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Overconfidence and Correlated Information Structures[*]

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ABSTRACT

This paper analyzes a model of multiple overconfident traders submitting market orders where traders' private information is subject to correlated errors, as well as its extension to endogenous information. We consider two standard types of overconfidence: overconfidence in own signals and underconfidence in others' signals. The analyses on the effects of overconfidence on traders' behavior and the equilibrium price suggest that these effects are richer than our typical understanding of overconfidence focusing on its positive effect on trading volume as follows: First, trading volume may increase or decrease with overconfidence depending on its type. Second, these different types of overconfidence may differ radically on the patterns of trading volume and price informativeness with respect to the number of traders. Third, overconfidence can cause equilibrium multiplicity in information acquisition.

Keywords: Overconfidence; Disagreement; Strategic trading; Information aggregation; Efficient market hypothesis

JEL Classification: G11, G14, G4.

1 Introduction

A lot of evidence supports the argument that traders in financial markets tend to be overconfident about their valuation and trading skills (e.g., Barber and Odean, 2001; Statman, Thorley and Vorkink, 2006), and even financial professionals are not the exception (Gloede and Menkhoff, 2014). This naturally generates a disagreement over the asset value, thereby motivating speculative trade. Such argument is consistent with the fact that investors who trade the most appear to lose from trade (e.g., Odean, 1999). This has been proposed as an explanation for the fact that the world-wide volume of trades in financial markets is too large to be justified only by rational investors' risk-hedging motive.[1] A line of analytic studies develop various equilibrium results regarding the effects of overconfidence on trading volume and price informativeness, largely supporting these observations (Benos, 1998; Garcia, Sangiorgi and Urosevic, 2007; Ko and Huang, 2007; Kyle and Wang, 1997; Odean, 1998).

Building on these well-established results, this study analyzes further implications of overconfidence in financial markets by deviating from the literature in the following ways. First, the presence of correlated signal errors is considered for the analyses of overconfident traders, in contrast to the standard assumption of independent signal errors used by previous studies. For instance, speculations about business cycles and policy directions often involve a correlation in errors due to the presence of common sources of information. While recognizing the correlation in errors, traders in financial markets still appear to be overconfident in their own overall competence in these speculations. This is the situation considered in this study, where overconfident traders form their expectations about others' private information based on their own private information. Second, this study examines how overconfidence changes the properties of price informativeness, which reflects the quality of information from prices, with respect to market size. Based on the long-held idea that financial markets convey information via prices, the notion of price informativeness can provide normative implications regarding changes in market size, which may arise from various trends of lower entry barrier in financial markets such as globalization and financial technology.

In this regard, this study considers a market with overconfident strategic traders who can buy or sell a risky asset based on private information involving correlated errors. I consider a class of symmetric information structures regarding strategic traders' private signals: While it is common knowledge that their signal errors are correlated each other's with a common correlation coefficient, these traders are overconfident in the overall precision of their private signals. This study considers two types of overconfidence: Under the first type of overconfidence ("overconfidence in own signals"), each strategic trader believes that his own private signal is more precise than its true precision, whereas, under the second type of overconfidence ("underconfidence in others' signals"), each strategic trader believes that others' private signals are noisier than their true precision. Trade occurs via market orders (e.g., Kyle, 1985): Strategic traders simultaneously submit their market orders to market makers who observe the total demand and then set the price. This trading mechanism allows for tractable analyses with the complex structures of information held by strategic traders.

The main focus of the analyses is the effects of the aforementioned types of overconfidence on trading volume and price informativeness. After providing a characterization of equilibrium for both types of confidence as well as that for the benchmark case without overconfidence (Proposition 1), we proceed to examine how these types of overconfidence affect trading volume, price informativeness and their properties with respect to the number of strategic traders.

The first set of main results answers the question of how the two types of overconfidence affects trading volume in the presence of correlated signal errors. Proposition 2 offers a clear answer to this question: On the one hand, overconfidence in own signals leads to an increase in trading volume, and this is even more pronounced with correlation in signal errors. On the other hand, "underconfidence" in others' signals causes a decrease in trading volume. This stems from the fact that each strategic trader incorrectly infers others' signals from his own private signal, thereby forming his conditional

[1] See Daniel and Hirshleifer (2015) for this fact in detail, as well as a comprehensive overview of the literature. Also, note that overconfidence is not the only explanation as there are other forms of disagreement (e.g., Eyster, Rabin and Vayanos, 2019).

expectation about the price. In contrast to the standard effect of overconfidence via each trader's expectation about the asset value, which unambiguously increases trading volume, this separate effect of overconfidence may decrease each trader's perceived trading opportunity by making him perceive the price to covary more with his own signal in the case of underconfidence in others' signals. These suggest that the relationship between overconfidence and trading volume qualitatively depends on the type of overconfidence and information structures. Overall, overconfidence does not necessarily increase trading volume in the presence of noise traders, in contrast to the intuitive idea that disagreement leads to more trade in general.[2] On the empirical side, these results can explain the mixed empirical and experimental findings regarding the relationship between overconfidence and trading volume (e.g., Biais, Hilton, Mazurier and Pouget, 2005; Fellner-Rohling and Krugel, 2014; Glaser and Weber, 2007; Merkle, 2017).

The second set of main results concerns how trading volume and price informativeness change with market size under the two types of overconfidence. In the benchmark case without overconfidence, (total) trading volume is inverse proportional to each strategic trader's market power. As a result, it increases toward infinity as the number of strategic traders increases so that each trader's market power becomes small. At the same time, price informativeness increases so that the price fully reveals all available information in large markets, given that it is equivalent to the relative proportion of strategic traders' trades in the market. We show that these results do not necessarily persist under the two types of overconfidence due to the interaction effects between these types of overconfidence and the number of strategic traders. In particular, in the case of overconfidence in own signals, trading volume increases with the number of strategic traders at a faster rate, going to infinity even as the number of strategic traders reaches a finite threshold. This causes the price to fully aggregate dispersed information in the market. In contrast, in the case of underconfidence in others' signals, trading volume does never approach infinity and may even decrease with the number of strategic traders. Accordingly, the price cannot fully aggregate dispersed information in large markets and price informativeness may even decrease with the number of strategic traders. Overall, the two types of overconfidence lead to radically different properties of price informativeness with respect to the number of strategic traders due to their opposing effects on trading volume through each trader's inference about others' signals.

Finally, this study analyzes an extension of the basic model where strategic traders simultaneously choose whether to costly acquire their signals before the trading stage identical to the basic model. The analyses focus on overconfidence in own signals, leading to a U-shaped relationship between the number of informed traders and the (perceived) expected payoff of becoming an informed trader. This is distinguished from the benchmark case featuring strategic substitutability of costly information acquisition in line with Grossman and Stiglitz (1980). The upward part of the U-shaped relationship leads to two possible equilibria, one of which features the property that strategic traders' private information can be fully aggregated by the price. This stands in contrast to the intuitive argument, which would hold without overconfidence, that the endogeneity of information undermines information aggregation by the price (e.g., Grossman and Stiglitz, 1980).

To summarize, the main contribution of the paper is to show that correlation in signal errors implies that two different types of overconfidence provide radically different implications regarding trading volume, price informativeness, and the incentive to acquire information. In particular, underconfidence in others' signals leads to lower trading volume and price informativeness, whereas overconfidence in own signals leads to higher trading volume and price informativeness. Such difference leads to radically different predictions regarding the relationship between price informativeness and market size as well as the incentive to acquire information. These implications stem from the formation of higher-order belief about each other's behavior (i.e., each trader's inference about others' signals), which can be represented by the covariances representing the joint distribution of traders' signals perceived by these traders, and generally highlight the interplay between overconfidence and information structures in financial markets.

In the next section, we discuss related literature, focusing on the theory side. Section 3 introduces

[2]In the absence of noise traders, no-trade theorem ensures that this argument holds in a wide variety of information structures (e.g., Milgrom and Stokey, 1982).

the basic model. Section 4 and 5 present the analyses of the basic model and its extension to endogenous information, respectively. Section 6 concludes the paper with discussions on various degrees of overconfidence observed empirically and alternative trading mechanisms.

2 Related literature

The first line of literature related to this study is the theoretical literature on overconfidence and other related biases in financial markets. Several existing studies analyze how overconfidence affects trading volume and price informativeness in a variety of trading mechanisms, as well as whether and how these effects of overconfidence can explain the observed pattern of these variables (e.g., Benos, 1998; Kyle and Wang, 1997; Odean, 1998). Others address whether these conclusions continue to hold with endogenous information acquisition (Garcia, Sangiorgi and Urosevic, 2007; Ko and Huang, 2007). Overall, these studies confirm that the positive effect of overconfidence on trading volume is rather robust. More recently, Eyster, Rabin and Vayanos (2019) consider "cursedness", which refers to traders not fully appreciating what prices convey about others' price information, and compare it with other biases including two types of overconfidence considered in this study.[3] As in this study, they analyze whether these biases possibly generate large trading volume as the number of traders grows large, and show that the answer is positive only on cursedness. Despite the differences in many aspects of the model, this study is largely complementrary to their main results, as explained in Subsection 4.2 in detail. While this paper borrows much of its analytic framework from the literature, two takeaway points from the literature are (i) consideration of correlated signal errors and (ii) the analyses on the relationship between price informativeness and market size. The current paper is the first one to analyze these points together, showing that they cause a richer set of possible patterns of trading volume and price informativeness than what would occur without either of them.

Another line of relevant literature is the literature on price informativeness and information aggregation in financial markets. In the literature, there is a disagreement about whether price informativeness increases as markets grow large, as well as whether it converges towards the level at which the price aggregates all traders' private information available in the market. On the one hand, many previous studies ask whether how price informativeness changes with market size when traders' private information is exogenously given (e.g., Rostek and Weretka, 2012; Rostek and Weretka, 2015). Among many others, Lambert, Ostrovsky and Panov (2018) generalizes Kyle's (1985) model to consider correlations among relevant variables, showing that information aggregation occurs in large markets under mild conditions. On the other hand, if such information is costly and endogenous, a well-known argument makes it difficult to interpret the price as information aggregators (e.g., Grossman and Stiglitz, 1980) in that more information in the market causes the price to be closer to the true value of the asset, thereby lowering incentives for traders to acquire further information. Compared to these previous studies without overconfidence, this paper considers both models with and without information acquisition and shows that underconfidence in others' signals can lead to a failure of information aggregation even with exogenous information, whereas overconfidence in own signals raises the possibility that information aggregation occurs with endogenous information.

This study is also related to the literature on strategic complementarities in trading activities and information acquisition. They have been a subject of long-standing interest in the literature, raising the possibility of multiple equilibria, as in this study's extended model. In the context of linear-quadratic global games, such complementarities in actions yield a number of implications regarding the externalities across players and the social value of public information (e.g., Colombo, Femminis and Pavan, 2014; Hellwig and Veldkamp, 2009; Myatt and Wallace, 2012). In the context of financial markets with asymmetric information, various sources of complementarities and equilibrium multiplicity have been identified in the literature (e.g., Dow, Goldstein and Guembel, 2017; Ganguli and Yang, 2009; Goldstein and Yang, 2015; Kawakami, 2016; Mondria, Vives and Yang, 2021; Rahi and Zigrand,

[3] In their terminology, "overconfidence" corresponds to overconfidence in own signals, and "dismissiveness" refers to a set of different types of biases including underconfidence in others' signals.

2018), generally explaining the extreme amplification of shocks to asset prices caused by small changes to fundamentals. This study offers a different mechanism whereby complementarities in information acquisition occur, possibly explaining fragility in financial markets with overconfidence combined with correlated information structures.

3 Basic model

A security is traded in a market whose value θ is not initially known to market participants. There are N strategic traders, noise traders, and competitive and risk-neutral market makers in the market.

The trading game proceeds as follows: Before trading, the asset value θ is drawn but it is not known to anyone. Instead, each strategic trader i observes a private signal s_i about the asset value. At the first stage, strategic and noise traders submit market orders to the market. While noise traders submit a random order ω, each strategic trader decides his market order based on his own belief about the joint distribution of all variables, which include the asset value θ, noise trade ω, his own signal s_i, and other strategic traders' signals $(s_j)_{j\in\{1,\cdots,N\}\setminus\{i\}}$. At the second stage, competitive market makers observe the total demand $X=\sum_{i=1}^{N} x_i+\omega$ and subsequently set a price P. At the final stage, the true value θ of the security is realized, and accordingly, each strategic trader i obtains profit $\pi_i = x_i(\theta - P)$.

As is common in the literature following Kyle (1985), we impose a joint normality on the true joint distribution of all variables. In particular, we assume that the asset value θ follows $\theta \sim N(\theta_0, \sigma_0^2)$, and noise trade ω follows $N(0, \sigma_\omega^2)$. They are independent with each other and of all other variables in the model. We also assume that strategic trader i's private signal s_i is the sum of the true asset value θ and a random error ϵ_i which may be correlated with those of other strategic traders. Assuming symmetry for the sake of simplicity, it is given by $s_i = \theta + \epsilon_i$, where $Var(\epsilon_i) = \sigma_\epsilon^2$ and $Corr(\epsilon_i, \epsilon_j) = \rho \in [0, 1]$ for every $j \in \{1, \cdots, N\}\setminus\{i\}$. Note that the correlation coefficient ρ is nonnegative and symmetric across strategic traders. Formally, the true joint distribution of the asset value θ, noise trade ω, and strategic traders' signal errors $(\epsilon_i)_{i=1}^N$ is described by the following mean vector and variance-covariance matrix:

$$E\begin{bmatrix}\theta\\ \omega\\ \epsilon_1\\ \epsilon_2\\ \vdots\\ \epsilon_N\end{bmatrix}=\begin{bmatrix}\theta_0\\ 0\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}\text{ and } Var\begin{bmatrix}\theta\\ \omega\\ \epsilon_1\\ \epsilon_2\\ \vdots\\ \epsilon_N\end{bmatrix}=\begin{bmatrix}\sigma_0^2 & 0 & 0 & 0 & \cdots & 0\\ 0 & \sigma_\omega^2 & 0 & 0 & \cdots & 0\\ 0 & 0 & \sigma_\epsilon^2 & \rho\sigma_\epsilon^2 & \cdots & \rho\sigma_\epsilon^2\\ 0 & 0 & \rho\sigma_\epsilon^2 & \sigma_\epsilon^2 & \cdots & \rho\sigma_\epsilon^2\\ \vdots & \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & \rho\sigma_\epsilon^2 & \rho\sigma_\epsilon^2 & \cdots & \sigma_\epsilon^2\end{bmatrix}.$$

Remark 1. Strategic traders' signal values $(s_i)_{i=1}^N$ could replace their signal errors $(\epsilon_i)_{i=1}^N$ only with a slight modification of the mean vector and variance-covariance matrix above. In fact, this would be interpreted more intuitively. However, it would unnecessarily complicate the formal presentation of variance-covariance matrix due to the correlation between the asset value θ and the signal values $(s_i)_{i=1}^N$.

