Government Debt Management and Inflation with Real and Nominal Bonds

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Introduction

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- Supply chain shortages due to Covid-19, rising energy prices due to war.
- COVID-19 is the largest fiscal shock since WWII.

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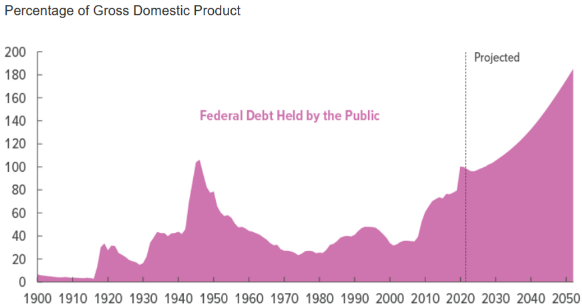

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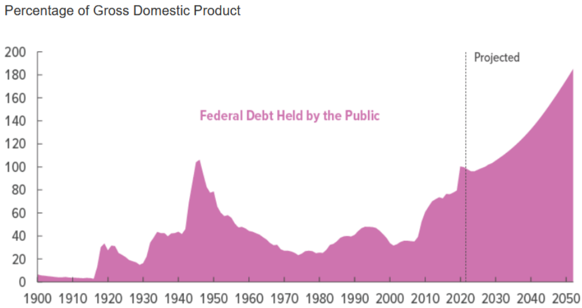

- *Debt is projected to rise in relation to GDP mainly because of increasing interest costs and growth in spending for Medicare and Social Security*, Congressional Budget Office, 2022
- Is this 'sustainable'?
- What about debt monetization?

Questions

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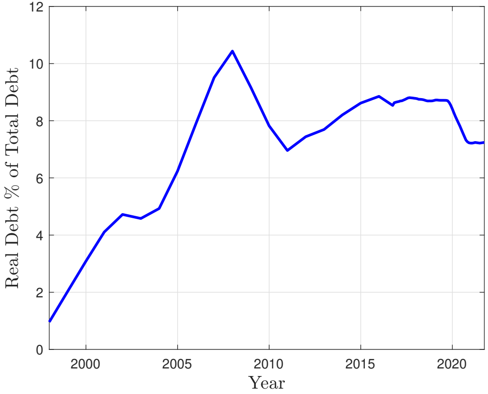

Source: fiscaldata.treasury.gov

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- We characterize
  1. the optimal policy under full commitment (i.e. the Ramsey equilibrium) and
  2. the optimal policy without commitment (i.e. the optimal time-consistent policy).

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- **Real debt** prices are higher and more stable, but such debt constitutes a real commitment ex-post.

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2. An active role for policy over the business cycle.
   - In periods of high inflation it is optimal to rebalance the portfolio to include more real debt.

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  - Internalizes the future government's behavior through the current nominal bond price, which reflects the elevated inflation expectations.
  - Reduces the borrowing costs ex-ante substituting nominal debt with real debt.
- Without commitment, the policies are quantitatively consistent with the US data.

Related Literature

- Optimal policy under Full Commitment.
  - With non contingent real debt: *Aiyagari et al., 2002; Angeletos, 2002; Buera and Nicolini, 2004; Faraglia et al., 2019; Bhandari et al., 2017*.
  - With non contingent nominal debt: *Chari and Kehoe, 1999; Siu, 2004; Schmitt-Grohe and Uribe, 2004; Lustig et al., 2008; Marcet et al., 2013; Leeper and Zhou, 2021*.
  - With non contingent real and nominal debt: *Barro, 2006*

- Optimal policy without Commitment.
  - Markov-Perfect Fiscal Policy: *Klein, Krusell, and Rıos-Rull, 2008; Debortoli and Nunes, 2013; Debortoli, Nunes, and Yared, 2017; Clymo and Lanteri 2020*.
  - With non contingent real and nominal debt: *Alvarez, Kehoe, and Neumeyer, 2004*

Model

Model: Household

- Representative household with utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \cdot U(c_t, l_t).$$

