them borrow more, capital stock increases. Note that I abstract from conditions which would make firms not to take more loans such as satiation of investment.

The increase in corporate lending that is associated with the portfolio effect does not only have a positive effect on capital stock. As higher capital translates into higher production, both households would reap the benefits of expanding production through higher wages. The saver would also receive profits from firms and the bank. These effects are possible because of the general equilibrium setup and, as I will show later, play a crucial role in determining welfare outcomes.

The secondary effect of the increase in the personal bankruptcy protection, the adjustment of

<table>
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<th>0</th>
<th>0.1</th>
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<th>0.3</th>
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<th>0.5</th>
<th>0.6</th>
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</thead>
<tbody>
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<tr>
<td>$\zeta^H$</td>
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Table 1: Household default decision contingent on the shock value in period 2, default = 1.

Figure 7: Borrower’s utility in different states.
the credit allocation in the economy, has bold implications for efficiency and economic growth. An increase in the level of personal bankruptcy protection shifts savings from investment in unproductive asset (housing) to investment in productive asset (capital). As a result, output is higher. From the perspective of total production the new allocation is more efficient. It follows that, interestingly enough and contrary to the popular view, higher borrower protection rather than higher creditor protection would make the allocation of resources more efficient (for some value of earnings exemptions).

Before concluding this section, I would like to note a few characteristics of the two types of assets available for the bank investment. To draw a difference between the productive asset and the unproductive asset, so far I have assumed that the productivity shock generates on average higher returns than the housing shock to reflect higher fundamental productivity of capital, but variance of both shocks is set equal. A higher variance of capital returns compared to a variance of returns on mortgage loans evolves endogenously and is the possible reason behind a rather slow shift towards financing corporate loans.

### 4.2 Effect on welfare

The borrowing household benefits from higher personal bankruptcy protection because of better consumption smoothing and even higher consumption opportunities. Higher consumption opportunities arise because in this general equilibrium setup a higher level of personal bankruptcy protection is also associated with the portfolio effect. The portfolio reallocation results in the increase in corporate lending and in turn higher equilibrium income. Both households benefit from
higher wages. The downside of being better protected against bad states is higher interest rates ex-ante. Higher interest rates may decrease the borrower’s utility in the states when she chooses to repay the loan. The decrease that I consider here is with respect to the borrower’s utility from repaying the mortgage in the economy with a lower level of personal bankruptcy protection. Further, I discuss why and when the benefits offset the downside of the reduced household access to credit and what does it imply for total expected welfare.

Evidence of better risk sharing is presented in Figure 10. It shows that sufficiently high earnings exemptions ($\kappa \geq 0.6$) decrease the difference between the borrower’s utility values at the two states. The utility in bad times improves considerably, because, if the low value of the housing stock occurs and $\kappa \geq 0.6$, the household defaults and reaps the benefits of the exemption. I see that her consumption in period 2 ($c^b_2$) increases with the level of earnings exemptions, see Figure 12, the top-left and the bottom-left panels. The increase in consumption smoothing is incorporated in the expected borrower’s utility value as well (Figure 11). When $\kappa \geq 0.6$, borrower’s consumption increases in the level of earnings exemptions in good states too, see the same figure, the top-right and the bottom-right panels. This is contrary to my prediction that higher earnings exemptions and the resulting higher mortgage interest rates would decrease borrower’s net worth and thus consumption opportunities in the states when the borrower chooses to repay the mortgage. The prediction was wrong because the economy with higher exemptions allows for higher non-defaulted borrower’s consumption because such economy produces more. Therefore, higher equilibrium income compensates for higher mortgage interest rates and the borrower becomes better-off not only in bad states but also in expectation.

![Figure 10: Utility across states.](image1)

![Figure 11: Total utility.](image2)
Moreover, higher earnings exemptions increase the household’s consumption in the last period (Figure 13) even though she is excluded from the credit market in period 2 and cannot smooth her consumption by borrowing. In the financial autarky she still benefits indirectly from the shift of the bank lending composition towards corporate loans and in turn higher income.