While strategic traders hold the true *prior* belief about the joint distribution of the asset value θ and noise trade ω, they may have different beliefs about the joint distribution of their signal errors, which naturally lead them to form differerent *posterior* expectations about the asset value θ and the equilibrium price P. Also, they "agree to disagree" in the sense that these different beliefs about the joint distribution of their signal errors are common knowledge.[4] Specifically, we consider three cases as follows:

1. Benchmark [common knowledge of correct beliefs]: It is common knowledge that all strategic traders hold the true belief about the joint distribution of all variables as described by the above mean vector and variance-covariance matrix.

[4] In fact, a deep theoretical argument generally rules out the possibility that there is no common knowledge at all among rational agents (Aumann, 1976).

2. κ-overconfidence [overconfidence in own signals]: Each strategic trader i perceives the variance of his own signal s_i to be smaller than the true variance by factor $\frac{1}{\kappa}$, i.e., $\frac{\sigma_\epsilon^2}{\kappa}$, where $\kappa > 1$ represents the extent to which each trader overestimates his own signal's precision. That is, as κ increases toward infinity, the extent of such κ-overconfidence becomes very strong. These strategic traders are unbiased about other traders' signal precision in that they perceive other traders' signal variances as the true variance σ_ϵ^2. They hold the true belief about the correlation coefficient of signal errors ρ. Also, they correctly recognize that the asset value θ and noise trade ω are independent with each other and of all strategic traders' signal errors. Formally, each strategic trader i's belief about the joint distribution of his signal error and those of others is represented by the following mean vector and variance-covariance matrix:

$$E_i \begin{bmatrix} \theta \\ \omega \\ \vdots \\ \epsilon_i \\ \epsilon_j \\ \epsilon_{j'} \\ \vdots \end{bmatrix} = \begin{bmatrix} \theta_0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \text{ and } Var_i \begin{bmatrix} \theta \\ \omega \\ \vdots \\ \epsilon_i \\ \epsilon_j \\ \epsilon_{j'} \\ \vdots \end{bmatrix} = \begin{bmatrix} \sigma_0^2 & 0 & \cdots & 0 & 0 & 0 & \cdots \\ 0 & \sigma_\omega^2 & \cdots & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \\ 0 & 0 & \cdots & \frac{1}{\kappa}\sigma_\epsilon^2 & \frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2 & \frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2 & \cdots \\ 0 & 0 & \cdots & \frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2 & \sigma_\epsilon^2 & \rho\sigma_\epsilon^2 & \cdots \\ 0 & 0 & \cdots & \frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2 & \rho\sigma_\epsilon^2 & \sigma_\epsilon^2 & \cdots \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \end{bmatrix},$$

where j and j' are any pair of indices other than i.

3. η-overconfidence [underconfidence in others' signals]: Each strategic trader i perceives the variance of other strategic traders' signals to be larger than the true variance by factor $\frac{1}{\eta}$, i.e., $\frac{\sigma_\epsilon^2}{\eta}$, where $\eta \in (0, 1)$ inversely represents the extent to which each trader underestimates the precision of other traders' signals. That is, as η decreases toward zero, the extent of such η-overconfidence becomes very strong. These strategic traders are unbiased about their own signal precision in that they perceive their own signal variances as the true variance σ_ϵ^2. Other assumptions are identical to the second case: Strategic traders hold the true belief about the correlation coefficient of signal errors ρ, and also correctly recognize that the asset value θ and noise trade ω are independent with each other and of all strategic traders' signal errors. Formally, each strategic trader i's belief about the joint distribution of his signal error and those of others is represented by the following mean vector and variance-covariance matrix:

$$E_i \begin{bmatrix} \theta \\ \omega \\ \vdots \\ \epsilon_i \\ \epsilon_j \\ \epsilon_{j'} \\ \vdots \end{bmatrix} = \begin{bmatrix} \theta_0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \text{ and } Var_i \begin{bmatrix} \theta \\ \omega \\ \vdots \\ \epsilon_i \\ \epsilon_j \\ \epsilon_{j'} \\ \vdots \end{bmatrix} = \begin{bmatrix} \sigma_0^2 & 0 & \cdots & 0 & 0 & 0 & \cdots \\ 0 & \sigma_\omega^2 & \cdots & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \\ 0 & 0 & \cdots & \sigma_\epsilon^2 & \frac{\rho}{\sqrt{\eta}}\sigma_\epsilon^2 & \frac{\rho}{\sqrt{\eta}}\sigma_\epsilon^2 & \cdots \\ 0 & 0 & \cdots & \frac{\rho}{\sqrt{\eta}}\sigma_\epsilon^2 & \frac{1}{\eta}\sigma_\epsilon^2 & \frac{\rho}{\eta}\sigma_\epsilon^2 & \cdots \\ 0 & 0 & \cdots & \frac{\rho}{\sqrt{\eta}}\sigma_\epsilon^2 & \frac{\rho}{\eta}\sigma_\epsilon^2 & \frac{1}{\eta}\sigma_\epsilon^2 & \cdots \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \end{bmatrix},$$

where j and j' are any pair of indices other than i.

Throughout all three cases, we assume that market makers hold the true belief about the joint distribution of all variables, and they correctly recognize strategic traders' beliefs about the joint distribution of their signals described above, as these beliefs are assumed to be common knowledge. This is in line with previous studies on overconfidence (e.g., Kyle and Wang, 1997) and can be justified with the fact that market makers in reality have a lot of professional experiences, making it less likely that they incorrectly recognize their own abilities.

Remark 2. The assumption of constant correlation coefficient ρ of signal errors is non-trivial. It draws on the premise that each strategic trader obtains a single piece of noisy signal which summarizes all available information possibly from different sources (e.g., data and experiences), whose perceived precision uniformly changes with overconfidence. In this sense, each strategic trader is uniformly overconfident in all available information in hand, rather than overconfident in a certain subset of information from "private" sources. This might be due to the fact that he cannot distinguish between various sources of information according to their accessibility to other traders. This point contrasts the current model with the alternative framework where agents endogenously choose between public and private signals to maximize utility, which draws on the premise that agents correctly recognize the publicity of the former signal.[5] Under this alternative framework, overconfidence would only affect the perceived precision of their private signals and thus effectively lower the overall correlation of their behaviors. Indeed, the assumption of constant correlation coefficient ρ of signal errors is crucial in the main results in what follows.

If strategic traders' signal errors are independent with each other (i.e., $\rho = 0$), our definitions of κ- and η-overconfidence match with those in the literature (e.g., Benos, 1998; Eyster, Rabin, and Vayanos, 2019; Kyle and Wang, 1997; Odean, 1998) as well as the informal idea that overconfidence refers to a person's own perception about his relative ability. It is already known by Kyle and Wang (1997) that κ-overconfidence increases trading volume, whereas η-overconfidence does not affect it.[6] These imply that trading volume unambiguously rises, assuming that the two types of overconfidence coexist in reality.

Our analyses focus on what additionally occurs under the two types of overconfidence in the presence of correlation in signal errors (i.e., $\rho > 0$). While the assumption of common correlation coefficient ρ might not be completely innocuous at this point (Remark 2), it can be regarded as neutrally capturing the potential effects of overconfidence on strategic traders' inferences about the covariance structures of signals.

Definition 1. An equilibrium consists of demand functions $\{x_i(s_i)\}_{i=1}^N$ and pricing rule $P(X)$ where the following two conditions hold:

1. At the second stage, the pricing rule set by market makers is equal to the expected value of the security conditional on total demand X, i.e., $P(X) = E[\theta|X]$, where the conditional expectation is taken with respect to the true joint distribution of all variables θ, ω, and $(\epsilon_i)_{i=1}^N$, which generates the true distribution of X combined with strategic traders' strategies $\{x_i(s_i)\}_{i=1}^N$;

2. For every strategic trader i, for every realization of signal s_i, demand $x_i(s_i)$ maximizes his expected profit. Formally, it is a best response given the profile of other strategic traders' strategies $(x_j(s_j))_{j\in\{1,\cdots,N\}\setminus\{i\}}$ and the pricing rule $P(X)$, as well as his perceived joint distribution of all variables θ, ω, and $(\epsilon_j)_{j=1}^N$.

Though the notion of equilibrium is mostly standard, it is worth describing the process of equilibrium formation, given the presence of players' disagreements over the structure of signals. Recall that strategic traders' and market makers' different beliefs about the distribution of their signals and errors are common knowledge. In equilibrium, each strategic trader i uses his conditional belief given his signal s_i to generate a joint distribution of θ, ω, and other strategic traders' signal errors $(\epsilon_j)_{j\in\{1,\cdots,N\}\setminus\{i\}}$. Combined with the correct conjecture about others' strategies $(x_j(s_j))_{j\in\{1,\cdots,N\}\setminus\{i\}}$, this translates to a joint distribution of total demand $X = x_i + \sum_{j\in\{1,\cdots,N\}\setminus\{i\}} x_j(s_j)$ perceived by the strategic trader i, which, combined with the correct conjecture about the pricing rule $P(X)$, eventually leads to a distribution of the strategic trader i's profit $\pi_i = x_i(\theta - P)$ perceived by him depending on his

[5] This is often adopted in linear-quadratic global games (e.g., Morris and Shin, 2002). See Han and Sangiorgi (2018) for a search-based microfoundation of the framework.

[6] While Kyle and Wang (1997) consider the case of two strategic traders, their result can easily extend to the general case of N traders, as formally shown by Proposition 2 later.

demand x_i.[7] Then he chooses $x_i(s_i)$ to maximize $E[\pi_i]$ under that distribution. Subsequently, market makers use their (true) belief about the distribution of strategic traders' signals $(s_i)_{i=1}^N$ and the correct conjecture about strategic traders' strategies $(x_i(s_i))_{i=1}^N$ to generate a distribution of the total demand X, which is in turn used to set the price $P = E[\theta|X]$ by market makers. Overall, these are in line with previous studies on overconfidence (e.g., Benos, 1998; Kyle and Wang, 1997; Odean, 1998).

As in Kyle (1985) and many other previous studies in the literature, this study focuses on the class of linear equilibria.[8]

Definition 2. Equilibrium $(\{x_i(\cdot)\}_{i=1}^N, P(X))$ is linear if demands x_i are linear in s_i, and pricing rule $P(X)$ is also linear in total demand X.

4 Analyses of the basic model

In this section, we first establish the existence of a unique equilibrium of the game. Then, restricting attention to the case where such unique equilibrium exists, we proceed to conduct comparative statics of trading volume and price informativeness with respect to overconfidence and the number of strategic traders. Throughout the section, we assume that $\sigma_\epsilon^2 < \sigma_0^2$. This assumption rules out the possibility of the uninteresting no-equilibrium outcome due to infinite trading of strategic traders who are highly κ-overconfident, as noted by Kyle and Wang (1997) and Odean (1998).

4.1 Equilibrium

The following proposition proves the existence of a unique equilibrium of the game and characterizes its trading coefficient for each of three cases in the model (i.e., benchmark, κ-overconfidence, and η-overconfidence):

Proposition 1. *The following statements hold true:*

(1) *In the benchmark case, there is a unique equilibrium of the game. In the equilibrium, strategic trader i submits $x_i^* = \beta_R^*(s_i - \theta_0)$, where his trading coefficient is given by*

$$\beta_R^* = \sqrt{\frac{\sigma_\omega^2}{N\left(\sigma_0^2 + \sigma_\epsilon^2\right)}}.$$

(2) *In the case of κ-overconfidence, there exists a unique equilibrium if and only if $N \in [1, \bar{N}_K)$, where $\bar{N}_K$ is defined as*

$$\bar{N}_K := 1 + \frac{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2}{\rho\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2}$$

for $\rho > 0$, and it takes infinity for $\rho = 0$. Whenever such an equilibrium exists, strategic trader i submits $x_i^* = \beta_K^*(s_i - \theta_0)$, where his trading coefficient is given by

$$\beta_K^* = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(N-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}}}.$$

[7] At this point, each strategic trader i can correctly conjecture other strategic traders' strategies (i.e., their mappings from s_j to x_j) because (i) strategic traders' different beliefs about the joint distribution of their signal errors are common knowledge, and (ii) these strategic traders correctly recognize that other strategic traders also maximize their profits given their own beliefs and signals.

[8] It is largely an open question whether a unique linear equilibrium of the model is also unique in a wider class of demand functions. See Mclennan, Monteiro and Tourky (2017) and Rochet and Vila (1994) for the case of a single strategic trader.

(3) *In the case of η-overconfidence, there is a unique equilibrium of the game. In the equilibrium, strategic trader i submits $x_i^* = \beta_E^*(s_i - \theta_0)$, where his trading coefficient is given by*

$$\beta_E^* = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \sigma_\epsilon^2 + \rho(N-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2\right\}}}.$$

The benchmark case is already known in the literature as a special case of Lambert, Ostrovsky and Panov (2018), and the existence and uniqueness of equilibrium are guaranteed for every $N \geq 1$ in this case. In the case of κ-overconfidence, the existence of equilibrium is guaranteed for every $N \geq 1$ only in the absence of signal error correlation (i.e., $\rho = 0$). If strategic traders' signal errors are correlated (i.e., $\rho > 0$), a unique equilibrium exists if and only if the number of strategic traders is not too large (i.e., $N < \bar{N}_K$). Finally, in the case of η-overconfidence, there exists a unique equilibrium for every $N \geq 1$, whether strategic traders' signal errors are correlated or uncorrelated, as in the benchmark case.