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$$P_t c_t + Q_t^N B_t^N + q_t^N b_t^N + p_t S_t = (1-\tau_t) P_t w_t A h_t + Q_t^{N-1} B_{t-1}^N + \Pi_{j=1}^N \pi_{t-j+1} q_t^{N-1} b_{t-1}^N + (p_t + d_t) S_t.$$

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- Optimality Conditions

$$Q_t^N = \mathbb{E}_t \left[\mathcal{M}_{t,t+N} \cdot \frac{1}{\Pi_{j=1}^N \pi_{t+j}} \right], \quad q_t^N = \mathbb{E}_t \left[\mathcal{M}_{t,t+N} \right],$$
$$U_{2,t} = U_{1,t} \cdot (1-\tau_t) A w_t.$$

Model: Firms

- An intermediate firm i (with production $Y_{i,t} = A \cdot h_{i,t}$) chooses prices and labor demand to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \mathcal{M}_{0,t} \cdot \left[\underbrace{P_{i,t} Y_{i,t} - P_t w_t h_{i,t} - P_t \Phi_t}_{\text{Dividend}} \right].$$

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- The intermediate goods demand is

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- The New Keynesian Phillips curve is

$$\frac{\nu - 1}{\nu} Y_t + \frac{Y_t}{\nu} \frac{w_t}{A} - \Phi'_t + \mathbb{E}_t [\mathcal{M}_{t,t+1} \cdot \Phi'_{t+1}] = 0.$$

Technology, Government and Central bank

- The government budget is

$$Q_t^{N-1}\frac{B_{t-1}^N}{\pi_t} + q_t^{N-1} b_{t-1}^N = \tau_t A h_t w_t - g_t + Q_t^N B_t^N + q_t^N b_t^N,$$

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- The central bank applies a Taylor rule

$$\left(\mathbb{E}_t\left[\mathcal{M}_{t,t+1}\cdot\frac{1}{\pi_{t+1}}\right]\right)^{-1} = i_t = \frac{1}{\beta}\pi\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}.$$

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- The resource constraint is

$$c_t + g_t + \Phi_t = A\cdot h_t.$$

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$$\overbrace{\frac{B_{t-1}^N}{\pi_t} \cdot \mathbb{E}_t \left[\mathcal{M}_{t,t+N-1} \cdot \frac{1}{\Pi_{j=1}^{N-1} \pi_{t+j}} \right]}^{\text{Buy-back of nominal bonds}} + \overbrace{b_{t-1}^N \cdot \mathbb{E}_t \left[\mathcal{M}_{t,t+N-1} \right]}^{\text{Buy-back of real bonds}} =$$

$$s_t + \underbrace{B_t^N \cdot \mathbb{E}_t \left[\mathcal{M}_{t,t+N} \cdot \frac{1}{\Pi_{j=1}^{N} \pi_{t+j}} \right]}_{\text{New issuance of nominal bonds}} + \underbrace{b_t^N \cdot \mathbb{E}_t \left[\mathcal{M}_{t,t+N} \right]}_{\text{New issuance of real bonds}} .$$

Optimal Policy under Full Commitment

Ramsey Equilibrium

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- the implementability constraint is satisfied,
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- the Taylor rule is satisfied.

Intuition

Outstanding liabilities at t...

$$\frac{B_{t-1}(s^{t-1})}{\pi_t(s^t)} + b_{t-1}(s^{t-1}) = \tilde{b}_t(s^t)$$

...are measurable wrt s^t!

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Can we exploit fluctuations in inflation to complete the market with real and nominal bonds with the same maturity?

- If we have as many bonds with non perfectly correlated returns as realization of the exogenous state, then yes.

Mechanism: Two-Periods Model

- Time is $t = 0, 1$.
- $u(c) = c$ and $v(h) = h^2/4$.
- Two realizations of exogenous shocks: (π^L, g^l) and (π^H, g^H).
- Initial conditions: B_0, b_0, g_0, π_0.