What is the optimal level of earnings exemptions? As I showed before, the borrower enjoys higher consumption insurance in bad states. Also, higher equilibrium income offsets the downside of higher mortgage interest rates. From the borrower’s welfare point of view, earnings exemptions should be as high as possible. Figure 11 shows that in the range of earnings exemptions for which household default can occur (for $\kappa \geq 0.6$) borrower’s expected utility indeed increases in earnings exemptions. The decline in the expected utility around $\kappa \in (0.5, 0.6)$ results from the sudden change in the household default pattern: household start defaulting in bad states when $\kappa \geq 0.6$. Then a marginal increase in the level of protection not only makes default attractive to borrowers in bad states, but also results in sufficiently small losses for the bank. It appears that for this (small) range of earnings exemptions the positive demand effect dominates despite the increase in borrowing costs and mortgage credit increases. This crowds-out corporate loans, creating negative effects on output and thus welfare. For the rest of the interval, the supply effect dominates in the mortgage market and the opposite result prevails.

Better consumption smoothing for the borrower, however, means larger losses for the bank and the saver who is the owner of the bank. Whenever the borrower defaults, negative bank profits erode saver’s income and limit consumption opportunities. Indeed, for $\kappa \geq 0.6$, i.e. the interval of exemptions when the borrower defaults in bad states, saver’s consumption in bad states depends negatively on the level of earnings exemptions, see Figures 12-13. Thus, consumption smoothing for the borrower is to some extent achieved by shifting losses to the saver. In a partial equilibrium setup this would likely result in lower total welfare. However, the decline in the expected saver’s utility is rather small in my setup because higher earnings exemptions also allow for the expansion in production and thus higher wage. Higher labor income makes up for some of the loss related to negative bank profits. Consequently, both expected borrower’s utility and total expected utility increase in earnings exemptions when $\kappa \geq 0.6$.

Even though total expected utility increases in earnings exemptions, this does not necessarily suggest that the optimal earnings exemption is full exemption, when more intricate characteristics of financial inefficiencies are taken into account. Allowing for over-borrowing or adverse selection among borrowers could affect the welfare result significantly.
4.3 Macroprudential policy: LTV ratio

As I have seen, increasing personal bankruptcy protection implies higher risks for the lender. The attempt at the welfare analysis shows that the increased demand for borrowing is not necessarily welfare enhancing for any increase in the level of personal bankruptcy protection. In this section I investigate the role of macroprudential policies in the presence of personal bankruptcy protection to determine whether increased risks on the lender’s side can be partially mitigated by adjusting macroprudential policies.

The main role of personal bankruptcy protection is provision of consumption insurance for insolvent borrowers, however, changing personal bankruptcy protection relaxes or tightens ex-ante borrowing conditions for all borrowers and in this respect personal bankruptcy law acts similarly to the tools at the disposal of the regulator. For instance, lower earnings exemptions reduce average default rates by making default conditions less favorable to the debtor. Similarly, the regulator would impose lower LTV ratios for mortgages\(^9\) to prevent households from excessive

\(^9\)Unless stated otherwise, by ‘LTV ratios’ I mean LTV ratios for mortgages in the remaining of the text.

Figure 12: Consumption in period 2.