Following Definitions 1 and 2, an equilibrium is determined by the following steps, which are mostly standard in the literature following Kyle (1985): First, given a conjecture on strategic traders' behavior (i.e., $x_i^* = \beta(s_i - \theta_0)$), the pricing rule set by market makers is linear in the total demand X, i.e.,

$$P(X) = E[\theta|X] = \theta_0 + \lambda X,$$

where its updating coefficient λ, which could be interpreted as market power, is given by[9]

$$\lambda = \frac{1}{N\beta}\frac{\sigma_0^2}{\sigma_0^2 + \frac{1+\rho(N-1)}{N}\sigma_\epsilon^2 + \frac{\sigma_\omega^2}{N^2\beta^2}}. \tag{1}$$

Second, each strategic trader i recognizes that the price P_i follows the above pricing rule $P(X)$ and is a function of his submitted demand x_i, others' signals $(s_j)_{j\in\{1,\cdots,N\}\setminus\{i\}}$, and noise trade ω, i.e.,

$$P_i\left(x_i, (s_j)_{j\in\{1,\cdots,N\}\setminus\{i\}}, \omega\right) = \theta_0 + \lambda\left\{x_i + \sum_{j\neq i}\beta(s_j - \theta_0) + \omega\right\}, \tag{2}$$

and he chooses the demand x_i so as to maximize his expected profit given by

$$E\left[\pi_i|\mathcal{I}_i\right] = E\left[x_i\left\{\theta - P_i\left(x_i, (s_j)_{j\in\{1,\cdots,N\}\setminus\{i\}}, \omega\right)\right\}|\mathcal{I}_i\right],$$

where the price P_i is obtained from Equation (2). Noting that the expected profit $E\left[\pi_i|\mathcal{I}_i\right]$ is hump-shaped with respect to x_i, its first-order condition is given by

$$\begin{aligned}\frac{dE\left[\pi_i|\mathcal{I}_i\right]}{dx_i} &= \underbrace{E\left[\theta - P_i|\mathcal{I}_i\right]}_{\text{Perceived trading opportunity}} - \underbrace{\lambda x_i}_{\text{Market power}} \\ &= \underbrace{E\left[\theta - \theta_0|\mathcal{I}_i\right]}_{\text{(i) Expected asset value}} \underbrace{- \lambda\left(x_i + \beta E\left[\sum_{j\neq i}(s_j - \theta_0)|\mathcal{I}_i\right]\right)}_{\text{(ii) Expected price}} - \lambda x_i = 0. \end{aligned} \tag{3}$$

Solving the first-order condition, we obtain strategic trader i's best response $B_i(\beta)$ as follows:

$$B_i(\beta) = \frac{1}{2\lambda}E\left[\theta - \theta_0|\mathcal{I}_i\right] - \frac{\beta}{2}E\left[\sum_{j\neq i}(s_j - \theta_0)|\mathcal{I}_i\right].$$

[9] Equation (1) is derived in the proof of Proposition 1.

Finally, an equilibrium is formed at the fixed point of trading coefficient β where the best response $B_i(\beta)$ equals the conjectured strategy $\beta(s_i - \theta_0)$.

To see how strategic traders' behavior differs across the three cases, we note that each strategic trader i's incentive to trade is represented by the above first-order condition in Equation (3), which consists of market power λ and his perceived trading opportunity (i.e., $E[\theta - P_i|\mathcal{I}_i]$), where the (perceived) price P_i is given by Equation (2). While both types of overconfidence do not affect market power λ, which hinges on market makers' inference about the asset value θ, they affect each trader i's inferences about both (i) the asset value θ and (ii) the price P_i in Equation (3). Here, the second effect (ii) arises from his inference about other strategic traders' signal values $(s_j)_{j\in\{1,\cdots,N\}\setminus\{i\}}$, despite the correct conjectures about market makers' pricing rule $P(X)$ and other strategic traders' strategies represented by trading coefficient β. These inferences are reflected on the best response $B_i(\beta)$ described above. In particular, κ-overconfidence affects each trader i's best response $B_i(\beta)$ via both effects (i) and (ii) in Equation (3) because it affects the perceived precision of his signal, which in turn affects his inferences about the asset value θ and others' signal values $(s_j)_{j\in\{1,\cdots,N\}\setminus\{i\}}$. On the other hand, η-overconfidence affects each trader i's best response $B_i(\beta)$ via the second effect (ii) in Equation (3) since it affects only his inference about others' signal values $(s_j)_{j\in\{1,\cdots,N\}\setminus\{i\}}$.

In the case of κ-overconfidence, there is no equilibrium if the number of strategic traders is sufficiently large (i.e., $N > \bar{N}_K$). This no-equilibrium outcome occurs due to the fact that each strategic trader has an infinite incentive to trade the asset regardless of other traders' trading strategies, so that there is no fixed point where each trader's best response equals any conjectured trading coefficient. This is in line with Kyle and Wang (1997) and Odean (1998). However, while such no-equilibrium outcome is attributed to highly overconfident traders in these previous papers assuming a fixed number of traders and the independence of signal errors (i.e., $\rho = 0$), the proposition indicates that it is also attributed to a large number of traders in the market. Indeed, even when traders have an arbitrarily small extent of overconfidence (i.e., κ just above one), there can always be no equilibrium with a sufficiently large number of traders.

It is worth contrasting such no-equilibrium outcome in the current model with that in previous studies such as Kyle (1989) and Rostek and Weretka (2012). In these studies, the absence of equilibrium occurs generally with a small number of traders having market power. This is due to the absence of incentive to trade, which could be interpreted as market failure resulting from adverse selection. This type of no-equilibrium outcome does not occur in the current framework based on Kyle (1985) due to the presence of risk-neutral market makers, who are informally interpreted as liquidity providers. In contrast, in our model, the no-equilibrium outcome which occurs with a large number of κ-overconfident traders is due to infinite trade carried out by these traders, rather than the absence of trade. Indeed, κ-overconfident traders a priori (incorrectly) expect infinite profits from their trades in this case, as shown later by Lemma 1 in the limit where $N \to \bar{N}_K$. However, such no-equilibrium outcome is not robust to a slight modification of their preferences preventing infinite trade (e.g., a quadratic utility with a slightly negative quadratic coefficient). This observation suggests that the absence of equilibrium for $N > \bar{N}_K$ appears to be an artifact of risk neutrality, rather than reflecting market failure from adverse selection.[10]

4.2 Effect of overconfidence on trading volume

In this subsection, we examine how overconfidence affects trading volume and its properties with respect to market size. Though trading volume by itself does not provide immediate implications on efficiency, the main objectives of these analyses are twofold: First, it provides various testable implications which are comparable with previous analytic studies (e.g., Eyster, Rabin and Vayanos, 2019; Kyle and Wang,

[10]In fact, it might be a sort of "market failure." Infinite trading which occurs as $N \to \bar{N}_K$ hurts these κ-overconfident traders in terms of actual utility. Though this is simply their loss from trading against market makers in the current framework abstracting from risk sharing, it could be broadly viewed as representing inefficient risk taking, which corresponds to the presence of negative externalities noted by Brunnermeier, Simsek and Xiong (2014). Still, it is distinguished from the sort of market failure from adverse selection seen in Kyle (1989) and Rostek and Weretka (2012).

1997; Odean, 1988), as trading volume is readily observable in financial markets. Second, given that trading volume is positively associated with price informativeness in our framework, it can serve as an intermediate step for the following analysis on price informativeness and its properties, which would provide various implications on efficiency.

Formally, trading volume is defined as follows:

Definition 3. Trading volume is defined as the sum of the expected absolute values of strategic and noise traders' demands as follows:

$$TV = N \cdot E\left[|x_i^*|\right] + E[|\omega|].$$

From now on, we restrict attention to the cases where a unique equilibrium exists, which rule out only the case of κ-overconfidence with correlated errors and too many strategic traders (i.e., $N > \bar{N}_K$). The below proposition compares trading volume in the cases of two types of overconfidence with that in the benchmark case:

Proposition 2. *Regarding the effects of κ- and η-overconfidence on trading volume compared with the benchmark case, the following statements hold true:*

(1) *For all $\rho \geq 0$, κ-overconfidence increases trading volume. Also, it increases trading volume even more with $\rho > 0$ than with $\rho = 0$.*

(2) *If $\rho = 0$, η-overconfidence does not affect trading volume. In contrast, if $\rho > 0$, it decreases trading volume.*

In order to get the intuition of these results through the lens of our equilibrium derivation, recall that both types of overconfidence affect each strategic trader i's behavior via his perceived trading opportunity (i.e., $E[\theta - P_i|\mathcal{I}_i]$ in Equation (3)), while they do not affect market power λ defined by Equation (1). The relevant conditional expectations are expressed using the standard Bayesian updating formula as follows:

$$E[\theta|\mathcal{I}_i] = \theta_0 + \frac{Cov\left(s_i, \theta\right)}{Var\left(s_i\right)}\left(s_i - \theta_0\right) \text{ and } E[P_i|\mathcal{I}_i] = \theta_0 + \lambda x_i + \lambda\beta(N-1)\frac{Cov\left(s_i, s_j\right)}{Var\left(s_i\right)}\left(s_i - \theta_0\right), \quad (4)$$

for any $j \in \{1, \cdots, N\}\backslash\{i\}$. The covariance term $\frac{Cov(s_i,\theta)}{Var(s_i)}$ ($\frac{Cov(s_i,s_j)}{Var(s_i)}$) in the first (second) expression of Equation (4) represents the extent to which the trader perceives the asset value θ (other traders' signal values s_j) to covary with a normalized unit change in his own signal s_i. These covariance terms are influenced by the two types of overconfidence, causing changes in trading volume as in the proposition.

With independent signal errors (i.e., $\rho = 0$), κ-overconfidence increases trading volume, whereas η-overconfidence does not affect trading volume, as shown by Kyle and Wang's (1997) model of two insiders, which corresponds to a special case of $\rho = 0$ and $N = 2$ in the current model. In particular, κ-overconfidence makes each strategic trader i perceive the asset value θ to covary more with a normalized change in his own signal s_i through an increase in its perceived precision. Though this is partially offset by the fact that it also makes him perceive the price P_i to covary more with a normalized change in his signal for the same reason, the former effect via the asset value θ is dominant, increasing his perceived trading opportunity $E[\theta - P_i|\mathcal{I}_i]$ and thus incentivizing him to trade more.[11] On the other hand, η-overconfidence does not affect trading volume. Indeed, both covariance terms in Equation (4) remain unchanged as they hinge on the inference about the asset value θ through his own signal s_i with independent signal errors. Overall, one could argue that the total effect of overconfidence on trading volume is positive when both types of overconfidence coexist, leading to the prevailing argument in the literature that overconfidence increases trading volume.

[11] The dominance of the direct effect via the asset value θ comes from the fact that the trader recognizes that the price P_i, which is equal to the conditional expectation of the asset value θ from the viewpoint of market makers, must respond to a change in the asset value θ less than one-for-one.

Now we move on to consider what additionally occurs with correlation in signal errors (i.e., $\rho > 0$). As strategic traders' signal errors are correlated, the interaction effect between κ-overconfidence (η-overconfidence) and correlation in signal errors makes each strategic trader i perceive the price P_i to covary less (more) with a normalized change in his signal s_i. To see this formally, we note that the second covariance term in Equation (4) is given by $\frac{Cov(s_i,s_j)}{Var(s_i)} = \frac{\sigma_0^2+\rho\frac{\sigma_\epsilon^2}{\sqrt{\kappa}}}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}}$ $(\frac{\sigma_0^2+\rho\frac{\sigma_\epsilon^2}{\sqrt{\kappa}}}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}})$, which increases less (decreases) with κ-overconfidence (η-overconfidence) with correlation in signal errors compared with independent errors. This increases (decreases) his perceived trading opportunity $E[\theta - P_i|\mathcal{I}_i]$, thereby incentivizing (disincentivizing) him to trade. These are summarized in the first (second) column of the below table, emphasizing what occurs only with correlation in signal errors (i.e., $\rho > 0$).

These opposite effects of two types of overconfidence stem from their effects on strategic traders' inference about others' signal errors, which influences their expectations about the price P_i via the second covariance term in Equation (4). In the case of κ-overconfidence, its effect on the inference about others' signal errors causes each strategic trader to expect that others are less likely to trade in the same direction due to lower perceived covariance between his own signal and others' signals, whereas, in the case of η-overconfidence, its effect on the inference about others' signal errors causes each strategic trader to expect that others are more likely to trade in the same direction due to higher perceived covariance between his own signal and others' signals. These opposite effects on each strategic trader's expectations about others' trading behavior lead to their opposite effects on trading volume. It is noteworthy that these opposite effects of the two types of overconfidence work through the covariance structure of signals perceived by each strategic trader. In a broad perspective, this reminds us of the role of covariances in the classical theory in asset pricing (e.g., Sharpe, 1964).

	Changes due to κ-overconfidence	Changes due to η-overconfidence
Market power	None	None
Perceived covariance of the asset value with signal value s_i	Increase	None
Perceived covariance of the price with signal value s_i	Increase	***Increase***
(i) Subeffect via the inference about other traders' expected asset value	Positive [covary more]	None
(ii) Subeffect via the inference about other traders' signal errors	***Negative [covary less]***	***Positive [covary more]***
Overall change in trading volume	Increase	***Decrease***

Remark 3. What type of disagreement in the model drives the results in the proposition? Though it seems to be challenging to formally distinguish between different types of disagreements that might coexist, the disagreement between each strategic trader and market makers is crucial in understanding the opposing effects of the two types of overconfidence on trading volume. Indeed, the second covariance term in Equation (4) reflects each strategic trader i's inference about the price P_i, which corresponds to market makers' expectation about the asset value θ. Recalling that η-overconfidence decreases each strategic trader i's perceived trading opportunity in Equation (3) via higher perceived covariance between his own signal and others' signals, we can see that this effect would be counteracted if market makers also agree with the strategic trader i over the distribution of other traders' signals so that they update the price less in response to the total demand (i.e., lower λ), thereby increasing his perceived trading opportunity $E[\theta - P_i|\mathcal{I}_i]$. In fact, if market makers believe that all strategic traders' signal errors have variance $\frac{\sigma_\epsilon^2}{\eta}$, η-overconfidence would *increase* trading volume in contrast to what occurs in the proposition.[12] In the case of κ-overconfidence, this point seems to be less clear due to the presence of various types of disagreements among strategic traders over the asset value θ and their signals. Nevertheless, it is intuitive that the positive effect of κ-overconfidence on each strategic trader i's perceived trading opportunity, which works through a higher perceived covariance between his own signal and others' signals, would be counteracted similarly if market makers perceive the strategic trader i to have higher signal precision so that they update the price more in response to the total demand (i.e., higher λ), thereby reducing his perceived trading opportunity.

