LIETUVOS BANKAS
EUROSISTEMA

WORKING PAPER SERIES

No 28 / 2016

LABOUR SHARES, FERTILITY AND LONGEVITY IN AN OLG MODEL

By Igor Fedotenkov

ISSN 2029-0446 (ONLINE)

WORKING PAPER SERIES
No 28 / 2016

LABOUR SHARES, FERTILITY AND LONGEVITY IN AN OLG MODEL

Igor Fedotenkov*

* Economics Department, Bank of Lithuania. E-mail: i.fedotenkov@gmail.com, ifedotenkov@lb.lt

Address
Totorių g. 4
LT-01121 Vilnius
Lithuania
Telephone (8 5) 268 0103

Internet
http://www.lb.lt

Working Papers describe research in progress by the author(s) and
are published to stimulate discussion and critical comments.

The Series is managed by Applied Macroeconomic Research Division of Economics Department
and Center for Excellence in Finance and Economic Research.

ISSN 2029-0446 (ONLINE)

Abstract

This paper studies the link between demographic factors and labour shares as well as tries to answer the question whether population ageing is responsible for the global decline in labour shares. We found that the link depends on the elasticity of substitution between labour and capital as production factors. Given the empirical estimates of this parameter, we conclude that population ageing is not responsible for the global decline in labour shares. On the contrary, it reduces the speed of this decline.

Santrauka

Šis straipsnis nagrinėja ryšį tarp demografinių veiksnių ir darbo užmokesčių dalies visose pajamose. Jis bando atsakyti į klausimą, kaip populiacijos senėjimas yra susijęs su pasauliniu darbo užmokesčio pajamų dalies sumažėjimu. Mes parodėme, kad šis ryšis priklauso nuo darbo ir kapitalo pakeitimo elastingumo dydžio. Empiriškai įvertinę šį parametrą mes padarėme išvadą, kad populiacijos senėjimas nėra visuotinio darbo užmokesčių pajamų dalies sumažėjimo priežastis. Priešingai, jis stabdo šio mažėjimo greitį.

JEL Classification: E25, C51, J14

Keywords: Labour shares, population ageing, CES production function

1 Introduction

The present study investigates the effect population ageing has on the allocation of income between capital and labor. More specifically, we analyze the impact of an exogenous fertility rate and longevity shock on the labor income share. We use a two-period overlapping generations model, where agents have the ability to predict the size and impact of the exogenous shock. We attempt to provide additional insights in labor shares of income by recognizing that population ageing may have an effect on labor shares through changes in the size of the labor pool as well as through effects caused by rebalancing the income proportions dedicated to consumption as opposed to savings. We, therefore, attempt to connect two strands of literature: one explaining the worldwide decline of labor income shares and the second linking population ageing and other demographic factors to labor market outcomes.

Without any policy reforms that would increase the pension age and target improvements in the flexibility of labor market institutions, a graying society is expected to lead to changes in the size of the labor pool. The shift in age profiles determines skill availability and productivity, which, together with a lower labor supply, may lead to changes in wages and thus different capital-labor ratios. These macroeconomic outcomes will depend on the elasticity of substitution between capital and labor (σ). We provide a way of assessing how ageing affects labor income given an empirical estimate of σ. We restrict our study to two cohorts, homogenous in all respects except for the probability of survival. The size of the labor pool is affected by shifts in the fertility rate. To explore the above-mentioned mechanisms, we develop a general equilibrium theoretical model and estimate it with cross-country data. Our results show that an older society is associated with higher labor income shares, therefore, population ageing is not only an unlikely driver of a decline in labor shares, but it may actually reduce the speed of it.

Two aspects of population ageing help clarify our findings: a lower fertility rate affects labor markets by reducing labor endowments, thus contributing to an increase in wages. Higher longevity leads to an increase in savings, resulting in greater capital endowments. Using cross-sectional data for approximately 100 countries, we estimate σ to be significantly below 1, which implies that population ageing leads to an increase in labor shares. To some extent, the effects of capital deepening on labor shares were already mentioned in literature discussed below, the direct effect of smaller labor force on capital-labor ratios being rather obvious. The main contribution of this paper is the inclusion of increasing longevity and the study of its effects on labor shares in the general equilibrium framework.

The most similar research paper to ours is that of Schmidt and Vosen (2013), who studied effects of demographic changes on the dynamics of labor shares. They implemented numerical simulations and found that the effects may depend on the elasticity of substitution between capital and labor as production factors, reaching a conclusion that population ageing leads to a decline in labor shares. We extend this contribution in two ways. First of all, we derive the link between demographic factors and labor shares analytically, which produces a stronger result.

Second, we have used demographic data to estimate the elasticity of substitution between labor and capital, and found that it is smaller than unity. This estimate is well in line with numerous empirical studies summarized by Klump et al. (2007b) and Chirinko (2008), yet it gives the opposite result to that of Schmidt and Vosen: population ageing leads to an increase in labor shares. Therefore, we have reached a conclusion that the decline in labor shares, observed worldwide, is determined by factors other than population ageing.

There is a wealth of studies on labor shares, however, we restrict our attention to studies that are the most relevant to our research question. Blanchard (1997) studied production factor shares in the OECD countries. In a partial equilibrium model, without capital accumulation, he argued that markups work as a tax on labor. An increase in markups reduces both wages and employment, leading to a decline in labor shares. An increase in profits results in the entry of new firms to the market, which, already having a certain fixed amount of capital, cause an increase in capital-labor ratios. In the long run, wages and unemployment return to their equilibrium levels, but labor shares remain lower due to higher capital – this is caused by the assumption that the elasticity of substitution between labor and capital is greater than 1. In general, a declining labor force in the above-mentioned model also leads to a decline in labor shares of income, given that labor and capital are substitutes. In contrast, rather than assuming a value for σ, in our paper we estimate the elasticity of substitution to be smaller than 1. Our result is also confirmed by a large number of other empirical studies (Eisner and Nadiri 1968; Clark 1993; Chirinko, Fazzari, and Meyer 1999; Ellis and Price 2004; Klump, McAdam, and Willman 2007a; Young 2013).