Note: Saver’s consumption is on the RHS y-axis.
leverage and hence higher sensitivity of household net worth to bad shocks. Lower LTV ratios and thus the imposed financial prudence for all borrowers mean that in bad times fewer borrowers become insolvent. From the first glance it looks that a lower LTV ratio achieves lower default rates by making insolvency less likely, while lower personal bankruptcy protection increases borrowers’ losses in the case of bankruptcy and reduces willingness to go bankrupt but does not mitigate insolvency as such. However, as an indirect effect of lower personal bankruptcy regulation, borrowing households would adjust their borrowing behavior ex-ante and reduce leverage and/or borrowing in absolute amount. This could lead to similar outcomes as lower LTV ratios: smaller loans would result in fewer insolvencies in bad times. Nevertheless, the main goal of the two tools is different and the similarity of the some of regulatory outcomes could be used to calibrate policies in order to increase their efficiency rather replace one policy with another. As my analysis shows, if personal bankruptcy protection is desired but comes at the expense of increased mortgage borrowing for some values of earnings exemptions, a tighter LTV ratio applied together with the higher level of earnings exemptions could remedy that.

\[\text{Note: Saver's consumption is on the RHS y-axis.}\]
I investigate the interaction of personal bankruptcy protection and LTV ratios for mortgages by varying regulatory LTV limits. I assume that the regulator imposes the limit on how much the borrowing household can borrow for every unit of housing she wants to acquire, e.g. what is the minimum amount of home equity, and the borrower borrows the maximum allowed amount. After discussing the credit outcomes, I check my results for the welfare scores and confirm that indeed coordinating LTV ratios for mortgages with personal bankruptcy protection can enhance welfare.

The interaction between different levels of personal bankruptcy protection and LTV ratios delivers a highly non-linear response of equilibrium mortgage credit to the changing set of the regulatory parameters. As expected, lower LTV ratios, if combined with higher personal bankruptcy protection, allow to achieve higher insurance in case of personal bankruptcy with the lower increase in the mortgage credit and thus even lower loss of output. However, this holds only for higher exemption levels, see Figure 14. If less than a half of earnings are exempt in case of bankruptcy, lower LTV ratios actually lead to a higher level of mortgage credit. This finding can be explained by the model structure which serves as a good example why a model extension should incorporate endogenous borrowing choices. Currently, LTV ratios directly determine household leverage. Thus, when the level of personal bankruptcy protection is low, implying that demand for mortgage credit is low, a lower LTV ratio allows the borrower to leverage less. A lower chance of default, which follows from lower leverage, is important in her borrowing decision because in case of bankruptcy, consumption insurance would be low in this region of $\kappa$. Thus lower leverage is better aligned with the borrower’s preferences given lower personal bankruptcy protection than higher leverage and this even leads to larger borrowing in absolute amount because the less-leveraged borrower can afford borrowing more and still avoid default. Higher LTV ratios impose higher leverage and the household chooses to borrow less in absolute amount in the range of lower personal bankruptcy protection to avoid default in this way. I give this explanation as opposed to the supply side effects because of the dynamics of mortgage interest rates: borrowing costs which is the reflection of the supply side effects do not vary by LTV ratio significantly for low $\kappa$ (Figure 15). It follows that the difference in borrowing across LTV ratios in this $\kappa$ region is more related to the household’s preferences than the bank losses. Further I will focus mostly on the right side of the exemption interval because, when earnings exemptions are sufficiently high, the household demand is high enough not to be dominated by the imposed LTV ratio and is more interesting for my research question.
Tighter LTV ratios affect not only credit volumes but also the default behavior of the borrowing household. Besides dampening the jump in mortgage demand when the level of personal bankruptcy protection increases, a lower LTV ratio also makes the borrowing household abstain from default unless the earnings exemption level is very high. The delayed default decision also explains why for the LTV ratio of 0.5 the jump in the demand for mortgages occurs at a higher exemption levels than when the LTV ratio is 0.7 or 0.9.