With regard to empirical relevance, the proposition predicts the presence of heterogeneity across markets regarding trading volume and its relationship with overconfidence. Indeed, the case of η-overconfidence suggests that a large amount of trade is not the only possible outcome from overconfidence. On the contrary, trading volume increases even further with κ-overconfidence as strategic traders' signal errors are correlated with each other. Overall, the relationship between overconfidence and trading volume greatly differs across information structures and the types of overconfidence.

Further, the proposition sheds light on empirical and experimental literature on the link between overconfidence and trading volume. Even though early strong results support the argument that overconfidence is a reason for high trading volume, these results largely hinge on the use of proxies for overconfidence, such as gender (e.g., Barber and Odean, 2001) and past returns (e.g., Statman, Thorley and Vorkink, 2006). In contrast, recent studies measure overconfidence more directly using survey data to capture individuals' beliefs and appear to provide mixed evidence on the relationship between overconfidence and trading volume. These studies employ so-called miscalibration scores, which are obtained by asking individuals to state confidence intervals for a number of general knowledge questions requiring numerical answers and then measuring the extent to which their confidence intervals are too narrow. Even though miscalibration scores seem to be the closest to the types of overconfidence considered in analytic studies in the literature, including "κ-overconfidence" in this study, these studies consistently find that such miscalibration scores do not increase trading volume even though they decrease trading profits (e.g., Biais, Hilton, Mazurier, and Pouget, 2005; Fellner-Rohling and Krugel, 2014; Glaser and Weber, 2007; Merkle, 2017). On the other hand, a different type of overconfidence focusing on positive self-illusions, which is called the better-than-average effect, appears to be related to trading volume (e.g., Glaser and Weber, 2007; Merkle, 2017). Also, Fellner-Rohling and Krugel (2014) propose an alternative measure to capture misconception of signal reliability based on the past observation of signals and actual outcomes and show that the proposed measure increases trading volume. According to our results on the effects of overconfidence, these mixed empirical and experimental findings are not necessarily surprising. Even if empirical evidence supports the positive effect of overconfidence on trading volume on average, this could be due to the prevalence of a particular form of overconfidence, such as κ-overconfidence, rather than indicating the presence of overconfidence in general. Also, we can say that the mixed findings on miscalibration scores can be explained with the proposition above. Even if miscalibration scores correspond to κ-overconfidence, which increases trading volume, it is expected that these scores are also strongly associated with η-overconfidence, which decreases trading volume. Therefore, we conclude that the link between overconfidence and trading volume is unlikely to be clear without identifying the form of overconfident beliefs about ability.

The next proposition examines how overconfidence affects the properties of trading volume with regard to the number of strategic traders N.

Proposition 3. *The following statements hold true:*

(1) *In the benchmark case and the cases of κ- and η-overconfidence with $\rho = 0$, trading volume increases with N, and goes to infinity at the rate of $\sqrt{N}$ as $N \to \infty$.*

(2) *In the case of κ-overconfidence with $\rho > 0$, trading volume increases with N, and goes to infinity as $N \to \bar{N}_K$.*

(3) *In the case of η-overconfidence with $\rho > 0$ and $\eta > (<) \left(\frac{\rho\sigma_\epsilon^2}{\sigma_0^2+\sigma_\epsilon^2+\rho\sigma_\epsilon^2}\right)^2$, trading volume increases (decreases) with N, and converges to a finite value as $N \to \infty$.*

[12] The formal proof of this statement is available upon request from the author.

Two properties of trading volume are considered: (i) A new strategic trader's entry increases trading volume, and (ii) trading volume approaches infinity in large markets. In the benchmark case, these properties hold true. They involve a general intuition, rather than being specific to the details of the model. To get the intuition, we use each strategic trader i's first-order equation given by Equation (3), which represents his incentive to trade the asset, as follows:

$$\begin{aligned} \frac{d}{dx_i} E[\pi_i|\mathcal{I}_i] &= E[\theta - P_i|\mathcal{I}_i] - \lambda x_i \\ &= E[\theta - \theta_0|\mathcal{I}_i] - \underbrace{\lambda\left(x_i + E[X_{-i}|\mathcal{I}_i]\right)}_{E[P_i - \theta_0|\mathcal{I}_i]} - \lambda x_i = 0, \end{aligned}$$

where x_i is the trader's demand, X_{-i} is the sum of other strategic traders' demands, and λ is market power given by Equation (1). Using the symmetry across strategic traders and taking the expectation over all variables except for θ, we can see that each strategic trader's trading coefficient in equilibrium is given by[13]

$$\beta_R^* = \frac{1}{\lambda(N+1)}. \tag{5}$$

Then both properties of trading volume, which can be represented by $N\beta_R^*$, follow from Equation (5), which is essentially the aggregation of individual traders' first-order conditions, combined with the fact that market power λ decreases toward zero with N. Intuitively, smaller market power caused by an increase in N increases each strategic trader's incentive to trade (i.e., $\frac{d}{dx_i}E[\pi_i|\mathcal{I}_i]$) given his individual demand (i.e., x_i) and total demand (i.e., $x_i + E[X_{-i}|\mathcal{I}_i]$), as seen in the above first-order condition. Given the symmetry, we can see that the size of total demand must increase to ensure that the first-order condition holds in equilibrium.[14] In addition, as N becomes large so that market power λ is very small, Equation (5) indicates that trading volume goes to infinity. Intuitively, the price would otherwise still be noisy due to a significant proportion of noise trade compared with strategic traders' trades, causing a non-negligible expected profit for each strategic trader in equilibrium. This leads to infinite trading volume, which is a contradiction. Note that the above derivation of Equation (5) involves the step of taking the expectation of the first-order condition over signal errors. This indicates that these properties of trading volume rely on the absence of disagreement between strategic traders and market makers over the distribution of signal errors.

Now we turn to the case of κ-overconfidence, focusing on what is changed compared with the benchmark case. Recall from Equations (3) and (4) that κ-overconfidence affects each strategic trader's trading aggressiveness by changing his perceived trading opportunity $E[\theta - P_i|\mathcal{I}_i]$, while it does not affect market power λ given by Equation (1). This effect of κ-overconfidence works through changes in two covariance terms in Equation (4) representing how strategic traders perceive the asset value θ and the price P_i to covary with normalized changes in their signals. To see this in detail, we use Equation (4) to represent his perceived trading opportunity as follows:

$$\begin{aligned} & E[\theta - P_i|\mathcal{I}_i] = E[\theta|\mathcal{I}_i] - E[P_i|\mathcal{I}_i] \\ = \ & \underbrace{\frac{\sigma_0^2}{\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}}(s_i - \theta_0)}_{E[\theta-\theta_0|\mathcal{I}_i]} - \lambda x_i - \lambda\beta(N-1)\underbrace{\left[\underbrace{\frac{\sigma_0^2}{\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}}(s_i - \theta_0)}_{E[\theta-\theta_0|\mathcal{I}_i]} + \underbrace{\frac{\rho\frac{\sigma_\epsilon^2}{\sqrt{\kappa}}}{\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}}(s_i - \theta_0)}_{E[\epsilon_j|\mathcal{I}_i]}\right]}_{E[s_j - \theta_0|\mathcal{I}_i] := E[\theta-\theta_0|\mathcal{I}_i] + E[\epsilon_j|\mathcal{I}_j]}. \end{aligned}$$

[13]We can represent these demands as $x_i = \beta_R^*(s_i - \theta_0)$ and $X_{-i} = \sum_{j\neq i}\beta_R^*(s_j - \theta_0)$ in equilibrium. Taking the expectation over all variables except for θ, we have $\theta - \theta_0 - \lambda(N+1)\beta_R^*(\theta - \theta_0) = 0$, which implies Equation (5).

[14]Formally, if the size of total demand decreases with an increase in N, his trading aggressiveness (i.e., β_R^*), which represents the size of his individual demand x_i given the distribution of his signal s_i, decreases as well by symmetry. However, the above first-order equation given by Equation (3) implies that his incentive to trade (i.e., $\frac{d}{dx_i}E[\pi_i|\mathcal{I}_i]$) must increase due to decreases in the size of both total demand and his individual demand. This contradicts our initial assumption.

With independent signal errors (i.e., $\rho = 0$), the absence of inference about others' errors (i.e., $E[\epsilon_j|\mathcal{I}_i] = 0$) implies that κ-overconfidence only scales up the above expression, which corresponds to his perceived trading opportunity, by increasing the denominators by the same rate regardless of the number of strategic traders N. As a result, trading volume still increases with N at the same rate as in the benchmark case. In contrast, with correlated signal errors (i.e., $\rho > 0$), the effect of κ-overconfidence on his inference about others' signal errors (i.e., $E[\epsilon_j|\mathcal{I}_i]$) increases the above expression proportionally to the number of other existing strategic traders (i.e., $N-1$). The intuition is that κ-overconfidence affects his inference about *each* of other strategic traders' signal errors, which separately influences his perceived covariance between his own signal and the price.

Indeed, the proposition indicates that the interaction effect between κ-overconfidence and the number of strategic traders N significantly changes the limiting property of trading volume with regard to N, causing it to increase toward infinity. In particular, as the number of strategic traders N increases toward its upper limit $\bar{N}_K$, two opposing effects are at work regarding each existing trader's perceived trading opportunity: First, an increase in trading volume leads to a decrease in the proportion of noise, causing the price to be less noisier and thus lowering each existing trader's (actual) trading opportunity, as in the benchmark case. Second, each existing trader believes that the price is now more strongly affected by systematic errors involved in others' signals, which he believes covary with his own signal less than theirs. The second effect, which corresponds to the aforementioned interaction effect via the inference about others' signal errors, increases his perceived trading opportunity in contrast to the first effect. Eventually (i.e., as $N \to \bar{N}_K$), the second effect dominates the first one, causing infinite trading.

In the case of η-overconfidence, recall from Equations (3) and (4) that η-overconfidence affects each strategic trader's trading aggressiveness only through a change in his perceived covariance between the price and his signal, which reflects his inference about other strategic traders' signals. This effect of η-overconfidence works through a change in the second covariance term in Equation (4) representing how strategic traders perceive the price to covary with normalized changes in their signals. To see this in detail, we use Equation (4) to represent his perceived trading opportunity as follows:

$$
\begin{aligned}
&E[\theta - P_i|\mathcal{I}_i] = E[\theta|\mathcal{I}_i] - E[P_i|\mathcal{I}_i] \\
= \;& \underbrace{\frac{\sigma_0^2}{\sigma_0^2+\sigma_\epsilon^2}(s_i-\theta_0)}_{E[\theta|\mathcal{I}_i]} - \lambda x_i - \lambda\beta(N-1) \underbrace{\left[\underbrace{\frac{\sigma_0^2}{\sigma_0^2+\sigma_\epsilon^2}(s_i-\theta_0)}_{E[\theta|\mathcal{I}_i]} + \underbrace{\frac{\rho\frac{\sigma_\epsilon^2}{\sqrt{\eta}}}{\sigma_0^2+\sigma_\epsilon^2}(s_i-\theta_0)}_{E[\epsilon_j|\mathcal{I}_i]} \right]}_{E[s_j|\mathcal{I}_i]:=E[\theta|\mathcal{I}_i]+E[\epsilon_j|\mathcal{I}_j]} .
\end{aligned}
$$

With correlated signal errors (i.e., $\rho > 0$), the effect of η-overconfidence on his inference about others' signal errors decreases the above expression proportionally to the number of other existing strategic traders (i.e., $N-1$). As in the case of κ-overconfidence, this is because η-overconfidence affects his inference about each of other strategic traders' signal errors, which separately influences his perceived covariance between his own signal and the price. As the number of strategic traders N increases, while its direct effect via smaller market power increases trading volume, it can be more than offset by the above interaction effect between η-overconfidence and the number of strategic traders N so that trading volume decreases. Indeed, this is the case when parameter η is low so that the extent of η-overconfidence is sufficiently large.

In line with Remark 3 in the previous subsection, these results rely on market makers' true belief about the distribution of private signals, which is reflected on the formation of price P_i. In particular, in the case of η-overconfidence, the possibility of negative relationship between the number of strategic traders N and trading volume would be overturned if market makers also agree with each strategic trader over the distribution of other traders' signals (i.e., having error variance $\frac{\sigma_\epsilon^2}{\eta}$) so that they update less in response to the total demand (i.e., lower λ). Here, as the number of strategic traders N increases, the above interaction effect between η-overconfidence and the number of strategic traders

N would persist, possibly leading to a decrease in trading volume. Nevertheless, such effect would be more than offset by the counterforce from market makers' perceived distribution of strategic traders' signals, which is also proportional to the number of strategic traders N in terms of its influence on perceived trading opportunity through market power λ. This argument generally reiterates Remark 3 concerning the importance of disagreement between each strategic trader and market makers.

In sum, these results generally indicate that overconfidence leads to a wide range of predictions regarding the properties of trading volume. In the benchmark case, the prevailing argument in the literature holds true in that trading volume increases with market size and becomes large in large markets. In contrast, in the case of η-overconfidence, trading volume cannot be arbitrarily large in large markets and may even decrease with a new trader's entry. On the other hand, κ-overconfidence makes it even much more likely that large trading volume occurs as markets grow large. Overall, these imply that the properties of trading volume with regard to the number of traders can vary across different markets in reality, depending on the type of overconfidence and information structures.

It is useful to compare between the predictions on trading volume under κ-overconfidence with correlated errors (i.e., $\rho > 0$) and independent errors (i.e., $\rho = 0$). The results with independent errors indicate that κ-overconfidence increases trading volume by the same rate throughout the number of traders so that the relationship between trading volume and the number of traders remains unchanged. This implies that a moderate extent of κ-overconfidence does not cause very large trading volume by itself. It appears to cause a difficulty in explaining large trading volume in the real world given the intuitive observation that market participants show overconfidence on average but to a moderate degree and working experience indeed reduces overconfidence (e.g., Gloede and Menkhoff, 2014). In contrast, the results on κ-overconfidence with correlated errors suggest that large trading volume possibly occurs even with a moderate extent of κ-overconfidence and a sufficient number of traders. Overall, large trading volume in the real world can be more easily explained with these results on κ-overconfidence with correlated errors.