Acemoglu (2003) presented evidence that, in the United States and France, labor shares of income are relatively constant. He argued that capital accumulation drives labor shares up because the elasticity of substitution between labor and capital is smaller than 1; however, reasons for capital deepening remain unclear. In the long run, the increase in labor shares is balanced out by an endogenous labor-augmenting technological process. Our work supplements Acemoglu's findings. We estimate the elasticity of substitution between labor and capital to be lower than 1 and show that changes in labor and capital markets driven by population ageing have a positive effect on labor shares.

In an empirical paper, Harrison (2002) argued that labor shares are eroded by globalization. If in the neighboring country wages are smaller, capital may go abroad and, as a result, labor's bargaining power falls; this leads to a decline in labor shares. Conversely, the presence of barriers to capital movements and higher government expenditures have a positive effect on labor shares. The author also noticed that higher capital-labor ratios are associated with higher labor shares, and guessed that the elasticity of substitution between labor and capital is smaller than 1. Our paper complements Harrison's results. Within a theoretical general equilibrium model, we render capital-labor ratios to be endogenous, linking them to demographic drivers of population ageing – fertility rates and longevity. Similarly to Harrison, the role of globalization in general and increasing capital mobility in explaining declining labor shares were also emphasized by Jaumotte and Tytell (2007), Guscina (2006) and Jayadev (2007).

Another reason for this process, according to Guscina, is a capital-augmenting technological process.

Bentolila and Saint-Paul (2003) found a link between labor shares and the capital-to-output ratio. They argued that the capital-augmenting technological process and declining prices of imported materials shift this ratio and affect labor shares. Karabarbounis and Neiman (2014) also focused on prices of materials and other capital goods. The authors collected a novel set of data on corporate labor shares not only for developed countries but also for less developed countries, and concluded that declining prices of investment goods is the cause for approximately 50 per cent of the decline in labor shares. In this paper, we use the same data on labor shares collected by Karabarbounis and Neiman, and compare our results with theirs. One essential difference between the two studies is that Karabarbounis and Neiman estimated the elasticity of substitution between labor and capital in the production function to be greater than 1. Our estimates are smaller than unity but exceed the conventional interval [0.4, 0.6] (Chirinko 2008). We attribute this difference to the inclusion of fertility rates in the estimation of σ.

Blanchard and Giavazzi (2003) linked the decline in labor shares in Europe to the deregulation of the labor market. In spite of the positive impact on labor shares of product market deregulation, the labor market reform has led to a reduction in the bargaining power of workers and thus to a decline in labor shares.

Suchanek (2009) created a model in which the bargaining power of production factors depends on market imperfections. The authors argued that capital market imperfections have a lower impact on income shares than imperfections in the labor market. This leads to a higher bargaining power of capital, which drives labor shares down. Dünhaupt (2013) also emphasized the role of financial markets. The author argued that the decline in labor shares is attributed to the increasing importance of the financial sector as it changes shareholders' value orientation. Namely, instead of empire-building and job creation, firms' management focuses on short-term economic indicators, such as increasing dividends and interest rate payments at the expense of long-term growth.

Other reasons for a decline in labor shares mentioned in literature are as follows: Azmat et al. (2012) suggested that a decline in labor shares is determined by privatization, which leads to a fall in employment. According to their calculations, privatization is responsible for approximately 20 per cent of the decline in labor shares in the OECD countries. Golin (2002) claimed that income of self-employed agents is often treated as capital income. If labor shares are adjusted for this factor, they become less volatile across countries. Even though it is not explicitly stated in the paper, using this logic it might be stated that "official" labor shares can decline if the number of self-employed agents increases. Diwan (1999) linked declining labor shares of income to financial crises. The author observed that labor shares decline sharply during financial crises and start growing in subsequent years; however, this growth is not sufficient for a full recovery of labor shares. Still, the exact reason why this is the case remains unclear.

In contrast to the previously-mentioned research, in this paper we make an attempt to link labor shares to population ageing. Our initial hypothesis is that there is a link between these two phenomena via the two channels described above. Our results show that population ageing leads to an increase in labor shares. The reviewed literature points to an array of factors which lead to a decline in labor shares. Our analysis indicates that population ageing may act as a counterbalancing force, reducing the potential impact of these factors. As population ageing is strongly connected with pension schemes, we have also studied the effects of pension system designs on labor shares, yet we found no statistically significant relation between pension systems and labor shares. For this reason, we employ a simple two-period overlapping generations model.We study the effects of population ageing in two settings: one with a declining fertility rate and another featuring an increase in longevity. The effect of declining fertility on labor shares in our model performs in two steps. First, it leads to capital deepening as a smaller number of agents share a higher amount of capital. Second, increasing capital-labor ratios lead to an increase in labor shares due to the elasticity of substitution between labor and capital that is smaller than 1.[2] Population ageing understood as higher longevity increases agents' desire to save more in the present in order to use savings in the subsequent period. This leads to capital deepening as well and, as a result, to the growth in labor shares.

The paper is structured as follows: in the next section a preliminary empirical analysis is performed in order to identify the factors affecting labor shares. In Section 3 a theoretical model is build to capture the effects discussed in Section 2. In Section 4 the theoretical model is estimated. In Section 5 robustness is discussed. Finally, Section 6 presents the conclusions.

2 Preliminary analysis

Before starting with the formal model, we present some preliminary regressions in order to illustrate the main relationships observed in the data.

2.1 Data

The data has been collected from several sources: Data on labor shares comes from Karabarbounis and Neiman (2014). They collected a novel set from the UN and OECD data, including printed material and Internet resources. The novelty of this data set lies in the focus on corporate records as a source for labor income. Our selected observations of the total labor shares are for 2010. For a few countries, if such observations were not available, we have selected the observations which are closest to 2010 in terms of time. All demographic data comes from the CIA world factbook 2010 as this data set, to the best of our knowledge, covers the largest number of countries.

[2]As a robustness check, we have also performed the same analysis with a variable elasticity of substitution (VES) production function (Revankar 1971), which led to similar results.