Previously I described crowding-in of capital when the level of personal bankruptcy protection is sufficiently high. Lower LTV ratios do not seem to strengthen this effect but allow for higher corporate lending regardless of the level of earnings exemptions. Here again I abstract from analyzing the left-hand side of the exemption interval as the ranking of outcomes on that side is partially caused by modelling choices. Higher corporate lending should be associated with lower LTV ratios because lower LTV ratios restrict mortgage credit by more in general ceteris paribus. Indeed, lower LTV ratios allow for higher corporate lending regardless of the level of earnings exemptions. At the same time, for any LTV ratio, when the level of earnings exemptions increases beyond 0.6 in this setup, loans to business start increasing in $\kappa$ as a result of the portfolio effect, see Figure 16. However, the bank does not shift to corporate lending at a higher rate for lower LTV ratios than higher LTV ratios. This follows from the observation that the slope of the curve barely changes across LTV ratios but the intercept increases visibly. Therefore, lower LTV ratios do not seem to strengthen the portfolio effect but combining lower LTV ratios with higher $\kappa$ would result in higher corporate lending than otherwise. In other words, higher corporate lending achieved under lower LTV ratios can be stimulated even more, if personal bankruptcy protection increases.
and if it is already sufficiently high. The expected total utility is unsurprisingly higher for lower LTV ratios. Tighter household borrowing rules increase corporate lending by redirecting bank credit to businesses and thus equilibrium income results in higher total utility, see Figure 17. Although total expected utility is higher for lower LTV ratios irrespective of the level of earnings exemptions, making personal bankruptcy more borrower-friendly grows total expected utility relatively more, if the LTV ratio is higher than lower. The result can be explained by the higher need of consumption insurance under higher LTV ratios: the household is more likely to default under higher LTV ratios (and in turn higher leverage) and thus values consumption insurance more. Besides these general patterns in total expected utility, Figure 17 shows that in the middle range of earnings exemptions, when the demand effect is strong enough to crowd out corporate lending, the total expected utility falls (due to income loss) but the fall still does not affect the general ranking of utility values across LTV ratios.

Before concluding this section, it is important to add that a lower LTV ratio for mortgages cannot be used as a substitute for personal bankruptcy protection: even under low LTV ratios consumption insurance still comes at a high value to the borrower and increases the total expected utility substantially. Of course, it is possible to choose such a value for the LTV ratio that the expected total utility would be higher even if earnings exemptions are not provided at all. However, such an LTV value would be very low as it is around 0.5 in my setup. For more reasonable levels of LTV ratios, lower LTV ratios just lift the level of total expected utility up for any level of earnings exemptions but not enough to make personal bankruptcy protection redundant from the welfare point of view. In general, combining the higher level of personal bankruptcy protection with tighter macroprudential rules for household borrowing not only dampens jumps in demand for mortgage credit while preserving insurance for insolvent individuals but also increases total welfare. For reasonable LTV ratios, this combination is more welfare-increasing than using the analyzed macroprudential policies to replace personal bankruptcy protection.

### 4.4 Effect on housing

In the current setup, demand for housing highly depends on the household's ability to borrow but the relationship is not linear. When a large enough increase in the level of personal bankruptcy protection impairs the household's ability to get mortgage credit, demand for housing eventually declines. However, if the level of earnings exemptions increases from a very low level, the effect
on equilibrium housing stock may be even positive. The findings suggest that an increase in the level of earnings exemptions would not necessarily cool down the housing market and prevent inflation of a housing bubble.

The non-linear relationship between the level of earnings exemptions and housing is illustrated in Figure 18. I observe that housing demand co-moves with mortgage credit stronger for higher $\kappa$ suggesting that, when earnings exemptions increase and thus mortgage credit becomes more expensive, demand for housing gets restricted too. Nevertheless, it takes a higher level of earnings exemptions to make equilibrium housing decrease than it takes for mortgage credit. The borrower values housing as a consumption smoothing tool and a utility-generating asset which is possibly driving the result.

An increase in earnings exemptions for low starting $\kappa$ ($\kappa < 0.6$) makes equilibrium housing increase. There the household starts borrowing more in response to less costly default and thus she can afford purchasing more housing. However, when higher earnings exemptions make access to credit even more restricted ($\kappa \geq 0.6$), household’s net worth is reduced by higher mortgage rates and the household cannot afford the same level of housing as before. Only then the housing stock starts declining.