It is also worth comparing these results with Eyster, Rabin and Vayanos' (2019) results on the pattern of trading volume with respect to the number of traders. As mentioned in Section 2, they consider the case where traders are "cursed" in the sense that they do not fully appreciate the informational content of the price. Using a different trading mechanism where a finite number of price-taking traders submit price-contingent demands and the standard assumption of independent errors, they show that cursedness generates infinite trading volume in large markets, whereas κ- and η-overconfidence do not. The intuition behind their results is as follows: As the number of traders grows large, despite their biased beliefs, κ- and η-overconfident traders recognize that the price fully reveals the average signal of all other traders. In large markets, therefore, these traders base their expectations of the asset payoff almost exclusively on the price, and as a result, the difference between any two traders' expectations converges to zero and so does their per-trader volume. In contrast, cursed traders give their signal nonnegligible weight even when the number of traders grows large so that the price reveals the average signal of other traders. As a result, cursed traders' per-trader volume does not converge to zero in large markets. Of course, given that their assumptions are rather different from those in this study, it appears to be difficult to compare Proposition 3 with theirs. For example, in their results, (total) trading volume is finite even in large markets in the benchmark case (i.e., without any bias of traders), whereas, in this study, it is infinite in large markets even in the benchmark case. This appears to be attributed to differences in the modeling framework.[15] Nevertheless, Proposition 3 can be thought of as being complementary to their results based on the assumption of independent signal errors in the sense that even κ-overconfidence combined with correlated signal errors could easily generate large trading volume which would be difficult to be explained otherwise.

[15]Kyle's (1985) framework is more likely to generate infinite trading volume in large markets due to the fact that traders are risk-neutral and the only factor that limits infinite trading volume is market power, which fades away as the number of traders grows large. In contrast, Eyster, Rabin and Vayanos adopt the assumption of constant absolute risk aversion to ensure that traders shy away from carrying out infinite trades to avoid a large amount of risk. Also, in their model, traders are price takers, making it possible that traders' beliefs converge toward the price so that these traders do not trade based on their own private information.

4.3 Effect of overconfidence on price informativeness and its properties

This subsection addresses the question of how overconfidence affects price informativeness, which measures the quality of information contained in the price, and its properties with respect to market size. The notion of price informativeness has been of significant interest as it represents the informational role of financial prices and thus can be regarded as part of the broad notion of market efficiency. Formally, we define price informativeness as follows:

Definition 4. Price informativeness is defined as the precision (or the inverse of variance) of the asset value conditional on the price as follows:

$$PI = \{Var(\theta|P)\}^{-1}.$$

Note that most previous studies (e.g., Rahi and Zigrand, 2018; Rostek and Weretka, 2012) define it slightly differently by normalizing the conditional variance of the asset value (i.e., $1 - \frac{Var(\theta|P)}{\sigma_0^2}$). However, it is easy to see that these definitions are equivalent to each other once we fix the prior σ_0^2. The best scenario for price informativeness is that the price fully aggregates all available signals held by traders in the market. We closely follow Rostek and Weretka's (2012) terminology to define this best scenario as follows:

Definition 5. The equilibrium price is privately revealing if price informativeness PI reaches $PI^* := \{Var(\theta|s_1, \cdots, s_N)\}^{-1}$.

A useful observation here is that price informativeness is positively associated with trading volume. To see this, note that knowing the price is equivalent to knowing total demand observed by market makers, which consists of strategic traders' demands and noise trade. The former corresponds to the informational content of the price, whereas the latter represents its "error" term. Given constant volume of noise trade, an increase in trading volume indicates an increase in the proportion of strategic traders' demands compared with noise trade, which corresponds to an increase in price informativeness. The following corollary confirms this point:

Corollary 1. *Regarding the effects of κ- and η-overconfidence on trading volume compared with the benchmark case, the following statements hold true:*

(1) *For all $\rho \geq 0$, κ-overconfidence increases price informativeness.*

(2) *If $\rho = 0$, η-overconfidence does not affect price informativeness. In contrast, if $\rho > 0$, it decreases price informativeness.*

Next, the following corollary concerns the relationship between price informativeness and market size:

Corollary 2. *The following statements hold true:*

(1) *In the benchmark case and the cases of κ- and η-overconfidence with $\rho = 0$, price informativeness increases with N and the price is privately revealing as $N \to \infty$.*

(2) *In the case of κ-overconfidence with $\rho > 0$, price informativeness increases with N and the price is privately revealing as $N \to \bar{N}_K$.*

(3) *In the case of η-overconfidence with $\rho > 0$ and $\eta > (<) \left(\frac{\rho\sigma_\epsilon^2}{\sigma_0^2 + 2\sigma_\epsilon^2}\right)^2$, price informativeness increases (decreases) with N and the price is not privately revealing for any N.*

In the benchmark case, a new strategic trader's entry affects price informativeness in two ways: First, the new trader brings his own information into the price by submitting his demand to the market. Second, the new trader's entry increases the trading volume of strategic traders (Proposition 3), thereby lowering the relative weight on noise trade in the price. Both of these effects lead to an increase in price informativeness, eventually making the price privately revealing due to infinite trading volume of strategic traders.[16]

The proposition indicates that these properties of price informativeness with respect to the number of strategic traders do not necessarily persist with overconfidence. This roughly follows from Proposition 3 combined with the point that price informativeness is positively associated with trading volume. Specifically, as a new strategic trader enters the market, the above two effects from his own information and smaller proportion of noise trade occur as in the benchmark case, leading to changes in price informativeness. However, under κ-overconfidence (η-overconfidence), the second effect from smaller proportion of noise trade becomes stronger (can be reversed), making the interaction effect between κ-overconfidence (η-overconfidence) and the number of strategic traders even more positive (possibly negative). In the case of η-overconfidence, such interaction effect can be negative and even dominate the first effect via his own information, leading to a negative relationship between price informativeness and the number of strategic traders. This is the case with with a sufficiently high degree of η-overconfidence (i.e., a sufficiently low value of η). The limiting properties of price informativeness naturally follow from the corresponding properties of trading volume in Proposition 3.

These results suggest that the two types of overconfidence significantly influence the relationship between price informativeness and market size in opposite directions. It is commonly believed that an increase in the size of financial markets, which possibly arises from the trends of globalization and financial technology, improves the functioning of these markets, and, on the side of the informational role of markets, this eventually leads to the best scenario that prices accurately summarize the dispersed information held by market participants.[17] These arguments hold in the benchmark case in the current model. However, in the world with overconfident traders, Corollary 2 illustrates the possibility that market prices still involve a noise in large markets, and, when the extent of overconfidence is severe, these prices even become noisier with additional investors. Even though it could be argued that every economic reasoning trivially breaks down with arbitrarily irrational players, overconfidence is thought of as a common form of irrationality which is empirically relevant. The lack of robustness of these properties to this particular deviation from the notion of Bayesian rationality undermines the argument that they persist in various economic environments in reality.

5 Endogenous information

In this section, we build and analyze an extension of the basic model where information is costly and endogenous. This extension is in parallel with Grossman and Stiglitz (1980), in the sense that strategic traders choose to costly acquire their private signals before the trading stage which corresponds to the basic model.

5.1 Model

As in the basic model, there are N strategic traders, noise traders, and competitive market makers in the market, and they trade a security whose value θ is not initially known and follows $N(\theta_0, \sigma_0^2)$. At the first stage of the model, each strategic trader $i \in \{1, \cdots, N\}$ simultaneously decides whether to acquire a private signal s_i with exogenous cost c that is common to all traders. Also, the number of strategic traders who chose to acquire their signals is publicly observed immediately after this stage.

[16] With regard to information aggregation in large markets, the benchmark case is a special case of the main result of Lambert, Ostrovsky and Panov (2018). Their paper describes a direct intuition behind this result as follows: Suppose that the price is not privately revealing in large markets so that a strategic trader's private information is still not fully incorporated into the price. Then each trader makes a nonnegligible profit in equilibrium, which implies that the sum of their expected profits goes to infinity. However, this cannot happen in any equilibrium because these profits must come from noise traders' random demand. This leads to a contradiction.

[17] This point could be interpreted as the promotion of competition in financial markets, as often argued informally. Indeed, in the current model, each strategic trader faces smaller market power, which corresponds to increased competition in reality, as the number of strategic traders N increases. However, it should also be noted that market size and competitiveness do not generally move together (e.g., Lee and Kyle, 2022; Rostek and Weretka, 2015).

The following stages are identical to the basic model: At the second stage, strategic and noise traders submit their demand to the market. Following the notation of the basic model, we denote by x_i each strategic trader i's demand, and by ω noise traders' demand, where ω follows $N(0, \sigma_\omega^2)$. At the third stage, the price is set by competitive market makers based on the total demand $X = \sum_{i=1}^N x_i + \omega$. At the final stage, the true asset value θ is realized, and each strategic trader i's profit is $\pi_i = x_i(\theta - P)$.

If a strategic trader i decides not to acquire his private signal at the first stage, he enters the second stage without any private information. In contrast, if he decides to acquire his signal, he observes $s_i = \theta + \epsilon_i$ at the second stage. All other assumptions are identical to the basic model, including the true distribution of signals held by market makers and how traders perceive their own signal and others' signals depending on three cases of benchmark, κ-overconfident and η-overconfident traders.

In what follows, we will call strategic traders simply "traders" for the sake of simplicity.

Definition 6. An equilibrium of the extended model consists of the number of traders who choose to be informed (i.e., M), informed and uninformed traders' demands (i.e., $(x_M^I(s_i), x_M^U(s_i))_{M=1}^N$ for each trader i), and the pricing rule (i.e., $(P_M(X))_{M=1}^N$) where the following three conditions hold:

1. At the third stage, the pricing rule set by market makers is equal to the expected value of the security conditional on X and M, i.e., $P_M(X) = E[\theta|X, M]$, where the conditional expectation is taken with respect to the true joint distribution of all variables θ, ω, and $(\epsilon_i)_{i=1}^N$, which generates the true distribution of the total demand X combined with traders' strategies $\{x_i(s_i)\}_{i=1}^N$;

2. At the second stage, for every realization of signal s_i, each trader i's demand $(x_M^I(s_i), x_M^U(s_i))_{M=1}^N$ maximize his expected profit depending on whether or not he is informed. Formally, it is a best response given the number of informed traders M, the profile of other traders' strategies $(x_M^I(s), x_M^U(s))$, the pricing rule $P_M(X)$, as well as his perceived joint distribution of all variables θ, ω, and $(\epsilon_j)_{j=1}^N$.

3. At the first stage, each trader i acquires a private signal s_i if and only if his expected net profit from obtaining the signal is higher than that from not doing so. Formally, it is a best response given others' strategies at the first stage (i.e., M) and later, as well as his perceived joint distribution of all variables θ, ω, and $(\epsilon_j)_{j=1}^N$.

The above definition extends Definition 1 to the extended model, maintaining its details about each trader's and market makers' beliefs generate their expectations about all relevant variables. Though symmetry is imposed on the concept of equilibrium, it does not hurt generality because the class of information structures is symmetric. It allows us to prevent redundancy arising from the presence of many equilibria which are symmetric to each other at the first stage.

Finally, as in the basic model, we restrict attention to the class of linear equilibria, as defined in Definition 2.

5.2 Solving for subgames

Following backward induction, we first solve for subgames starting from the second stage, and then, proceed to analyze how many traders choose to be informed at the second stage. As is clear in Lemma 1 below, a subgame is characterized by the number of informed traders M where $M \in [1, N]$ since it is equivalent to the basic model with M strategic traders. In line with basic model, we restrict attention to the cases where a unique equilibrium exists, which only rule out the case of κ-overconfidence with correlated errors and too many informed traders $M > \bar{N}_K$, where $\bar{N}_K$ is defined in Proposition 1. The following definition will be useful in what follows:

Definition 7. For a subgame with M informed traders, each informed trader's expected profit from the subgame is denoted by $U_R(M)$, $U_K(M)$, and $U_E(M)$, for the benchmark case, the case of κ-overconfidence, and the case of η-overconfidence, respectively.

The lemma provides a characterization of equilibrium of subgames in all three cases.

Lemma 1. *In all three cases (i.e., benchmark, κ-overconfidence, and η-overconfidence), each subgame has a unique equilibrium where uninformed traders submit zero demand (i.e., $x_M^U = 0$), while informed traders submit the same demand as they would submit in the basic model with M traders in the corresponding case, as stated in Proposition* 1*. Also, the following statements hold true:*

(a) *In the benchmark case and the case of η-overconfidence, each informed trader's expected profit (i.e., $U_R(M)$ and $U_E(M)$) decreases with M, and converges to zero as $M \to \infty$;*

(b) *In the case of κ-overconfidence with $\rho = 0$, each informed trader's expected profit (i.e., $U_K(M)$) has the same properties as in statement (a), whereas, in that case with $\rho > 0$, each informed trader's expected profit (i.e., $U_K(M)$) is U-shaped with M, and goes to infinity as $M \to \bar{N}_K$;*

(c) *The expected profit for each informed trader is always higher in the case of κ-overconfidence compared with the benchmark case. That is, $U_R(M) < U_K(M)$ for every $M \in [1, \bar{N}_K)$.*

The above lemma verifies the argument that a subgame with M informed traders is equivalent to the basic model with the same number of traders in the same case, as uninformed traders in the subgame behave as if they are "inactive". In this sense, the total number of traders N is irrelevant for what occurs in the subgame.

The proposition indicates that the properties of informed traders' expected profit with respect to the number of informed traders M differ between the benchmark case and the case of κ-overconfidence. In the benchmark case, it decreases to zero as the number of informed traders increases to infinity. This is consistent with the argument that there is a strategic substitutability in information acquisition across traders, as seen in Grossman and Stiglitz (1980). In contrast, statement (b) indicates that these properties with respect to the number of informed traders do not necessarily persist under κ-overconfidence. In line with the basic model (Subsection 4.2), the U-shaped curve of the expected profit for each informed trader follows from two opposing effects that arise as more κ-overconfident traders choose to be informed: (i) each existing trader's trading opportunity from noise trade decreases due to increased trading volume of other informed traders; (ii) each existing trader's perceived trading opportunity from other informed traders increases due to the interaction effect between κ-overconfidence and the number of informed traders noted in the basic model. Consistent with the basic model, the proposition shows that the second effect dominates the first one as the number of informed traders M is sufficiently large. This implies complementarity in trading incentive among traders in that each trader expects to earn more from choosing to be informed when more traders choose to be informed.