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Optimal Nominal and Real Bonds choices:

$$
\begin{aligned}
\mu_0 \cdot \mathbb{E}_0\left[\frac{1}{\pi_1}\right] &= \mathbb{E}_0\left[\mu_1 \cdot \frac{1}{\pi_1}\right], \\
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Optimal Fiscal Policy implies:

$$2(1 - h_t) \cdot \mu_t = 2 - h_t.$$

Mechanism: Two-Periods Model

(Debt Management, Labor and Tax Smoothing). Given initial conditions B_0, b_0, g_0, π_0, optimal nominal and real debt management and tax management are such that smoothing of taxes and leisure is achieve across states

$$l_1^H = l_1^L \iff \tau_1^H = \tau_1^L, \tag{1}$$

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where l_1^L and l_1^H denote leisure at time 1 in the low and high state. respectively. Moreover, smoothing of taxes and leisure is achieved across time

$$l_1^x = l_0 \iff \tau_1^x = \tau_1^0, \tag{2}$$

where $x \in \{L, H\}$.

Mechanism: Two-Periods Model

(Optimal Nominal and Real Debt Management). Given the initial conditions, optimal nominal debt management is such that

$$B_1^* = \frac{g_1^H - g_1^L}{\pi_1^H - \pi_1^L} \cdot \pi_1^L \pi_1^H,$$

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Optimal real debt management is such that

$$b_1^* = \frac{1}{1+\beta}\left[\frac{B_0}{\pi_0} + b_0 - \left(\frac{1}{\pi_0} + \beta \mathbb{E}_0\left[\frac{1}{\pi_1}\right]\right) B_1^*\right],$$

satisfy the inter-temporal smoothing condition (2).

Quantitative Analysis: Recursive Solution

Ramsey Problem with incomplete markets and bonds with $N = 5$...

$$\mathcal{I}_t = \{g_t, \{B^N_{t-k}\}^N_{k=1}, \{b^N_{t-k}\}^N_{k=1}, \{\mu_{t-k}\}^N_{k=1}, \{\lambda^T_{t-k}\}^N_{k=1}, \{\lambda^\pi_{t-k}\}^N_{k=1}\}$$

...requires to solve for 10 policy functions of 26 state variables.

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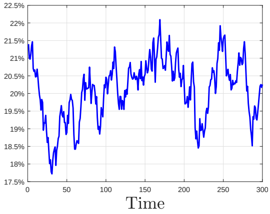
- We use a stochastic simulation approach...
  - den Haan and Marcet (1990), Faraglia et al. (2019), Judd et al. (2011).
- ...combined with machine learning.
  - Duarte (2018), Azimovich et al.(2019), Maliar et al.(2021).

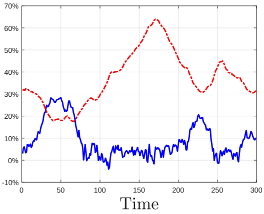
Algorithm

An Extract of the Equilibrium Path

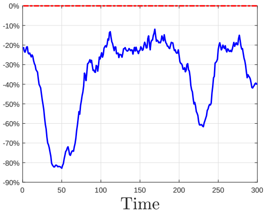
(a) Gvt. Exp. g_t



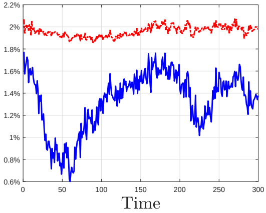
(b) Nominal Bonds B_t^N



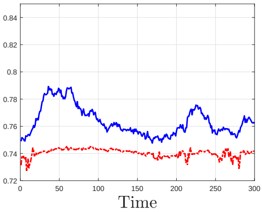
(c) Real Bonds b_t^N



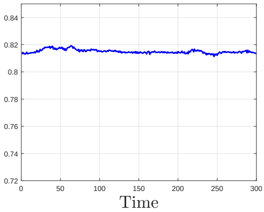
(d) Inflation π_t, %



(e) Nominal Price Q_t^N



(f) Real Price q_t^N



- Blue - Baseline model. Red - Model without TIPS.