Table 1: Linear regression, dependent variable: Labor share

	Model 1	Model 2	Model 3	Model 4
Intercept	–0.2825 (0.2004)	0.1702 (0.2109)*	0.3565 (0.0414)***	0.1451 (0.1301)
Life expectancy	0.0010 (0.0021)	0.0009 (0.0020)		
Fertility/woman	–0.0394 (0.0114)***	–0.0259 (0.0117)**	–0.0276 (0.0099)***	
65 years+, %		0.0068 (0.0023)***	0.0068 (0.0022)***	
PAYG taxes	0.0002 (0.0011)	–0.0005 (0.0011)		
Pension age	0.0106 (0.0028)***	0.0020 (0.0029)		
GDP/capita PPP	1.5e-06 (6.8e-07)**	1.7e-06 (7.0e-07)***	1.5e-06 (8.1e-07)*	2.61e-07 (5.4e-07)***
Oil/GDP		–5.5e-06 (2.2e-06)**	–8.0e-06 (2.3e-06)***	–6.3e-06 (2.0e-06)***
FDI in-FDI out			0.0001 (0.0002)	
Net migration			0.0035 (0.0033)	
Median age				0.0009 (0.0003)**
Birth rate/1000				–0.0002 (0.0002)
Death rate/1000				0.0007 (0.0032)
R^2	0.5202	0.5847	0.5406	0.5458
N	95	95	86	106

* $p < 0.1$

** $p < 0.05$

*** $p < 0.01$ significance level

Demographic factors are also very closely linked with pension systems, as population ageing increases the pay-as-you-go (PAYG) tax burden for the young, and/or reduces pension benefits for the old, and, thus, affects savings and investment. Information on contributions to the first pillars of pension schemes comes from the World Bank Social Protection & Labor data. Data on the pension age is taken from two online sources.[3] We have only taken the observations for 2010. If they were not available, observations for 2009 or 2011 have been used instead. Macroeconomic factors employed in the regression analysis are: GDP/capita PPP, nominal GDP, oil production, incoming and outgoing foreign direct investments (FDI) as a percentage of GDP. All this data, apart from FDI, comes from the CIA world factbook 2010; the observations for the FDI have been taken from the World Bank development indicators. As FDIs are highly volatile, we use arithmetic averages for the period 2005–2010.

Table 1 presents the estimates of several linear regression models for the labor shares on demographic and macroeconomic variables. We use a robust regression (M-estimator) as we expect to have outliers in the data. We use a specification which is default in EViews 8.1.[4] The first regression suggests that fertility per woman (total fertility rate) has a significant impact (at 1% level) on the labor shares, higher fertility rate being associated with lower labor shares. GDP per capita PPP captures the effects of a country's economic development. Richer countries have, in general, higher labor shares. Surprisingly, the coefficients corresponding to life expectancy and PAYG contributions are insignificant. Pension age has a significant positive impact on labor shares.

The data shows that countries with a large oil production relative to their GDP (OPEC, Russia) have a smaller pension age. The correlation among these two variables is –0.43 (see the Appendix). In Model 2, we include oil production (in million tons) divided by nominal GDP. The results indicate that oil production per GDP is significant at the 5 per cent significance level, while the pension age becomes insignificant. Furthermore, we include the percentage of agents aged 65 or older. The coefficient corresponding to this value is significant at the 1 per cent significance level. PAYG contribution rates make no statistically significant contribution in explaining the cross-sectional variation in labor shares.

Given the surveyed literature that links ageing to developments in the investment and capital markets, we also test whether international capital flows (as measured by FDI volumes) and migration are connected. We exclude portfolio flows since they are highly volatile and most of the time represent short-term investments without productivity enhancing effects. In Model 3, we remove all insignificant variables from Model 2 and add a difference between incoming and outgoing FDI (as a share of GDP) and net migration. We observe that both FDI and migration have no significant impact on labor shares. This is not surprising as international capital and labor mobility are far from perfect (Lucas 1990; Alfaro, Kalemli-Ozcan, and Volosovych

[3] http://www.ssa.gov/policy/docs/progdesc/index.html
http://chartsbin.com/view/2468

[4] M settings: weight=Bisquare, tuning=4.685, scale=Huber, Huber Type I Standard Errors & Covariance

2008; Batista 2008), which implies that international flows of production factors are much less important than their domestic counterparts.

The result for capital flows is very robust to specifications of the econometric model, and the coefficient corresponding to the differences in FDI remains insignificant with other specifications of regressions. We have also tried to include only the ingoing FDI, because it may represent the ingoing flow of technology to the country. In this case, the p-value of the corresponding coefficient declines; however, the coefficient remaines insignificant at the 10 per cent significance level.

Migration is correlated with GDP per capita PPP, and if one of these variables is removed, the other becomes highly significant, the inclusion of GDP PPP per capita to the model resulting in a higher R^2 than migration.

In Model 4, we include demographic factors other than those used in Models 1–3: median age and numbers of births and deaths per thousand of population. Indeed, the median age and the number of births are highly correlated (Table 5). In this case, the median age is significant at the 5 per cent significance level, while the number of births is not.

Labor shares of income are defined in the interval [0,1]. As a result, a logistic regression may also be used for their analysis. If regressors are correlated with the error term in the linear model, a nonlinear transformation sometimes solves this problem. Table 2 presents similar regression models, yet a logistic transformation for the dependent variable is used. The dependent variable is $log(l_i/(1-l_i))$, where l_i denotes the labor share in a country i. The results are virtually the same as in the case of the linear model, but in the fourth regression the birth rate per thousand of population is significant and the median age is not.

All in all, the Tables indicate that higher fertility rates are associated with a smaller labor share of income, and a higher number of agents over 65 results in higher labor shares. In the next section, we build a theoretical model to explain these observed relationships.

3 The model

In this section, a two-period overlapping generation model is developed. We omit country-specific indexes i, because we create a one-country model.

3.1 Firms

Firms produce output Y_t with the use of labor L_t and capital K_t as production factors:

$$Y_t = A\Big(\gamma K_t^{1-\frac{1}{\sigma}} + (1-\gamma)L_t^{1-\frac{1}{\sigma}}\Big)^{\frac{\sigma}{\sigma-1}} = L_t A\Big(\gamma k_t^{1-\frac{1}{\sigma}} + (1-\gamma)\Big)^{\frac{\sigma}{\sigma-1}} = L_t y_t, \tag{1}$$

where A denotes the country's technological level (it may also include institutional, historical and cultural peculiarities of the country). k_t denotes capital-labor ratios, y_t stands for output