On the other hand, in the case of η-overconfident traders, the second effect described above is reversed: As more η-overconfident traders choose to be informed, each existing informed trader believes that the price has a higher covariance with his own signal, meaning that he updates his expectation about the price even more in response to a change in his signal. This effect further decreases his expected profit from the subgame, reinforcing the first effect described above.

Finally, statement (c) shows that κ-overconfidence increases each informed trader's expected profit given the number of informed traders. This result is intuitive, as his expected profit increases with what he believes is the precision of his own signal, which in turn increases with κ-overconfidence. The formal proof in the Appendix is complicated by strategic interaction among informed traders, which turns out not to reverse the aforementioned intuition. This statement is the main driving force leading to the analyses on the effects of overconfidence in what follows.

5.3 Information acquisition in equilibrium

Now we proceed to analyze information acquisition at the first stage. At the first stage, each trader chooses to be informed if and only if his expected profit from the following trading stage is higher than the cost of information c. As is implied by Lemma 1, only the case of κ-overconfident traders is qualitatively different from the benchmark case in terms of informed traders' expected profit. In other cases, the standard properties of information acquisition that hold in the benchmark case continue to hold. Therefore, we hereafter focus on the comparison between the benchmark case and the case of κ-overconfidence with correlated errors.

In the benchmark case, the following proposition provides a characterization of traders' behavior and price informativeness in equilibrium:

Proposition 4. *Consider the benchmark case.*

If $c \in (0, U_R(1))$, then there exists a unique equilibrium. In this equilibrium, there exists $N_R^*(c) \in (1, \infty)$ such that the following statements hold true: (a) *If $N \in [1, N_R^*(c))$, then all traders choose to be informed at the first stage;* (b) *If $N > N_R^*(c)$, then only $N_R^*(c)$ traders choose to be informed at the first stage.*

If $c > U_R(1)$, then there exists a unique equilibrium where all traders choose not to be informed at the first stage.

In equilibrium, the number of informed traders is uniquely determined at the first stage. If the total number of traders is small, all traders choose to be informed at the first stage. However, as it exceeds the limit denoted by $N_R^*(c)$, the number of informed traders remains constant at $N_R^*(c)$ and the rest of them choose to be uninformed. Even though price informativeness would increase toward its maximum *if all traders were informed*, the endogeneity of information prevents the outcome of all traders choosing to be informed, thereby undermining information aggregation. Overall, the above proposition reiterates the idea that endogeneity of information prevents information aggregation.[18]

In contrast, in the case of κ-overconfidence with correlated signal errors, the following proposition suggests that their behavior and the properties of the price are qualitatively distinct:

Proposition 5. *Consider the case of κ-overconfidence with correlated signal errors $\rho > 0$. Denote by M_K^* the number of traders minimizing informed traders' expected profit as shown in Lemma 1.*

If $c < U_K(M_K^*)$, there is a unique equilibrium where all N traders choose to be informed at the first stage.

If $c \in (U_K(M_K^*), U_K(1))$, then there exist $N_K^*(c)$ and $N_K^{**}(c)$ such that $1 < N_K^*(c) < N_K^{**}(c) < \bar{N}_K$ and the following statements hold true: (a) *If $N \in [1, N_K^*(c))$, there is a unique equilibrium where all N traders choose to be informed at the first stage;* (b) *If $N \in (N_K^*(c), N_K^{**}(c))$, there is a unique equilibrium where only $N_K^*(c)$ traders choose to be informed at the first stage;* (c) *If $N \in (N_K^{**}(c), \bar{N}_K)$, there are two equilibria: In one equilibrium, only $N_K^*(c)$ traders choose to be informed at the first stage, whereas, in the other equilibrium, all N traders choose to be informed at the first stage.*

Finally, it holds that $N_K^*(c) > N_R^*(c)$ for every $c \in (U_K(M_K^*), U_K(1))$. This implies that κ-overconfidence weakly increases the number of informed traders in both possible equilibria described above.

Throughout the range considered in the proposition (i.e., $N \in [1, \bar{N}_K)$), there are two different equilibria in information acquisition: One equilibrium exists for all N in this range, where no more than $N_K^*(c)$ traders choose to be informed. This feature arises from the left side of the U-shaped curve of κ-overconfident informed traders' expected profit in Lemma 1. While this equilibrium is similar to the benchmark case at this point, its upper bound on the number of informed traders (i.e., $N_K^*(c)$) is higher than that in the benchmark case (i.e., $N_R^*(c)$). This implies that κ-overconfidence weakly increases the number of informed traders in this equilibrium. In contrast, the other equilibrium exists only with a sufficient number of traders (i.e., $N \in (N_K^{**}(c), \bar{N}_K)$), where all traders choose to be informed. This feature arises from the right side of the U-shaped curve of κ-overconfident informed

[18] Note that this point is related to but is still distinguished from the Grossman-Stiglitz paradox that perfectly informationally efficient markets would collapse as traders would not incur the costs of acquiring information (Grossman and Stiglitz, 1980). It refers to the limiting case where small noise leads to nonexistence of equilibrium due to the lack of incentive to acquire information, which leads to the lack of trade. The driving force behind this paradox is similar to the failure of information aggregation in large markets, which holds in their model as well, as increasing the number of traders is effectively the same as reducing noise. Also, though Grossman and Stiglitz (1980) consider a different trading mechanism allowing for the observation of prices, strategic substitutability of information acquisition, which causes the failure of information acquisition, appears to hold more broadly than the particular mechanism considered by Grossman and Stiglitz.

traders' expected profit in Lemma 1, and it is qualitatively distinct from the unique equilibrium in the benchmark case. The number of informed traders equals the number of all traders, which is trivially higher than that in the benchmark case in this equilibrium.

It is worth commenting on the range of the number of traders which is not considered in the proposition. If the number of traders is so large that it exceeds the range (i.e., $N \geq \bar{N}_K$), the absence of equilibrium in subgames complicates the interpretation of the formal analyses. In particular, Lemma 1 implies that there is no equilibrium for subgames with more than $\bar{N}_K$ informed traders, and thus, there cannot be any equilibrium where these subgames are realized. Accordingly, the "new" equilibrium does not survive, whereas the other equilibrium still persists. However, note that such no-equilibrium outcome is due to infinite trading by κ-overconfident traders, which appears to be an artifact of risk neutrality and is not robust to a slight modification of preferences, as noted in Subsection 4.1. Therefore, we refrain from interpreting the absence of the new equilibrium entailing large volume as a decrease in trading volume on average.

This sort of equilibrium multiplicity reminds us of Mondria, Vives and Yang (2021). They consider a model in which investors cannot costlessly process information from asset prices. It then naturally follows that investors optimally choose sophistication levels by balancing the benefit of beating the market against the cost of acquiring sophistication. They show that there can exist strategic complementarities in the choice of sophistication levels, leading to multiple equilibria. Compared with their results, the proposition identifies another potential channel, whereby equilibrium multiplicity is caused by overconfidence in financial markets. This channel is distinguished from that of Mondria, Vives and Yang in that it does not require the notion of sophistication in processing information and instead combines two notions of correlation in errors and overconfidence, which are, to the best of my knowledge, addressed separately in the literature as reviewed in Section 2, to raise the possibility of multiple equilibria.

5.4 Effects of overconfidence on trading volume and price informativeness

κ-overconfidence has two effects on trading volume and price informativeness: (i) increasing trading volume and price informativeness given the number of informed traders, as implied by Proposition 2 and Corollary 1, respectively; (ii) (weakly) increasing the number of informed traders, as implied by Proposition 5. Both of these effects lead to increases in trading volume and price informativeness in both equilibria which possibly occur with κ-overconfidence. Overall, the endogeneity of information causes additional indirect effects of κ-overconfidence on both trading volume and price informativeness through an increase in the number of informed traders, but such effects do not reverse the results that hold in the basic model (i.e., Proposition 2 and Corollary 1) because they operate in the same direction with its direct effects on trading volume and price informativeness given the number of informed traders.

In parallel with Propositions 3 and Corollary 2 in the basic model, the following corollary concerns how the properties of trading volume and price informativeness with respect to the number of traders are changed with κ-overconfidence.

Corollary 3. *Fix $c \in (U_K(M_K^*), U_K(1))$, where M_K^* is defined in Proposition 5. With $N_R^*(c)$, $N_K^*(c)$ and $N_K^{**}(c)$ defined in Propositions 4 and 5, the following statements hold true:*

(1) *In the benchmark case, trading volume and price informativeness increase with N for $N \in [1, N_R^*(c))$, but then stay constant for $N > N_R^*(c)$.*

(2a) *In the case of κ-overconfidence, one equilibrium exists for $N \in [1, \bar{N}_K)$, and in this equilibrium, trading volume and price informativeness increase with N for $N \in [1, N_K^*(c))$, but then stay constant for $N \in (N_K^*(c), \bar{N}_K)$.*

(2b) *In the other equilibrium which exists for $N \in (N_K^{**}, \bar{N}_K)$ in the case of κ-overconfidence, trading volume and price informativeness increase with N throughout the range of N. As $N \to \bar{N}_K$, trading volume goes to infinity and price informativeness approaches its maximum.*

Statement (1) regarding the benchmark case follows from combining Proposition 3 (for trading volume) and Corollary 2 (for price informativeness) with Proposition 4. In particular, if all traders were informed, Proposition 3 and Corollary 2 would apply. However, Proposition 4 implies that as the number of traders increases above N_R^*, only N_R^* traders choose to be informed, thus imposing an upper bound in trading volume and price informativeness even with an increase in the number of traders. This is consistent with the idea that the endogeneity of information prevents information aggregation, as noted in Section 2.

Statements (2a) and (2b) consider two different possible equilibria in the case of κ-overconfident traders with correlated errors. The equilibrium in statement (2a) resembles the benchmark case in that the number of informed traders stays at N_K^* as the number of traders increases beyond N_K^*, and thus, trading volume and price informativeness are bounded above as in the benchmark case. However, in the other equilibrium in statement (2b), which occurs with a sufficient number of traders (i.e., $N \in (N_K^{**}, \bar{N}_K)$), all traders choose to be informed. Therefore, Proposition 3 and Corollary 2 directly apply so that trading volume and price informativeness increase with the number of traders and they converge toward infinity and its maximum, respectively, as the number of traders approaches $\bar{N}_K$.

The equilibrium in statement (2b) which distinctly occurs in the case of κ-overconfident traders tells us that the standard argument that endogenous information prevents information aggregation hinges on the assumption of common belief about the asset value (e.g., the benchmark case), and it may not apply otherwise. It is notable that such equilibrium occurs even with an arbitrarily small extent of κ-overconfidence, indicating that the validity of the mechanism is sensitive to a slight deviation from Bayesian rationality. Overall, information aggregation in markets can be explained by a large number of κ-overconfident traders with correlated signal errors, even when information is costly and endogenous.

6 Concluding remarks

The main objective of this study is to analyze the implications of overconfidence in financial markets. Two important features of the model are that strategic traders are overconfident in the relative precision of their signals in two possible ways (i.e., overconfidence in own signals and underconfidence in others' signals) and that their imperfect signals are correlated with each other with a common correlation coefficient. The analyses suggest that the direction of implications of overconfidence on trading volume and price informativeness differs radically across two different types of overconfidence. Given that these two types of overconfidence are likely to coexist in reality, one could argue that the implications of overconfidence depend on which type of overconfidence prevails in markets. As noted in Subsection 4.2, this is indeed consistent with the mixed empirical and experimental results on the relationship between overconfidence and trading volume (e.g., Biais, Hilton, Mazurier, and Pouget, 2005; Fellner-Rohling and Krugel, 2014; Glaser and Weber, 2007: Merkle, 2017).

It is noteworthy that the implications of overconfidence are complicated further by the observation that the degree of overconfidence is only moderate on average and even negative in many cases. Indeed, the degree of overconfidence varies across a lot of individual-specific and time-specific factors, such as working experience and ambiguity (e.g., Gloede and Menkhoff, 2014; Yang and Zhu, 2016). Though we do not formally analyze the effects of "negative" overconfidence here, these effects appear to be symmetric to those of overconfidence of the corresponding type. For example, the results in Subsection 4.2 immediately imply that it does not necessarily decrease trading volume, depending on its type, whereas the results in Subsection 4.3 raise a possibility of the price becoming noisier with the number of traders. On the theoretical side, the results presented in this study could be regarded as a rich set of possible outcomes arising from a reasonable range of forms of disagreements over the structure of information possibly observed in reality, rather than being specific to overconfidence. On the empirical viewpoint, these illustrate a difficulty in explaining the pattern of trading volume and price informativeness with overconfidence, as it is rather hard to observe the form of investors' beliefs about information structures.

Finally, we discuss the robustness of the main results to alternative trading mechanisms. In the basic model and its variant considered in this study, traders submit market orders conditional only on their own private information. This particular trading mechanism abstracts from learning from the price, thereby allowing for tractable analyses even with complex information structures involving overconfidence and correlated signal errors.

Nevertheless, it is worth mentioning that the possibility of learning from the price featured in price-contingent trading mechanisms (e.g., Grossman and Stiglitz, 1980; Kyle, 1989) could influence traders' perceived trading opportunities through their inferences about the asset value and the price, accordingly providing different implications of overconfidence in the presence of correlated errors. In fact, the specific intuition in the current model is shut down under price-contingent trading mechanisms where each trader directly observes the price. However, he still faces a different type of inference about the asset value based on his private information and the price, which is reflected on his perceived trading opportunity. Accordingly, in line with our model with market orders, the two types of overconfidence change each strategic trader's perceived covariances between the asset value, the price, and traders' private signals through his inferences about the asset value and others' signal errors. As in our model with market orders, the effects of overconfidence on these perceived covariances are generally unclear, potentially leading to a non-obvious answer on the relationship between overconfidence and trading volume. Moreover, our conclusion on information acquisition of overconfident traders is expected to be robust to the details of trading mechanism including the observation of price. Even if each trader observes the price, he still perceives the price to be noisier as the market grows large, thereby incentivizing him to acquire his private information, as in Lemma 1 and Proposition 5. It is a possible avenue for future research to verify these speculations regarding the effects of overconfidence and correlated information structures under price-contingent trading mechanisms.