The optimal level of $\kappa$ as suggested by my welfare comparison exercise would actually lead to a higher level of housing than without earnings exemptions ($\kappa = 0$). Thus higher earnings exemptions do not decrease equilibrium housing monotonically. This is an important take-away for the regulator who aims at reducing housing demand by fine-tuning personal bankruptcy regulations.

Macroprudential policy indeed has a more efficient tool to adjust housing demand, namely, the
LTV ratio. A comparison of housing outcomes for different levels of LTV as presented in Figure 19 suggests that increasing the required equity amount when purchasing housing would shift equilibrium housing down for any level of earnings exemptions.

![Figure 18: Housing and mortgages.](image1)

![Figure 19: Housing and LTV ratios.](image2)

5 Robustness check

The portfolio effect triggered by the increase in the level of personal bankruptcy protection is robust for a series of different parameters. Mortgage credit remains a decreasing function in the level of earnings exemptions and different parametrization mostly affects the size of portfolio adjustment.

Uncertainty with respect to returns on mortgages plays a crucial role in determining the bank portfolio choice, therefore, I check the result for a different probability distribution of the housing value shock. In particular, I increase the variance of the housing value shock by assuming the bad state to be more likely. I increase the probability of the bad value of the housing value shock $p_L$ from 0.2 to 0.3 and adjust other parameters of this shock such that the expected value of the housing value shock is not affected. Notably, the portfolio effect and the pattern of the resulting credit allocation prevails. I see that the imposed higher uncertainty of the housing value shock reduces the level of mortgage credit and increases the level of corporate lending for any $\kappa \geq 0.6$ (Figures 20-21 in the appendix). This suggests that the negative supply effect becomes only stronger if bad states are more likely.

I introduce deadweight bankruptcy loss as a fixed share of housing value $\chi$ that is lost in case
of bankruptcy. This loss can be attributed to fire sales, monitoring or legal costs. The price at fire sale can get discounted because the highest potential bidder may be facing financial constraints as well (Shleifer and Vishny 1992, 2011). Foreclosed housing is a type of distressed asset.\footnote{Research shows that discounts on foreclosed homes can be as high as 27\% (Campbell, Giglio, and Pathak, 2011).} The introduced deadweight loss does not affect the borrower directly because in case of bankruptcy the housing asset is seized by the creditor. However, the lender would take the loss into account when setting the interest rate and mortgage credit would become more expensive. Then under the exogenous deadweight bankruptcy costs of 30\% the bank's response to higher earnings exemptions becomes stronger and the supply effect dominates the dynamics of equilibrium mortgage credit even more: equilibrium mortgage credit decreases in $\kappa$ by more when $\kappa$ is higher than 0.6 (Figure 22 in the appendix). Consequently, the general equilibrium effect is stronger under positive deadweight bankruptcy costs (Figure 23 in the appendix).

A lower endowment reduces the amount of intermediated credit but the pattern of the decline in mortgage credit in the earnings exemption level prevails. I see that firstly a lower endowment translates into lower equilibrium savings which restricts equilibrium capital stock (Figure 24 in the appendix). Nevertheless, the composition of the bank portfolio starts shifting in favor of corporate loans when $\kappa$ is higher than 0.6 (Figure 25 in the appendix). I observe that the lower endowment makes the borrower borrow less and purchase less housing. An exogenous LTV ratio (which is binding by assumption) makes demand for mortgages a function of demand for housing, so that lower demand for housing results in lower mortgage credit. This is the reason I do not observe heavier leveraging at the household in order to compensate for low income and achieve the desired level of consumption and housing.

6 Future extensions

Results are driven by the general equilibrium effect which rewards households with higher income when the level of personal bankruptcy protection increases. This makes the borrower better off in good times too, because the resulting losses from higher mortgage interest rates get offset by gains from larger production in the economy. The income gain offsets a fraction of the income loss of the saver too and makes personal bankruptcy protection welfare-increasing in my setup.