Table 2: Logistic regression, dependent variable: Labor share

	Model 1	Model 2	Model 3	Model 4
Intercept	–2.6690 (0.8656)	–0.5749 (0.9319)	–0.5483 (0.1856)***	–0.5824 (0.5675)
Life expectancy	–0.0015 (0.0089)	0.0004 (0.0087)		
Fertility/woman	–0.2227 (0.0493)***	–0.1564 (0.0516)***	–0.1397 (0.0044)***	
65 years+, %		0.0292 (0.0102)***	0.0275 (0.0097)***	
PAYG taxes	–0.0006 (0.0047)	–0.0047 (0.0047)		
Pension age	0.0437 (0.0120)***	0.0009 (0.0129)		
GDP/capita PPP	6.5e-06 (2.9e-06)**	7.8e-06 (3.1e-06)**	6.4e-06 (3.6e-06)*	7.7e-06 (2.4e-06)***
Oil/GDP		–2.6e-05 (9.8e-06)***	–3.3e-05 (1.1e-05)***	–2.7e-05 (9.1e-06)***
FDI in-FDI out			0.0006 (0.0011)	
Net migration			0.0153 (0.0148)	
Median age				0.0082 (0.0145)
Births/1000				–0.0277 (0.0116)**
Deaths/1000				0.0174 (0.0140)
R^2	0.4606	0.5390	0.4535	0.4798
N	95	95	86	106

* $p < 0.1$
** $p < 0.05$
*** $p < 0.01$ significance level

per labor unit, σ is the elasticity of substitution, $\sigma > 0$, and γ is a parameter $\gamma \in (0, 1)$. Labor and capital markets are competitive; therefore, wages are equal to marginal returns to labor:

$$w_t = A(1-\gamma)\left(\gamma k_t^{1-\frac{1}{\sigma}} + 1 - \gamma\right)^{\frac{1}{\sigma-1}}. \tag{2}$$

Interest rates are defined as marginal returns to capital adjusted by the annuity factor ψ; ψ denotes the probability to survive until the second period of life. To close the model, we assume that perfect annuity markets exist, reallocating savings made by agents who passed away at the end of the first period to agents of the same generation. This increases returns to savings of the agents who survived. Capital fully depreciates in one period.

$$1 + r_t = \frac{A\gamma}{\psi}\left(\gamma + (1-\gamma)k_t^{\frac{1-\sigma}{\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{3}$$

3.2 Households

We suppose that the country is populated by representative agents that live for two periods: the "old" and the "young". They supply one unit of labor when they are young. We denote consumption of the young agents at time t as C_t^y, C_t^o is consumption when old, and s_t is savings made at time t, ρ denotes a discount factor. Agents maximize their lifetime utilities U_t

$$U_t = \log C_t^y + \frac{\psi}{1+\rho}\log C_{t+1}^o, \tag{4}$$

with budget constraints

$$C_t^y = w_t - s_t, \tag{5}$$

$$C_{t+1}^o = (1 + r_{t+1})s_t. \tag{6}$$

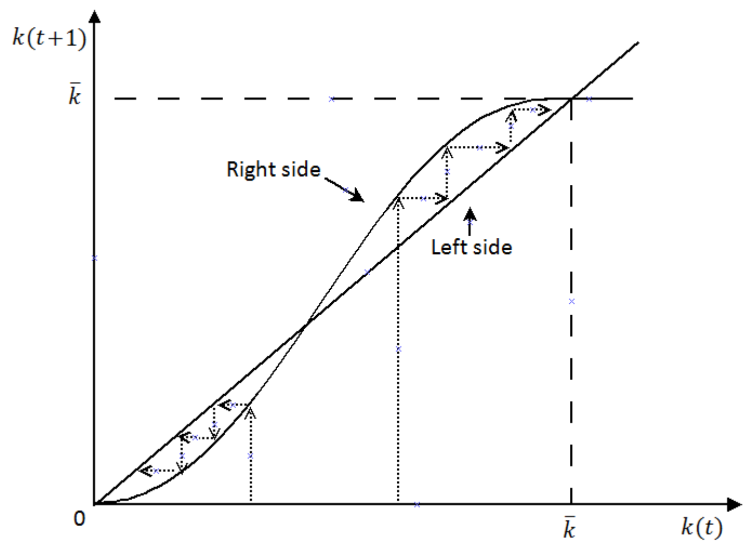
We assume that there are no pension schemes and in the second period of life agents use their savings only. Inserting budget constraints into the utility function and maximizing it with respect to savings, we find that

$$s_t = \frac{\psi w_t}{1+\rho+\psi}. \tag{7}$$

3.3 Equilibrium and dynamics

We assume that capital depreciates in one period; therefore

Figure 1: Steady state (Eq. 9, $\sigma < 1$)



$$K_{t+1} = L_t s_t. \tag{8}$$

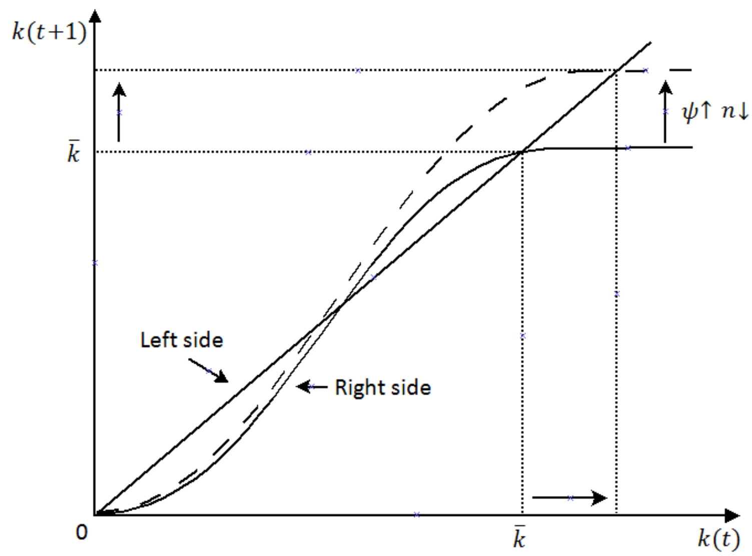
$$k_{t+1} = \frac{\psi A(1-\gamma)\left(\gamma k_t^{\frac{\sigma-1}{\sigma}} + 1 - \gamma\right)^{\frac{1}{\sigma-1}}}{(1+n)(1+\rho+\psi)}. \tag{9}$$

The dynamics of capital-labor ratios depends on whether σ is smaller or larger than unity. Fig. 1 presents left and right side of equation (9) when $\sigma < 1$; however, σ is not too close to 0. In this case, the left and right side of this equation intersect three times. One of the stable equilibria is in the origin. It corresponds to the case when the capital-to-labor ratio is equal to 0, and the economy does not operate. There is another stable equilibrium denoted by $\bar{k}$. An intersection of the curves between these two stable equilibria corresponds to an unstable equilibrium.[5] If σ is close to 0, the curve, which corresponds to the right side of the equation, shifts to the right and hence does not intersect with the curve corresponding to the left side of the equation. In this case, no internal equilibrium exists.