Appendix

Proof of Proposition 1

Suppose that every strategic trader i follows a linear strategy given by $x_i = \alpha + \beta(s_i - \theta_0)$. As in the main text of the paper, this strategy profile constitutes an equilibrium if two conditions in Definitions 1 and 2 are satisfied. We first show that the first condition corresponds to a linear price with slope given by Equation (1), and then, proceed to obtain each trader i's best response, which corresponds to the second condition. Finally, we prove the proposition by solving the fixed point of the best response.

We first show that the pricing rule is given by $P(X) = \theta_0 + \lambda(X - N\alpha)$, where λ is given by Equation (1). Note that market makers believe that total demand is given by $X = N\alpha + \beta\sum_{j=1}^{N}(s_j - \theta_0) + \omega$, which implies

$$\frac{X - N\alpha}{N\beta} = \frac{1}{N}\sum_{j=1}^{N}(s_j - \theta_0) + \omega = \theta - \theta_0 + \frac{1}{N}\sum_{j=1}^{N}\epsilon_j + \frac{\omega}{N\beta}.$$

Thus, market makers regard $\theta_0 + \frac{X-N\alpha}{N\beta}$ as an unbiased signal on the asset value with error variance given by

$$\begin{aligned} Var\left(\frac{X - N\alpha}{N\beta}|\theta\right) = Var\left[\frac{1}{N}\sum_{j=1}^{n}\epsilon_i + \frac{\omega}{N\beta}\right] &= \frac{1}{N^2}Var\left[\sum_{j=1}^{N}\epsilon_j\right] + \frac{\sigma_\omega^2}{N^2\beta^2} \\ &= \frac{1}{N^2}\left\{N\sigma_\epsilon^2 + N(N-1)\rho\sigma_\epsilon^2\right\} + \frac{\sigma_\omega^2}{N^2\beta^2} \\ &= \frac{1+\rho(N-1)}{N}\sigma_\epsilon^2 + \frac{\sigma_\omega^2}{N^2\beta^2}. \end{aligned} \tag{6}$$

The price is then Bayesian updated by market makers as follows:

$$P(X) = E[\theta|X] = \theta_0 + \gamma\left(\frac{X - N\alpha}{N\beta}\right),$$

where updating weight γ is obtained by the Projection Theorem as follows:

$$\begin{aligned} \gamma &= \frac{Cov\left(\theta, \theta_0 + \frac{X-N\alpha}{N\beta}\right)}{Var\left(\theta_0 + \frac{X-N\alpha}{N\beta}\right)} \\ &= \frac{Cov(\theta,\theta)}{Var(\theta) + Var\left(\frac{1}{N}\sum_{j=1}^{n}\epsilon_i + \frac{\omega}{N\beta}\right)} = \frac{\sigma_0^2}{\sigma_0^2 + \frac{1+\rho(N-1)}{N}\sigma_\epsilon^2 + \frac{\sigma_\omega^2}{N^2\beta^2}}. \end{aligned} \tag{7}$$

This implies that slope $\lambda := \frac{\gamma}{N\beta}$ is given by Equation (1).

Now we proceed to obtain each trader i's best response given others' strategy described by trading coefficient β, where all other traders j submit $\beta(s_j - \theta_0)$). With his correct belief on the pricing rule $P(X)$, we have $P_i\left(x_i, (s_j)_{j\in\{1\cdots,N\}\setminus\{i\}}, \omega\right) = \theta_0 + \lambda(x_i + T_i + (N-1)\alpha + \omega - N\alpha)$, where $T_i := \sum_{j\neq i}(x_j - \alpha) = \beta\sum_{j\neq i}(s_j - \theta_0)$, as in Equation (2) in the main text. Consistent with Equation (3) in the main text, trader i's expected profit is

$$\begin{aligned} E[\pi_i|\mathcal{I}_i] &= E\left[x_i\left\{\theta - P_i\left(x_i, (s_j)_{j\in\{1\cdots,N\}\setminus\{i\}}, \omega\right)\right\}|\mathcal{I}_i\right] \\ &= x_iE\left[\theta - \theta_0|\mathcal{I}_i\right] - x_i\lambda E\left[T_i|\mathcal{I}_i\right] + x_i\lambda\alpha - \lambda x_i^2. \end{aligned}$$

By solving the first-order condition, trader i's optimal demand is given by

$$\begin{aligned} x_i^* &= \frac{1}{2\lambda}\left\{E\left[\theta - \theta_0|\mathcal{I}_i\right] - \lambda E\left[T_i|\mathcal{I}_i\right] + \lambda\alpha\right\}. \\ &\equiv \alpha_i + \beta_i(s_i - \theta_0). \end{aligned}$$

Note that the expectation terms (i.e., $E[\theta-\theta_0|\mathcal{I}_i]$ and $E[T_i|\mathcal{I}_i] = E\left[\beta\sum_{j\neq i}(s_j-\theta_0)|\mathcal{I}_i\right]$ are proportional to $s_i-\theta_0$, as is clear in Equations (8) and (9) below. This implies that only the last term (i.e., $\lambda\alpha$) is constant. Thus, we have $\alpha_i = \frac{\alpha}{2}$, which implies $\alpha = 0$ in equilibrium. Note that, as each trader's signal is decomposed into the asset value and his signal error, T_i is decomposed into two terms, i.e.,

$$\begin{aligned} T_i &= \sum_{j\neq i}\beta(s_j-\theta_0) \\ &= \sum_{j\neq i}\beta(\theta-\theta_0) + \sum_{j\neq i}\beta\epsilon_j := T_{i\theta} + T_{i\epsilon}, \end{aligned}$$

where $T_{i\theta}$ and $T_{i\epsilon}$ represent the true-value component of other traders' signals and the error component of these signals, respectively. Plugging this into trader i's optimal demand above, we can see that an equilibrium is formed at the fixed point of each trader i's best response function given by

$$B_i(\beta) = \frac{1}{2\lambda}E[\theta-\theta_0|\mathcal{I}_i] - \frac{1}{2}\beta(N-1)E[\theta-\theta_0|\mathcal{I}_i] - \frac{1}{2}\beta(N-1)E[\epsilon_j|\mathcal{I}_i]. \tag{8}$$

Finally, we prove the proposition by calculating the expectation terms in Equation (8) and then solving for the fixed point of Equation (8). To prevent redundancies, we consider the "combined" case nesting all three cases. In particular, suppose that each strategic trader believes that his own signal error has variance $\frac{\sigma_\epsilon^2}{\kappa}$, and other traders' signal errors have variance $\frac{\sigma_\epsilon^2}{\eta}$. By the Projection Theorem, the expectation terms in Equation (8) are given by

$$E[\theta-\theta_0|\mathcal{I}_i] = \frac{Cov(\epsilon_j,s_i)}{Var(s_i)}(s_i-\theta_0) = \frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}}(s_i-\theta_0); \tag{9}$$

$$E[\epsilon_j|\mathcal{I}_i] = \frac{Cov(\epsilon_j,s_i)}{Var(s_i)}(s_i-\theta_0) = \frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\epsilon^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}}(s_i-\theta_0). \tag{10}$$

Applying these into Equation (8), we have

$$\begin{aligned} x_i^* &= \frac{1}{2\lambda}\left[\{1-\lambda\beta(N-1)\}E[\theta-\theta_0|\mathcal{I}_i] - \lambda\beta(N-1)E[\epsilon_j|\mathcal{I}_i]\right] \\ &= \frac{1}{2\lambda}\left[\{1-\lambda\beta(N-1)\}\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} - \lambda\beta(N-1)\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\epsilon^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}}\right](s_i-\theta_0) \\ &\equiv \beta_i(s_i-\theta_0). \end{aligned}$$

This implies

$$\begin{aligned} \beta_i &= \frac{1}{2\lambda}\left[\{1-\lambda\beta(N-1)\}\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} - \lambda\beta(N-1)\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\epsilon^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}}\right] \\ &= \left\{\frac{1}{2\lambda} - \frac{\beta(N-1)}{2}\right\}\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} - \frac{\beta(N-1)}{2}\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\epsilon^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} \\ &= \left\{\frac{N\beta}{2}\frac{\sigma_0^2+\frac{1+\rho(N-1)}{N}\sigma_\epsilon^2+\frac{\sigma_\omega^2}{N^2\beta^2}}{\sigma_0^2} - \frac{\beta(N-1)}{2}\right\}\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} - \frac{\beta(N-1)}{2}\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\epsilon^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} \\ &= \left[\frac{\beta}{2} + \frac{\beta}{2}\{1+\rho(N-1)\}\frac{\sigma_\epsilon^2}{\sigma_0^2} + \frac{1}{2}\frac{\sigma_\omega^2}{N\beta\sigma_0^2}\right]\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} - \frac{\beta(N-1)}{2}\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\epsilon^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} \\ &= \frac{\beta}{2}\frac{\sigma_0^2+\sigma_\epsilon^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}} + \frac{\beta}{2}(N-1)\rho\frac{\sigma_\epsilon^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}}\left(1-\frac{1}{\sqrt{\kappa\eta}}\right) + \frac{1}{2N\beta}\frac{\sigma_\omega^2}{\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}}. \end{aligned} \tag{11}$$

At the fixed point, we have $\beta_i = \beta$. Applying this into Equation (11) and then arranging the terms with respect to β, we have

$$\beta^2 = \frac{\sigma_\omega^2}{N} \frac{1}{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(N-1)\sigma_\epsilon^2 \left(1 - \frac{1}{\sqrt{\kappa\eta}}\right)}.$$

We then determine the unique trading coefficient β^* satisfying the equilibrium conditions (if it exists) as follows:

$$\beta^* = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(N-1)\left(1 - \frac{1}{\sqrt{\kappa\eta}}\right)\sigma_\epsilon^2\right\}}}. \tag{12}$$

Note that the above equation nests all three cases considered in the proposition. The cases of rational, κ-overconfident, and η-overconfident traders correspond to (i) $\kappa = \eta = 1$, (ii) $\kappa > 1$ and $\eta = 1$, and (iii) $\kappa = 1$ and $\eta \in (0, 1)$, respectively. The existence of equilibrium is non-trivial only in the second case of κ-overconfident traders with correlated signal errors: If $\rho > 0$, Equation (12) implies that there exists such β^* satisfying the equilibrium conditions if and only if

$$N < \bar{N}_K := 1 + \frac{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2}{\rho\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2},$$

as stated in the proposition.

Proof of Proposition 2

For the sake of convenience later, denote by $TV_R(\rho, N)$, $TV_K(\rho, N)$, and $TV_E(\rho, N)$ trading volume in the cases of benchmark, κ-overconfident, and η-overconfident traders, respectively. To prove the first statement of the proposition, we will show that if $N \geq 2$, we have

$$TV_R(0, N) = TV_R(\rho, N) < TV_K(0, N) < TV_K(\rho, N)$$

for $\rho > 0$. Also, the second statement corresponds to

$$TV_E(\rho, N) < TV_E(0, N) = TV_R(0, N) = TV_R(\rho, N),$$

for $\rho > 0$. Note first that given the number of traders N, trading volume is generally proportional to trading coefficient β^* in equilibrium, which is described in Proposition 1. Therefore, both statements in the proposition are easily shown by comparing trading coefficients β^* among the cases of benchmark, κ-overconfident, and η-overconfident traders

In the benchmark case, β_R^* does not depend on ρ as seen in Proposition 1. That is, $TV_R(0, N) = TV_R(\rho, N)$ for $\rho > 0$. In the case of κ-overconfident traders with $\rho = 0$, we have

$$\beta_K^*(0, N) = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2\right\}}},$$

which is larger than β_R^* as defined in Proposition 1. It follows that $TV_K(0, N) > TV_R(0, N)$. In the case of κ-overconfident traders with $\rho > 0$, we have

$$\beta_K^*(\rho, N) = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(N-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}}},$$

which is larger than $\beta_K^*(0, N)$ as above. This implies that $TV_K(\rho, N) > TV_K(0, N)$.

In the case of η-overconfident traders with $\rho = 0$, $\beta_E^*(0, N)$ is the same as $\beta_R^*(0, N)$ as above by Proposition 1. This yields $TV_E(0, N) = TV_R(0, N)$. Finally, in the case of η-overconfident traders with $\rho > 0$, Proposition 1 implies that

$$\beta_E^*(\rho, N) = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \sigma_\epsilon^2 + \rho(N-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2\right\}}},$$

which is smaller than $\beta_E^*(0, N)$. This implies $TV_E(\rho, N) < TV_E(0, N)$.

Proof of Propositions 3

Note that TV is proportional to the number of traders times their trading coefficient (i.e., $N\beta^*$) in equilibrium where $\beta^* = \beta_R^*$ in the benchmark case, $\beta^* = \beta_K^*$ in the case of κ-overconfident traders, and $\beta^* = \beta_E^*$ in the case of η-overconfident traders. All statements in the proposition follow from the properties of $N\beta^*$ with respect to N. The only non-trivial statement among them is that in the case of η-overconfident traders with $\rho > 0$, trading volume increases with N for $\eta \in \left(\left(\frac{\rho\sigma_\epsilon^2}{\sigma_0^2+\sigma_\epsilon^2+\rho\sigma_\epsilon^2}\right)^2, 1\right)$, whereas it decreases with N for $\eta \in \left(0, \left(\frac{\rho\sigma_\epsilon^2}{\sigma_0^2+\sigma_\epsilon^2+\rho\sigma_\epsilon^2}\right)^2\right)$. Applying Proposition 1, and then arranging the terms, we have

$$\begin{aligned} N\beta_E^* &= \sqrt{\frac{N\sigma_\omega^2}{\sigma_0^2 + \sigma_\epsilon^2 + \rho(N-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2}}, \\ &= \sqrt{\frac{\sigma_\omega^2}{\rho\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2 + \frac{1}{N}\left\{\sigma_0^2 + \sigma_\epsilon^2 - \rho\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2\right\}}}. \end{aligned}$$

For $\eta \in \left(\left(\frac{\rho\sigma_\epsilon^2}{\sigma_0^2+\sigma_\epsilon^2+\rho\sigma_\epsilon^2}\right)^2, 1\right)$, it is easy to see that $\sigma_0^2 + \sigma_\epsilon^2 - \rho\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2 > 0$, which implies that trading volume, which is equivalent to $N\beta_E^*$, increases with N. Otherwise, we have $\sigma_0^2 + \sigma_\epsilon^2 - \rho\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2 < 0$ so that trading volume decreases with N.