Portfolio

Extensions

Full Commitment results robust to multiple model extensions:

- Maturity,
  - Spread inflationary distortion over longer periods, even more leveraged positions. Link
- Slope of the Philips Curve,
  - Matters more in the model with only nominal bonds. Link
- Monetary Policy Tightness,
  - Tighter monetary policy implies less volatile inflation and, therefore, more leveraged bond portfolio. Link
- Alternative Shocks.
  - Correlation between the net present values and inflation is what matters. Link

Optimal Policy without Commitment

Symmetric Markov-Perfect Equilibrium

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- The government in power at t seeks (B', b', τ, g) to strategically best respond to the future government.
- We focus on the Symmetric Markov-Perfect Equilibrium of the associated infinite-horizon game.

Mechanism: Two-Period Model

- $t = 0, 1$.
- Gov. $t = 1$ chooses π_1.
- Gov. $t = 0$ chooses B_1 and b_1.
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$$-\Phi_\pi(\pi_1) + \mu_1 \frac{B_1}{\pi_1^2} = 0.$$

Planner at $t = 0$ internalizes the effect of higher B_1 on current prices through coupled Generalized Euler Equations:

$$\mu_0 \left(Q + \frac{\partial Q}{\partial B_1} + \frac{\partial q}{\partial B_1} b_1 \right) = \beta \mathbb{E}_0 \left[\frac{\mu_1}{\pi_1} \right],$$

$$\mu_0 \left(q + \frac{\partial Q}{\partial b_1} B_1 + \frac{\partial q}{\partial b_1} \right) = \beta \mathbb{E}_0 [\mu_1].$$

Quantitative Results

- Each gov. chooses (B', b', τ, g).
- Preference shocks calibrated to have equilibrium stochastic properties of g similar as in FC.
- We solve for the symmetric MPE using an algorithm similar in spirit to Clymo and Lanteri, 2020.

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Description	Moments	NC	Data
Real Portfolio Weight	$\mathbb{E}[b/(b+B)]$	**0.08**	**0.07**
Nominal Portfolio Weight	$\mathbb{E}[B/(b+B)]$	**0.92**	**0.93**
Corr. Tax and GDP	$\rho(\tau, Y)$	0.89	0.17
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.92	0.18
Corr. Inflation and Real	$\rho(\pi, b)$	0.77	0.25
Corr. Inflation and Nominal	$\rho(\pi, B)$	-0.57	-0.54

Conclusions

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  - Leveraged portfolio of positive nominal and negative real debt.
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  - Policy quantitatively consistent with the US data.

Thank You!

Parameter Values

Parameter	Value	Description
β	0.96	Discount factor
γ	2	Relative risk aversion
η	1.8	Leisure utility parameter
A	1.0	Technology level
χ	4.3276	Labor utility parameter
$-\frac{1}{\nu}$	-10	Price elasticity of demand
φ	4.375	Rotemberg adj. cost, Sbordone, 2002
ϕ_π	1.2	Taylor rule response to inflation
Π	1.02	SS inflation, Fed target
ρ, σ_ϵ	0.977, 0.0161	g_t Persistence and std, BEA
$\mu(1-\rho)$	0.2	Ratio of gvt. expenditure to GDP, BEA
N	5	Maturity of gvt. debt
ϕ_1, ϕ_2	0.00001, 5.7143×10^{-7}	Adjustment cost

⏩ Quantitative Analysis

Bonds Optimality

The first order condition with respect to nominal bonds is

$$\mu_t = \left[\mathbb{E}_t[U_{1,t+N}/\Pi_{j=1}^N \pi_{t+j}]\right]^{-1} \left[\mathbb{E}_t[\mu_{t+1} U_{1,t+N}/\Pi_{j=1}^N \pi_{t+j}] + \frac{\xi_{U,t}}{\beta^N} - \frac{\xi_{L,t}}{\beta^N}\right]$$

where $\xi_{U,t}$ and $\xi_{L,t}$ are the Lagrange multipliers on the upper and lower bounds, respectively.

The first order condition with respect to real bonds is

$$\mu_t = [\mathbb{E}_t [U_{1,t+N}]]^{-1} \left[\mathbb{E}_t[\mu_{t+1} U_{1,t+N}] + \frac{\xi^T_{U,t}}{\beta^N} - \frac{\xi^T_{L,t}}{\beta^N}\right]$$

Algorithm

Algorithm: Part II

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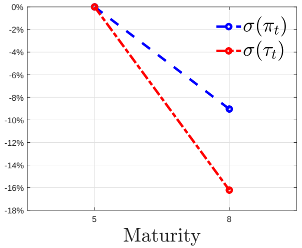
This reduces the complexity of the algorithm significantly since the maximum number of combinations are $\sum_{k=2}^{N} C_{N,k}$. We gain in speed and scalability.