Fig. 2 presents shifts of the curves resulting from demographic developments. If agents' longevity ψ increases, it shifts the left side of equation (9) upwards, leading to an upward shift

[5]Indeed, the Figure is schematic and, for reasonable parameter values, the point corresponding to the unstable equilibrium is very close to 0.

Figure 2: Effects of demographic developments (Eq. 9, $\sigma < 1$)



of the stable equilibrium point, in turn resulting in higher capital-labor ratios.[6] If fertility n declines, then the curve shifts upwards, leading to an increase of equilibrium capital-labor ratios values corresponding to the internal stable equilibrium point. Therefore, we can conclude that when $\sigma < 1$ and an internal equilibrium exists, population ageing leads to capital deepening in the long run.

Fig. 3 presents the dynamics of the model when $\sigma > 1$. In this case, there is only one equilibrium point in the region $k \geq 0$. It is stable. The curve corresponding to the left side of the equation (9) does not intersect the origin: as in this case capital and labor are substitutes, even if capital is equal to 0, production is larger than 0, and thus part of production is saved, leading to capital accumulation.

If $\sigma = 1$, the CES function simplifies to the Cobb-Douglas case, and the curve corresponding to the left side of equation (9) goes through the origin, leading to two equilibria points. The equilibrium point corresponding to the intersection in the origin is unstable. Everything else is the same as in the case when $\sigma > 1$.

In Fig. 4 the long-run effects of demographic shocks when $\sigma > 1$ are depicted. An increase in longevity (ψ) and/or a decline in the fertility rate (n) shifts the left side of equation (9) upwards, leading to an increase in the equilibrium capital-labor ratio. Therefore, exactly like in the case $\sigma < 1$, population ageing leads to capital deepening.

[6]The shift of the left side of equation (9) can be easily seen if the nominator and denominator are divided by ψ.

Figure 3: Steady state (Eq. 9, $\sigma > 1$)

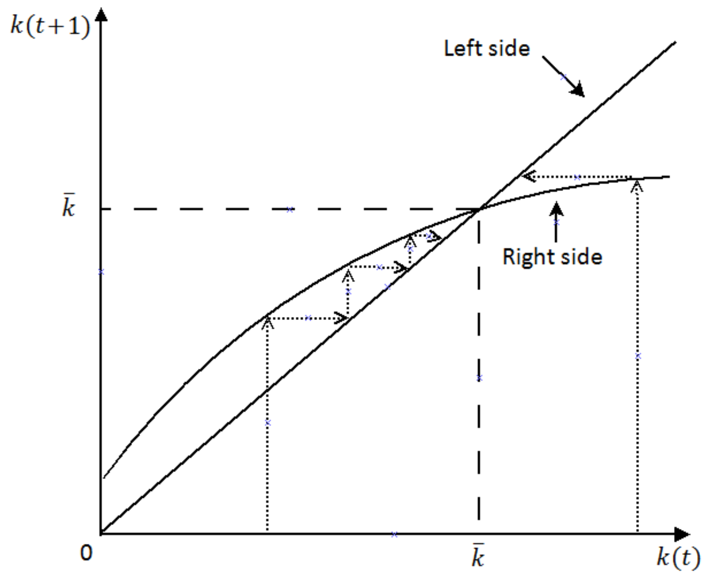
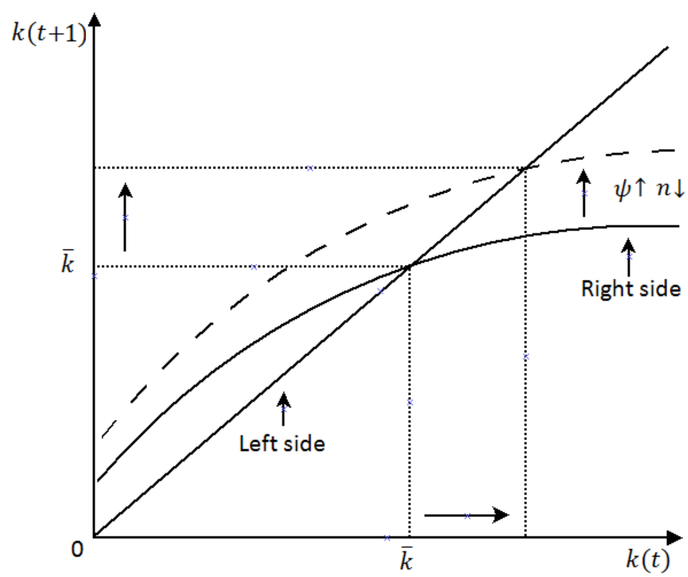


Figure 4: Effects of demographic developments (Eq. 9, $\sigma > 1$)



3.4 Labor shares

Total income in the model consists of labor income and capital income: $Y_t = L_t w_t + K_t(1+r_t)$. Labor share is defined as a share of wage income in the total income:

$$l_t = \frac{L_t w_t}{Y_t} = \frac{1-\gamma}{\gamma k_t^{\frac{\sigma-1}{\sigma}} + 1 - \gamma}. \tag{10}$$

In the previous subsection we have shown that population ageing leads to an increase in capital-labor ratios. Therefore, according to equation (10), the effect of population ageing on labor shares depends on the σ value. If $\sigma > 1$, an increase in capital-labor ratios leads to a decline in labor shares, whereas if $\sigma < 1$, the effect is the opposite. In the next section we perform estimations and show that, according to our estimates, $\sigma < 1$.