The general equilibrium effect could be dampened by several realistic extensions.

Firstly, the current setup suffers from the limited number of options for bank investment. If
the bank invested in a risk-free asset (i.e., put deposits at the central bank), a change in the mortgage credit supply would translate into a smaller increase in the supply of corporate loans. Second, currently featured shocks are independent. Positive correlation between the two shocks would make the bank account more for a scenario when both mortgages and corporate loans underperform. Then shifting to capital loans would be less profitable for the bank.

Homogeneity in labor income profiles among agents leads to the outcome where what was lost through a smaller mortgage loan can be recouped in the form of higher wage to some extent. With more heterogeneity the general equilibrium gains would be redistributed unevenly and borrowing-constrained agents would not necessarily benefit from larger returns on capital.

Although the current setup potentially overstates the efficiency gains, the credit reallocation channel would not disappear. The remaining question is about the power of this channel.

Employing households to produce housing goods may have a qualitative effect on my findings. Once the increase in the level of personal bankruptcy protection reduces mortgage demand, for some values of earnings exemptions housing demand would decrease too. This would have a negative effect on wage income from working at the housing sector and act as an opposite effect to the increase in wage income received from financially constrained firms. However, given that the share of the housing production in total output in emerging Europe (and everywhere else) sector is less than half, the decrease in wage income from the housing sector is unlikely to offset the increase in wage income from the rest of the economy.

Another direction of useful extensions is the introduction of regular default (as opposed to strategic default) to make the model reflect the more frequent causes of mortgage default such as job loss, illness, etc., and test the credit allocation result in such an environment.

A more detailed model of the housing market would enrich my analysis of the personal bankruptcy effect on demand for housing and allow us to bridge the literature on housing bubble formation and the literature on personal bankruptcy protection.

### 7 Conclusions

I develop a general equilibrium model with mortgage default and the bank portfolio choice to illustrate how an increase in the level of personal bankruptcy protection can shift the bank portfolio composition in favor of corporate loans. Higher lending to financially constrained firms increases capital stock and leads to the positive feedback effect into production. However, the effects are
non-linear. I find that for a low starting level of personal bankruptcy protection a marginal increase in the level of protection would lead to a sufficiently small increase in borrowing costs such that a strong positive demand effect on mortgages would dominate and lead to higher mortgage credit, crowding out corporate lending. The non-linearity of credit outcomes in response to the protection changes is an important take-away for the regulator who aims at not only providing consumption insurance for insolvent borrowers but also takes total output into consideration.

I find that the optimal level of earnings exemptions in the considered setup is strictly larger than zero because welfare measures take into account both consumption sharing and higher equilibrium income. Ignoring the bank portfolio choice would omit the spillover effect on production and the benefits of higher personal bankruptcy protection would be underestimated considerably. However, more inefficiencies in borrowing and lending should be taken into account to give a definite answer about the welfare outcome.

Higher earnings exemptions do not decrease equilibrium housing monotonically. Again this is an important case to consider for the regulator who aims at reducing housing demand by fine-tuning personal bankruptcy regulations. Also, even for a sufficiently high level of personal bankruptcy protection, a marginal increase in the level of personal bankruptcy protection would cool down the housing market by little. Housing demand would respond to the level of personal bankruptcy slowly due to two opposite effects. On one hand, higher exemptions would imply tighter household credit constraints. On the other hand, housing is the only consumption smoothing tool for the borrower which would keep the demand for housing relatively stable.