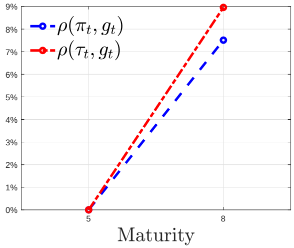
Algorithm

Extensions: ↑ Maturity → ↓ Inflation Volatility

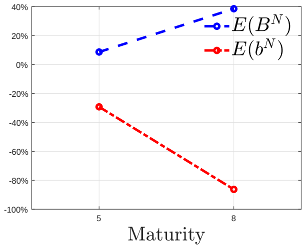
(a) Policy Volatility



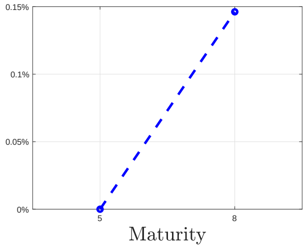
(b) Policy Correlation



(c) Nominal and Real Debt

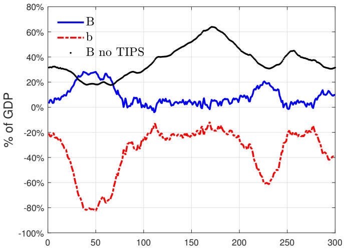


(d) Welfare Consumption Equivalent

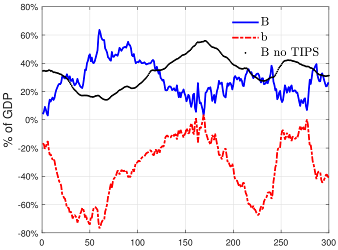


Extensions: Slope of the Philips Curve

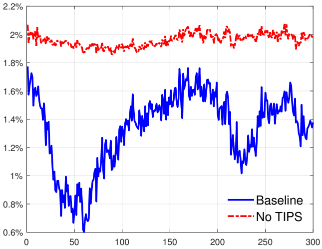
(a) Bonds



(b) Bonds



(c) Inflation π_t [%]



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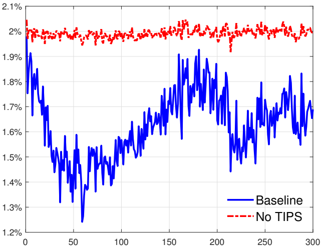

Extensions: Alternative Shocks

Table: Comparison with a model with TFP shocks

Description	Moments	No TIPS *g shocks*	Baseline *g shocks*	Baseline *TFP shocks*
Avg. Real to GDP	$\mathbb{E}(b^N/Y)$	-	**-0.28**	**-0.37**
Avg. Nominal to GDP	$\mathbb{E}(B^N/Y)$	0.40	**0.24**	**0.40**
Corr. Tax and GDP	$\rho(\tau, Y)$	0.54	0.3	-0.84
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.39	0.39	-0.66
Corr. Tax and Inflation	$\rho(\tau, \pi)$	0.84	0.96	0.81
Corr. Inflation and Real	$\rho(\pi, b^N)$	-	**0.93**	**0.45**
Corr. Inflation and Nominal	$\rho(\pi, B^N)$	0.68	**-0.69**	**-0.22**
Corr. Real and Nominal	$\rho(b^N, B^N)$	-	-0.84	-0.70

Extensions: Monetary Policy Tightness

	$\rho(b_t^N, B_t^N)$	$\rho(B_t^N - b_t^N, g_t)$	$\mathbb{E}(B_t^N / Y_t)$	$\mathbb{E}(b_t^N / Y_t)$	$\sigma(\pi_t)$
$\phi_\pi = 1.2$	-0.8545	-0.8046	0.0912	-0.3054	0.0040
$\phi_\pi = 1.25$	-0.9171	-0.8332	0.4972	-0.2780	0.0032

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