4 Estimation of the model

In order to perform estimations, we rewrite the model in a form that is suitable for this task. First of all, we add country-specific indexes i to the equations derived in the previous section. From equation we (10) express capital-labor ratios:

$$k_{i,t} = \left(\frac{1-\gamma}{\gamma}(l_{i,t}^{-1} - 1)\right)^{\frac{\sigma}{\sigma-1}}. \tag{11}$$

Having expressed capital-labor ratio, we insert it to equation (1) in order to express country-specific technological levels $A_{i,t}$:

$$A_{i,t} = y_{i,t}\left(\frac{l_{i,t}}{1-\gamma}\right)^{\frac{\sigma}{\sigma-1}}. \tag{12}$$

Now, we plug equations (11–12) to (9):

$$\left(\frac{1-\gamma}{\gamma}(l_{i,t+1}^{-1} - 1)\right)^{\frac{\sigma}{\sigma-1}} = \frac{\psi y_{i,t} l_{i,t}}{(1+n)(1+\rho+\psi)}. \tag{13}$$

We make a simplifying assumption that labor shares decline in all countries at the same rate: $l_{i,t+1} = \eta l_{i,t}$, with $0 < \eta < 1$.

As we do not take into account price differences in the model, income per capita $y_{i,t}$ can be approximated by GDP PPP per capita $y_{i,t}^{PPP}$. However, it should be noted that $y_{i,t}$ is expressed in terms of "conceptual" goods produced in the model, and the units of measurements of $y_{i,t}^{PPP}$ are US dollars. Thus, we suppose that $y_{i,t} = \phi y_{i,t}^{PPP}$, where $\phi > 0$ is a parameter that transfers US dollars into the units of goods used in the model. Now, taking the logarithm of equation (13) we write an econometric model:

$$\log l_{i,t} y_{i,t}^{PPP} = \beta_0 + \beta_1 \log((\eta l_{i,t})^{-1} - 1) + \log(1 + n_i) + \log((1 + \rho)/\psi_i + 1) + \varepsilon_{i,t}, \tag{14}$$

where $\varepsilon_{i,t}$ is a residual term, and

$$\beta_0 = \frac{\sigma}{\sigma - 1} \log \frac{1 - \gamma}{\gamma} - \log \phi, \tag{15}$$

$$\beta_1 = \frac{\sigma}{\sigma - 1}. \tag{16}$$

We are interested in β_1 estimation, since it will give us information about σ. Coefficients ϕ and γ cannot be estimated separately.

The model includes fertility rate $1 + n$. Alternatively, the econometric model can include a term $\psi/(1 + n)$, which has an interpretation of the old-age dependency ratio:

$$\log l_{i,t} y_{i,t}^{PPP} = \beta_0 + \beta_1 \log((\eta l_{i,t})^{-1} - 1) - \log \frac{\psi_i}{1 + n_i} + \log(1 + \rho + \psi_i) + \epsilon_{i,t}. \tag{17}$$

4.1 Data

The data used in this Section is the same as in section 2.1 with a few additions described bellow.

Since in this data population growth is yearly, and in our model the period is around 35 years, in order to calculate the growth of labor force $1 + n$ we have taken the data for labor force available in the CIA world factbook 2010 and 1995. We denote observations for labor force taken for a country i from the factbook 2010 as L^{CIAnew_i} and L_i^{CIAold}, whereas the years corresponding to the observations – as $YEARnew_i$ and $YEARold_i$.[7] If observations for 1995 were missing, we have chosen the oldest observations from CIA world factbooks 1996, 1997, 1998, etc. Then, the growth of labor force is calculated as

$$1 + n_i = (L_i^{CIAnew}/L_i^{CIAold})^{\frac{35}{YEARnew_i - YEARold_i}} \tag{18}$$

Finally, a few observations, namely for Micronesia, Nigeria, San Marino, Siera Leone, Qatar, and United Arab Emirates, have been eliminated as outliers. It should also be mentioned that since in 1995 Serbia and Montenegro made up one one country and in 2010 observations for them are given separately, we have summed observations for Serbia and Montenegro for 2010 and assigned their joint growth in labor force to Serbia. Montenegro is not analyzed, because other observations, such as labor shares, are missing.

The probabilities of survival before the second period can be calculated from the World Health Organization data. It provides age-specific probabilities of dying, grouped in 5-year

[7] For a number of countries older observations from CIA world factbooks have been used, if the latest data was not available.

intervals. We discard the probabilities of dying before the age of 20 because individuals enter our model at the working age. Then we calculate the probabilities of survival until the age of 65 in a straightforward way.

Table 3: Average dummy for ψ_i per region

Region	ψ(65 years)	ψ(pension age)
East Asia & Pacific	0.776208	0.853160
Europe & Central Asia	0.795986	0.842995
High income: OECD	0.895550	0.896655
Latin America & Caribbean	0.819024	0.857625
Middle East & North Africa	0.847628	0.893511
South Asia	0.766285	0.867328
Sub-Saharan Africa	0.599737	0.667617

Another way to calculate the probabilities of survival before the next period is to suppose that the second period of life starts not at 65 years but at the retirement age. In this case, we take an arithmetic average of male and female retirement ages. If this retirement age ends with 5 or 0 (55, 60, 65, 70 years) – the calculation of survival probabilities is straightforward – multiplication of survival probabilities in each period. However, in a number of countries pension age is not expressed in such a round number. Consider, for example, Hungary, where the pension age is 62 years. In this case, we multiply 5-year probabilities of survival in the interval 20–60. Then, the probability of survival in the period of 60-61 years is calculated as $1-(1-\tilde{\psi})^{2/5}$, where $\tilde{\psi}$ denotes probability of survival in the period of 60–64 years. In terms of other countries, calculations have been made in an analogical way. These probabilities are summarized in Table 3. We consider not only probabilities of dying before retirement, but also before the age of 65, since often pension age does not require agents to retire and they can work longer, if they want.

4.2 Estimation and calibration

The parameters of econometric models (14) and (17) are β_0, β_1, ρ and η. We cannot estimate all of them. Karabarbounis and Neiman (2014) estimated that labor shares decline at the speed of approximately 5 per cent in 35 years. We use their set of data for labor shares; therefore, we set $\eta = 0.95$. Another parameter that cannot be estimated is ρ. Often it is assumed that the annual discount rate is in the range 1–8 per cent (Börsch-Supan et al., 2006; Schotman, 1997). Giglio et al. (2015) estimated 100 year discount rates smaller than 2.6 per cent, implying that 35-years discount rates are smaller than 1 per cent. Brent (1993) argued that even negative discount rates are possible. Therefore, we estimate the model using a number of different discount rates. Namely, we have chosen annual discount rates of –2%, –1%, 0%, 1%, 2%, 4%, 6%, 10%, and transferred them into the 35-year period using formula $1+\rho = (1+\rho_a)^{35}$, where ρ_a stands for the annual discount rate.