Besides presenting the credit allocation channel, I study one of the macroprudential tools at the regulator’s disposal to reduce demand for mortgage credit. This may be necessary, if a higher level of personal bankruptcy protection, in general a welfare-increasing outcome, turns the increased demand for mortgage credit to over-borrowing. I show that combining the higher level of personal bankruptcy protection with tighter LTV ratios for household borrowing can not only dampen jumps in demand for mortgage credit while preserving insurance for insolvent individuals but also increases total welfare. Total welfare increases mainly because lower LTV ratios restrict mortgage credit by more in general. Lower demand for mortgages makes lenders shift to corporate lending and thus investment in productive asset, capital, as opposed to unproductive housing.
References


A Solution algorithm

In the description of the algorithm we introduce uncertainty in the form of the housing quality shock $\zeta$ and productivity shock $z$ and allow both the firm and the borrowing household to default. We employ the backward induction method which makes use of the assumptions $d_3 = 0$ and $m_3 = 0$.

The housing value shock occurs in period 2 and has two realization values $\{\zeta^L, \zeta^H\}$. The low value occurs with a probability $p^L_\zeta$. The productivity shock occurs in period 2 as well and has two realization values $\{z^L, z^H\}$. The low value occurs with a probability $p^L_z$.

1. Generate two grids: one for savings and one for housing.
2. Take $y_1$, $d_3$ and $m_3$ as given. Also, due to 3-periods setup, $k_3 = 0$.
3. Make a guess for risk-free interest rates $\{r^m_1, r^{mLL}_2, r^{mHL}_2, r^{mLH}_2, r^{mHH}_2\}$, mortgage interest rates $\{r^m_1, r^{mLL}_2, r^{mHL}_2, r^{mLH}_2, r^{mHH}_2\}$ and corporate interest rates $\{r^l_1, r^{lLL}_2, r^{lHL}_2, r^{lLH}_2, r^{lHH}_2\}$ for all states in period 1 and period 2. $r_1$ is the risk-free interest rate in period 1 and $\{r^{mLL}_2, r^{mHL}_2, r^{mLH}_2, r^{mHH}_2\}$ correspond to interest rates in period 2 given different realizations of the housing quality shock and the productivity shock such that the first letter in the superscript (H or L) means the value of the housing shock and the second letter denotes the materialized value of the productivity shock. Since default is possible only in period 2, only $r^{m}_1$ and $r^{m}_1$ will be different from the corresponding $r$ value.
4. $w_1$ and $k_1/n_2$ follow.
5. Compute the resulting prices $\{w^{LL}_2, w^{HL}_2, w^{LH}_2, w^{HH}_2\}$ and $k^{LL}_2/n_3, k^{HL}_2/n_3, k^{LH}_2/n_3$ and $k^{HH}_2/n_3$ by solving the firm’s optimization problem (equation (1)).
6. Since labor supply is inelastic, $k_1, k^{LL}_2, k^{HL}_2, k^{LH}_2$ and $k^{HH}_2$ can be determined immediately.
7. Start defining value functions from period 3.
8. Determine $V^S_3$ for every value of savings in period 2 $d_2$ (take $d_2$ values from the grid). Then compute $V^{BP}_3$ and $V^{BD}_3$ for every value of housing accumulated in period 2 $h_2$. To do it take $h_2$ values from the grid and make use of $m_2 = \rho(1 - \delta)h_2 / (1 + r^m_2)$. This yields $V^S_3 (d_2)$, $V^{BP}_3 (h_2)$ and $V^{BD}_3 (h_2)$.
9. Express functions in period 2 $V^S_2$, $V^{BP}_2$ and $V^{BD}_2$ as functions of instantaneous utility in period 2, given the realizations of shocks (four possible states), and discounted values of $V^S_3$, $V^{BP}_3$ and $V^{BD}_3$, respectively. Conditional on shock values, compute the value for the
saver in period 2 \((V_{2S}^S)\) for every possible combination of \(d_1\) and \(d_2\). Compute the value of the borrower who repaid in period 2 \((V_{2BP}^B)\) for every possible combination of \(h_1\) and \(h_2\) (make use of \(m_1 = \rho(1-\delta)\zeta h_1/(1+r_m^u)\)). Compute the value of the borrower who defaulted in period 2 \((V_{2BD}^B)\) for every possible combination of \(h_1\) and \(h_2\) (make use of \(m_1 = \rho(1-\delta)\zeta h_1/(1+r_m^u)\)). This yields \(V_{2S}^S(d_1,\zeta,z), V_{2BP}^B(h_1,\zeta,z)\) and \(V_{2BD}^B(h_1,\zeta,z)\).

10. Find the maximum values of \(V_{2S}^S(d_1,\zeta,z)\) for every realization of shocks and savings stock in period 1. This yields decision rules for every possible \(d_1\) the agent brings to period 2 given shock values.

11. Find the maximum values of \(V_{2BP}^B(h_1,\zeta,z)\) and \(V_{2BD}^B(h_1,\zeta,z)\) for every possible \(h_1\), given the realization of the shock. Then determine the optimal default choice for the borrowing household by computing \(V_2^B(h_1,\zeta,z) = \max \{V_{2BP}^B(h_1,\zeta,z), V_{2BD}^B(h_1,\zeta,z)\}\). This yields decision rules for every possible \(h_1\) the agent brings to period 2, given the materialization of shocks in period 2.

12. Express \(V_1^S\) as a function of instantaneous utility in period 1 and the respective value in period 2. To find the corresponding optimal value in period 2, the realizations of \(V_{2S}^S(d_1,\zeta,z)\) have to be not only discounted but also weighted with respective probabilities of each shock occurring. Do an analogical exercise for \(V_1^B\). Note that to find the corresponding \(V_2^S(d_1,\zeta,z)\) and \(V_2^B(h_1,\zeta,z)\) here we already make use of the retrieved decision rules for every possible \(d_1\) and \(h_1\) that connect them to \(d_2\) and \(h_2\), given the realization of the shock.

13. Compute the value of the saver in period 1 \((V_{1S}^S)\) for every possible \(d_1\). Compute the value of the borrower in period 1 \((V_{1B}^B)\) for every possible \(h_1\).

14. Find the maximum value of \(V_{1S}^S(d_1)\) and \(V_{1B}^B(h_1)\). This yields optimal \(d_1\) and optimal \(h_1\) with the already determined future choices that follow from the optimal choice in period 1.

15. Compute expected returns on the mortgage \(\tilde{r}_m^m\) and on the corporate loan \(\tilde{r}_l^m\) by weighting returns under different materializations of the shock with the respective probabilities.

16. Compute \(\text{var} (\tilde{r}_m^m), \text{var} (\tilde{r}_l^m)\) and \(\gamma_m\) as given by equation (3). Check if \(m_1 - \gamma_m d_1\) is different from zero.

17. If yes, change \(r_m^m\) and repeat steps 9-17 until \(m_1 \approx \gamma_m d_1\) under the set tolerance level \(fminsearch\).

18. Compute \(E_1\tilde{r}_2\) from equation (2) and check if \(E_1\tilde{r}_2 - (1+\tau)r_1\) is different from zero.
19. If yes, change $r_1$ and repeat steps 9-19 until $E_1 \tilde{r}_2 \approx r_1$ under the set tolerance level (fminsearch).

20. For the developed sequences $\{d_1, d_2^{LL}, d_2^{HL}, d_2^{HH}\}$ and $\{m_1, m_2^{LL}, m_2^{HL}, m_2^{HH}\}$ compute $d - m - k$.

21. If the deviation from the clearing condition for savings is different from zero, change the vector of $r$ and repeat steps 4-21 until the deviation from the clearing condition disappears (fminsearch).

B Additional figures

Figure 20: Mortgages at different state of nature.

Figure 21: Capital stock at different state of nature.
Figure 22: Mortgages and deadweight bankruptcy costs.

Figure 23: Capital and deadweight bankruptcy costs.

Figure 24: Capital and endowments.

Figure 25: Mortgage credit ratio and endowments.
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<th>Symbol</th>
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Table 2: Baseline parameters.