Table 4: $\hat{\beta}_1$ for different ρ_a.

	Equation (14)		Equation (17)	
ρ_a	ψ(65 years)	ψ(pension age)	ψ(65 years)	ψ(pension age)
–2%	–2.0400 (0.1395)	–2.0383 (0.1378)	–2.7491 (0.1609)	–2.7802 (0.1645)
–1%	–2.0490 (0.1403)	–2.0466 (0.1383)	–2.7484 (0.1622)	–2.7861 (0.1650)
0%	–2.0592 (0.1413)	–2.0548 (0.1389)	–2.7564 (0.1632)	–2.7923 (0.1655)
1%	–2.0710 (0.1423)	–2.0624 (0.1394)	–2.7613 (0.1643)	–2.7805 (0.1663)
2%	–2.0816 (0.1432)	–2.0692 (0.1399)	–2.7690 (0.1652)	–2.7889 (0.1667)
4%	–2.0978 (0.1446)	–2.0836 (0.1409)	–2.7848 (0.1666)	–2.7917 (0.1675)
6%	–2.1079 (0.1455)	–2.0902 (0.1414)	–2.7880 (0.1675)	–2.7886 (0.1681)
10%	–2.1158 (0.1463)	–2.0960 (0.1418)	–2.7919 (0.1684)	–2.7923 (0.1686)
max R^2	0.50716 (ρ_a =0%)	0.49861 (ρ_a =2%)	0.55391 (ρ_a =2%)	0.5555 (ρ_a =10%)
min R^2	0.50209 (ρ_a =10%)	0.49506 (ρ_a =6%)	0.54991 (ρ_a =–2%)	0.54556 (ρ_a =–2%)
$\Rightarrow \hat{\sigma}$	0.6711–0.6791	0.6709–0.6770	0.7333–0.7442	0.7354–0.7363

In the first two columns of Table 4 we present estimates of β_1 in regression (14). Standard deviations are presented in parenthesis. All estimates are statistically significant at the 1 per cent significance level. In the last row of the Table, the estimates of σ, which are calculated using equation (16) are presented. Probabilities of survival before the age of 65, as well as the retirement age give almost identical results for σ– around 0.67–0.68.

In the third and fourth columns of Table 4, the model is estimated using regression specification (17). The estimates are very similar for both probabilities of survival, resulting in the elasticity of substitution approximately equal to 0.73–0.74.

R^2 in the first column is maximal when the annual discount rate is equal to 0%. In the second and third columns, the maximum is achieved when $\rho_a = 2\%$. In the fourth column, R^2 is maximal for a rather unusual value of $\rho_a = 10\%$. However, in all four cases, ρ_a has a very limited impact on R^2, which indicates that the discount rate cannot be estimated precisely. Fortunately, the choice of ρ_a does not affect the estimate of σ much.

The importance of the use of $\sigma < 1$ in research on the dynamics of labor shares was emphasized by Acemoglu (2003). Chirinko (2008) summarized a number of studies related to the estimation of the elasticity of substitution between labor and capital and concluded that σ lies in the interval [0.4, 0.6]. Our estimation results suggest that it is slightly higher. Nonetheless, these estimates are much lower than those received by Karabarbounis and Neiman (2014), whose data on labor shares we use. They received a value of about 1.25. We believe that this difference appeared due to the inclusion of demographic factors in the estimation of σ.

Our empirical estimates of the elasticity of substitution suffer from the fact that only a cross-sectional data has been used. However, the results found in literature indicate that estimates of σ based on time series and panel data are also usually below 1. Apart from literature discussed above, we should also mention the papers of Panik (1976), Caballero et al. (1995) and Klump et al. (2006), whose estimates were based on the US and EU panel data and are very similar to our results (around 0.7–0.75).

5 Robustness

Our result depends on number of assumptions, such as a correct specification of the aggregate production function and time-invariant elasticities of substitution between capital and labor. As a robustness check we have also considered a VES production function (Revankar 1971), leading to similar results. Regarding the assumption about the time-invariant elasticity of substitution, in fact, our results are valid as long as it is smaller than 1. An elasticity of substitution greater than unity would lead to the opposite conclusion. However, the case of $\sigma < 1$ is more consistent with empirical literature. Our results are rather robust to the assumption of log-linear utility function, since the result that population ageing leads to capital deepening holds also with more general utility functions.

Both empirical and theoretical parts of the paper suffer from the fact that, in reality, demographic factors, such as fertility and longevity, are endogenous (see, for example, empirical studies of Caudill and Mixon 1995, Rodgers 2002, Schnabel and Eilers 2009,). The same in-built two-way causality can also be a problem when including oil production in the model. However, all macroeconomic and macroeconometric models suffer from endogeneity problems to some extent; therefore, we believe that it is impossible to solve this problem completely in the model. Another source of endogeneity in the model comes from the heterogeneity of country-specific factors. Indeed, country-specific factors may affect both labor shares and demographics, leading to a bias in estimates.

6 Conclusions

In this paper we have shown that population ageing leads to capital deepening, and the elasticity of substitution between labor and capital as production factors is smaller than unity. As a result, according to equation (10), population ageing leads to an increase in labor shares. Reality suggests the opposite: labor shares decline slowly but permanently. Thus, the decline of labor shares is determined by other factors discussed in the introduction, and population ageing reduces the speed of this decline.

Conflict of Interest

The author declares that he has not received any grant, honorarium or financial support (apart from the official salary at the Bank of Lithuania). The author declares that he has no conflict of interest.

Acknowledgments

I would like to thank Mihnea Constantinescu for his help in writing this paper. I would also like to thank Dmitrij Celov for discussion and anonymous reviewers for their useful comments. I am also grateful to Laura Stundžaitė, for her help with the language editing of this paper.

References

Acemoglu, D. (2003). Labour- and capital-augmenting technical change. *Journal of the European Economic Association 1*(1), 1–37.

Alfaro, L., S. Kalemli-Ozcan, and V. Volosovych (2008). Why doesn't capital flow from rich to poor countries? An empirical investigation. *Review of Economics and Statistics 90*(2), 347–368.

Azmat, G., A. Manning, and J. van Reenen (2012). Privatization and the decline of labour's share: International evidence from network industries. *Economica 79*(315), 470–492.

Batista, C. (2008). Why doesn't labour flow from poor to rich countries? micro evidence from the european integration experience. Economic Series Working Papers, No 402, University of Oxford Department of Economics.

Bentolila, S. and G. Saint-Paul (2003). Explaining movements in the labour share. *B. E. Journal of Macroeconomics(Contributions) 3*(1), 1–33.

Blanchard, O. J. (1997). The medium run. *Brookings Papers on Economic Activity 28*(2), 89–158.

Blanchard, O. J. and F. Giavazzi (2003). Macroeconomic effects of regulation and deregulation in goods and labor markets. *The Quarterly Journal of Economics 118*(3), 879–907.

Börsch-Supan, A., A. Ludwig, and J. Winter (2006). Ageing, pension reform, and capital flows: a multicountry simulation model. *Economica 73*(292), 625–658.

Brent, R. J. (1993). Country estimates of social discount rates based on changes in life expectancies. *Kyklos 46*, 399–409.

Caballero, R. J., E. Engel, and J. Haltiwanger (1995). Plant-level adjustment and aggregate investment dynamics. *Brookings Papers on Economic Activity 28*(2), 1–48.

Caudill, S. B. and F. G. J. Mixon (1995). Modeling household fertility decisions: Estimation and testing of censored regression models for count data. *Empirical Economics 20*(2), 183–196.

Chirinko, R. S. (2008). σ: The long and short of it. *Journal of Macroeconomics 30*(2), 671–686.

Chirinko, R. S., S. Fazzari, and A. Meyer (1999). How responsive is business capital formation to its user cost?: An exploration with micro data. *Journal of Public Economics 74*(1), 53–80.

Clark, P. K. (1993). Tax incentives and equipment investment. *Brookings Papers on Economic Activity 1*, 317–347.

Diwan, I. (1999). Labor shares and financial crises. Mimeo, The World Bank.

Dünhaupt, P. (2013). The effect of financialization on labor's share of income. Working paper No. 17/2013. Institute for International Political Economy. Berlin.

Eisner, R. and M. I. Nadiri (1968). Investment behavior and neo-classical theory. *The Review of Economics and Statistics 50*(3), 369–382.

Ellis, C. and S. Price (2004). UK business investment and the user cost of capital. *The Manchester School 72*(s1), 72–93.

Giglio, S., M. Maggiori, and J. Stroebel (2015). Very long-run discount rates. *The Quarterly Journal of Economics 130*(1), 1–53.

Golin, D. (2002). Getting income shares right. *Journal of Political Economy 110*(2), 458–474.

Guscina, A. (2006). Effects of globalization on labor's share in national income. IMF Working Paper WP/06/294.

Harrison, A. (2002). Has globalization eroded labor's share? Some cross-country evidence. Mimeo, University of California, Berkeley.

Jaumotte, F. and I. Tytell (2007). How has the globalization of labor affected the labor income share in advanced countries? IMF Working Paper WP/07/298.

Jayadev, A. (2007). Capital account openness and the labour share of income. *Cambridge Journal of Economics 31*(3), 423–443.

Karabarbounis, L. and B. Neiman (2014). The global decline of the labor share. *The Quarterly Journal of Economics 129*(1), 61–103.

Klump, R., P. McAdam, and A. Willman (2006). A supply-side diagnosis of growth in the euro area: Factor substitution, productivity and unemployment. Mimeo.

Klump, R., P. McAdam, and A. Willman (2007a). Factor substitution and factor augmenting technical progress in the U.S.: A normalized supply-side system approach. *The Review of Economics and Statistics 89*(1), 183–192.

Klump, R., P. McAdam, and A. Willman (2007b). The long-term sucCESs of the neoclassical growth model. *Oxford Review of Economic Policy 23*(1), 94–114.

Lucas, R. E. J. (1990). Why doesn't capital flow from rich to poor countries? *The American Economic Review 80*(2), 92–96.

Panik, M. (1976). Factor learning and biased factor efficiency growth in the United States, 1929-1966. *International Economic Review 17*(3), 733–739.

Revankar, N. S. (1971). A class of variable elasticity of substitution production functions. *Econometrica 39*(1), 61–71.

Rodgers, G. B. (2002). Income and inequality as determinants of mortality: an international cross-section analysis. *International Journal of Epidemiology 31*(3), 533–538.

Schmidt, T. and S. Vosen (2013). Demographic change and the labour share of income. *Journal of Population Economics 26*, 357–378.

Schnabel, S. K. and P. H. C. Eilers (2009). An analysis of life expectancy and economic production using expectile frontier zones. *Demographic Research 21*, 109–134.

Schotman, P. C. (1997). Small sample properties of the regression test of the expectations model of the term structure. *Economics Letters 57*(2), 129–134.

Suchanek, L. (2009). Labour shares and the role of capital and labour market imperfections. Bank of Canada Discussion Paper 2009-2.

Young, A. T. (2013). US elasticities of substitution and factor-augmentation at the industry level. *Macroeconomic Dynamics 17*(4), 861–897.

Appendix

Table 5: Correlation matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13
1 Labor share	1												
2 Life expectancy	0.597	1											
3 Fertility/woman	–0.565	–0.806	1										
4 65 years+, %	0.647	0.671	–0.708	1									
5 PAYG taxes	0.232	0.257	–0.382	0.421	1								
6 Pension age	0.477	0.559	–0.392	0.551	0.039	1							
7 GDP/capita PPP	0.507	0.668	–0.484	0.571	0.176	0.475	1						
8 Oil/GDP	–0.382	–0.283	0.171	–0.294	0.050	–0.433	–0.019	1					
9 FDI in-FDI out	–0.129	–0.111	0.041	–0.095	–0.001	–0.139	–0.464	0.021	1				
10 Net migration	0.099	0.262	–0.135	0.238	0.092	0.022	0.378	0.209	–0.294	1			
11 Median age	0.674	0.780	–0.842	0.948	0.441	0.513	0.659	–0.264	–0.103	0.294	1		
12 Births/1000	–0.617	–0.340	0.980	–0.809	–0.397	–0.458	–0.572	0.220	0.067	–0.199	–0.923	1	
13 Deaths/1000	0.075	–0.847	0.983	0.389	0.208	–0.035	–0.112	–0.012	0.024	–0.022	0.219	0.031	1