Proof of Corollary 1

Recall from the proof of Proposition 1 that the pricing rule is represented by

$$P(X) = \theta_0 + \lambda X,$$

where λ is given by Equation (1). Thus, knowing the price is equivalent to knowing the total demand X. Thus, $\frac{X}{N\beta} = \theta - \theta_0 + \frac{1}{N}\sum_{j=1}^N \epsilon_j + \frac{\omega}{N\beta}$ is regarded as an unbiased signal about θ. Applying the standard Bayesian updating formula for normal distribution into the definition of price informativeness, we have

$$\begin{aligned} PI &= \{Var(\theta|P)\}^{-1} = \left\{Var\left(\theta|\frac{X}{N\beta}\right)\right\}^{-1}. \\ &= \left[\frac{1}{\sigma_0^{-2} + \left\{Var\left(\frac{X}{N\beta}|\theta\right)\right\}^{-1}}\right]^{-1} = \sigma_0^{-2} + \left\{Var\left(\frac{X}{N\beta}|\theta\right)\right\}^{-1} \\ &= \sigma_0^{-2} + \left\{\frac{1+\rho(N-1)}{N}\sigma_\epsilon^2 + \frac{\sigma_\omega^2}{N^2\beta^2}\right\}^{-1}, \end{aligned} \tag{13}$$

where the last line follows from Equation (6) combined with $\alpha = 0$ in the proof of Proposition 1. This implies that PI increases with trading coefficient β^* in equilibrium, immediately implying all statements in the corollary.

Proof of Corollary 2

We first prove the following claim, and then proceed to prove the proposition.

Claim 1. It holds that

$$PI^* = \sigma_0^{-2} + \left\{ \frac{1 + \rho(N-1)}{N} \sigma_\epsilon^2 \right\}^{-1}.$$

Proof. By the definition of $PI^* = \{Var(\theta|s_1, \cdots, s_N)\}^{-1}$, it suffices to show that

$$Var(\theta|s_1, \cdots, s_N) = \frac{\sigma_0^2 \frac{1+\rho(N-1)}{N} \sigma_\epsilon^2}{\sigma_0^2 + \frac{1+\rho(N-1)}{N} \sigma_\epsilon^2}.$$

To show this, define $s = (s_1, \cdots, s_N)'$. Its variance-covariance matrix is given by

$$[Var(s)]_{ii} = \sigma_0^2 + \sigma_\epsilon^2; \;\; [Var(s)]_{ij} = \sigma_0^2 + \rho\sigma_\epsilon^2 \text{ for every } i \neq j.$$

Applying the formula for conditional variance of multivariate normal distribution, we have

$$Var(\theta|s_1, \cdots, s_N) = Var(\theta) - Cov(\theta, s) Var(s)^{-1} Cov(s, \theta).$$

By straightforward calculation, it holds that $Cov(\theta, s) = (\sigma_0^2, \cdots, \sigma_0^2)$, $Cov(\theta, s) = (\sigma_0^2, \cdots, \sigma_0^2)'$, and $Var(s)^{-1}$ is given by

$$\left[Var(s)^{-1}\right]_{ii} = \frac{1}{(1-\rho)\sigma_\epsilon^2} - \frac{1}{C(1-\rho)^2(\sigma_\epsilon^2)^2}; \;\; \left[Var(s)^{-1}\right]_{ij} = -\frac{1}{C(1-\rho)^2(\sigma_\epsilon^2)^2} \text{ for every } i \neq j,$$

where $C := (\sigma_0^2 + \sigma_\epsilon^2)^{-1} + N\left\{(1-\rho)\sigma_\epsilon^2\right\}^{-1}$. Plugging these into the above equation, we get the conditional variance $Var(\theta|s_1, \cdots, s_N)$ as in the claim.[19] □

To prove the proposition, first consider the benchmark case and the cases of κ- and η-overconfident traders with $\rho = 0$. In all these cases, Proposition 3 implies that $N\beta^*$ increases with N, and converges to infinity as $N \to \infty$. Applying the former to Equation (13), we can easily see that PI increases with N. Applying the latter to Equation (13), we have, as $N \to \infty$,

$$PI \to \sigma_0^{-2} + \left\{ \frac{1 + \rho(N-1)}{N} \sigma_\epsilon^2 \right\}^{-1},$$

which equals PI^* by Claim 1.

Next we consider the case of κ-overconfident traders with $\rho > 0$. By Proposition 3, $N\beta_K^*$ increases with N, and converges to infinity as $N \to \bar{N}_K$. Applying the former to Equation (13), PI increases with N. Applying the latter to Equation (13), we have, as $N \to \infty$,

$$PI \to \sigma_0^{-2} + \left\{ \frac{1 + \rho(N-1)}{N} \sigma_\epsilon^2 \right\}^{-1} = PI^*.$$

Finally, consider the case of η-overconfident traders with $\rho > 0$. By Proposition 1, we have

$$N\beta_E^* = \sqrt{\frac{N\sigma_\omega^2}{\sigma_0^2 + \sigma_\epsilon^2 + \rho(N-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2}}.$$

[19] The detailed proof of this part is available upon request. It uses basic linear algebra but is tedious.

Plugging this into Equation (13), we have

$$
\begin{aligned}
PI &= \sigma_0^{-2} + \left\{ \frac{1+\rho(N-1)}{N}\sigma_\epsilon^2 + \frac{\sigma_\omega^2}{(N\beta_E^*)^2} \right\}^{-1} \\
&= \sigma_0^{-2} + \left\{ \rho\frac{\sigma_\epsilon^2}{\eta} + \frac{1}{N}\left(\sigma_0^2 + 2\sigma_\epsilon^2 - \rho\frac{\sigma_\epsilon^2}{\eta}\right) \right\}^{-1}. \qquad (14)
\end{aligned}
$$

If $\eta \in \left(\left(\frac{\rho\sigma_\epsilon^2}{\sigma_0^2+2\sigma_\epsilon^2}\right)^2, 1\right)$, then $\sigma_0^2 + 2\sigma_\epsilon^2 - \rho\frac{\sigma_\epsilon^2}{\eta} > 0$, which implies that PI increases with N, as stated in the proposition. Otherwise, we have $\sigma_0^2 + 2\sigma_\epsilon^2 - \rho\frac{\sigma_\epsilon^2}{\eta} > 0$, which implies that PI decreases with N, as stated in the proposition. Finally, for every $\eta \in (0,1)$, Equation (14) implies that PI does not converge to PI^* as $N \to \infty$.

Proof of Lemma 1

This proof is mostly parallel with that of Proposition 1, except that uninformed traders' demand is shown to be zero. As in the proof of Proposition 1, we consider the combined case where each strategic trader believes that his own signal error has variance $\frac{\sigma_\epsilon^2}{\kappa}$, and other traders' signal errors have variance $\frac{\sigma_\epsilon^2}{\eta}$. Consider a subgame with M informed traders involved in the extended model with N strategic traders. Without loss of generality, we rearrange strategic traders' indices so that traders $i \in \{1, \cdots, M\}$ are informed and other traders are uninformed.

First consider the pricing rule, which is given by $P_M(X) = \lambda_M\left(X - M\alpha_M^I - (N-M)\alpha_M^U\right)$. The total demand of these strategic traders and noise traders is given by

$$
X = M\alpha_M^I + \beta_M^I \sum_{i=1}^{M}(s_i - \theta_0) + (N-M)\alpha_M^U + \omega.
$$

This is normalized to an unbiased signal about θ as follows:

$$
\frac{X - M\alpha_M^I - (N-M)\alpha_M^U}{M\beta_M^I} = \theta - \theta_0 + \frac{1}{M}\sum_{i=1}^{M}\epsilon_i + \frac{\omega}{M\beta_M^I}.
$$

The price is then set by market makers as:

$$
\begin{aligned}
P_M(X) &= E[\theta|X, M] \\
&= \theta_0 + \gamma_M\left(\frac{X - M\alpha_M^I - (N-M)\alpha_M^U}{M\beta_M^I}\right), \qquad (15)
\end{aligned}
$$

where updating coefficient γ_M is given by

$$
\gamma_M = \frac{\sigma_0^2}{\sigma_0^2 + \frac{1+\rho(M-1)}{M}\sigma_\epsilon^2 + \frac{\sigma_\omega^2}{M^2(\beta_M^I)^2}},
$$

which is parallel with Equation (7). Therefore, we have $\lambda_M = \frac{\gamma_M}{M\beta_M^I}$.

Next, consider uninformed traders' demand (i.e., α_M^U). Each uninformed trader's expected profit is given by

$$
\begin{aligned}
E[\pi_U] &= E\left[x_u\left\{\theta - \theta_0 - \frac{\gamma}{M\beta_M^I}\left(X - M\alpha_M^I - (N-M)\alpha_M^U\right)\right\}\right] \\
&= x_u E\left[\theta - \theta_0 - \frac{\gamma}{M\beta_M^I}\left(-\alpha_M^U + \beta_M^I\sum_{i=1}^{M}(s_i - \theta_0)\right)\right] - \frac{\gamma}{M\beta_M^I}x_u^2 \\
&= \frac{\gamma}{M\beta_M^I}\alpha_M^U x_u - \frac{\gamma}{M\beta_M^I}x_u^2
\end{aligned}
$$

where the second equality follows from

$$X = x_U + (N - M - 1)\alpha_M^U + M\alpha_M^I + \beta_M^I \sum_{i=1}^{M}(s_i - \theta_0),$$

and the third equality holds because

$$E[\theta] = E[s_i] = \theta_0$$

for every informed trader $i \in \{1, \cdots, M\}$. Differentiating his expected profit with respected to x_U, we have the first-order condition as follows:

$$\frac{\gamma_M}{M\beta_M^I}\alpha_M^U - \frac{2\gamma_M}{M\beta_M^I}x_U = 0.$$

This yields

$$x_U^* \equiv \alpha_M^U = \frac{1}{2}\alpha_M^U,$$

which implies $\alpha_M^U = 0$. That is, uninformed traders demand zero in any equilibrium, as stated in the lemma.

Given that uninformed traders are "inactive" in equilibrium, this subgame is equivalent to the combined case of the basic model with M strategic traders, described in the proof of Proposition 1. Thus, $\alpha_M^I = 0$ holds, and β_M^I is given by Proposition 1, only replacing N by M. This holds for all three cases, as stated in the lemma.

Now we want to obtain each informed trader i's expected profit from each subgame (i.e., given M). By applying the pricing rule$P_M(X)$ which is given by Equation (15) to informed trader i's expected profit, we have

$$\begin{aligned}
E[\pi_i] &= E\left[\beta_M^I(s_i - \theta_0)\left\{\theta - \theta_0 - \frac{\gamma_M}{M\beta_M^I}\beta_M^I(s_i - \theta_0) - \frac{\gamma_M}{M\beta_M^I}\sum_{j\in\{1,\cdots,M\}\setminus\{i\}}\beta_M^I(s_j - \theta_0)\right\}\right] \\
&= E\left[\beta_M^I(s_i - \theta_0)\left\{\theta - \theta_0 - \frac{\gamma_M}{M}(s_i - \theta_0) - \frac{\gamma_M}{M}\sum_{j\in\{1,\cdots,M\}\setminus\{i\}}((\theta - \theta_0) + \epsilon_j)\right\}\right] \\
&= \beta_M^I\sigma_0^2 - \frac{\gamma_M\beta_M^I}{M}\left(\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\right) - \frac{\gamma_M\beta_M^I}{M}\sigma_0^2(M-1) - \frac{\gamma_M\beta_M^I}{M}\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\epsilon^2(M-1) \\
&= \sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa\eta}}\right)\sigma_\epsilon^2\right\}}}\left\{\sigma_0^2 - \frac{\gamma_M}{M}\left(M\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa} + (M-1)\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\epsilon^2\right)\right\} \\
&= \sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa\eta}}\right)\sigma_\epsilon^2\right\}}} \\
&\quad\times\left[\sigma_0^2 - \frac{\sigma_0^2}{2\left(\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\right) + (M-1)\left(\sigma_0^2 + \frac{\rho}{\sqrt{\kappa\eta}}\sigma_\epsilon^2\right)}\left(M\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa} + (M-1)\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\epsilon^2\right)\right] \\
&= \sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa\eta}}\right)\sigma_\epsilon^2\right\}}}\frac{\sigma_0^2\left(\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\right)}{2\left(\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\right) + (M-1)\left(\sigma_0^2 + \frac{\rho}{\sqrt{\kappa\eta}}\sigma_\epsilon^2\right)}, \qquad (16)
\end{aligned}$$

where the third equality follows from

$$E[(s_i - \theta_0)^2] = \sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}, E[(s_i - \theta_0)(\theta - \theta_0)] = \sigma_0^2, \text{ and } E[(s_i - \theta_0)\epsilon_j] = \frac{\rho}{\sqrt{\kappa\eta}}\sigma_\epsilon^2,$$

and the fifth equality holds by Equation (15).

Consider the benchmark case (i.e., $\kappa = \eta = 1$). Plugging these parameters into Equation (16), we have

$$U_R(M) = \sqrt{\frac{\sigma_\omega^2}{M\left(\sigma_0^2+\sigma_\epsilon^2\right)}}\frac{\sigma_0^2\left(\sigma_0^2+\sigma_\epsilon^2\right)}{2\left(\sigma_0^2+\sigma_\epsilon^2\right)+(M-1)\left(\sigma_0^2+\rho\sigma_\epsilon^2\right)}. \tag{17}$$

It is easy to see that it decreases with M, and approaches zero as M goes to infinity, as stated in the lemma.

In the case of κ-overconfident traders (i.e., $\kappa > 1$ and $\eta = 1$), Equation (16) implies that

$$U_K(M) = \sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\epsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}}}\frac{\sigma_0^2\left(\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}\right)}{2\left(\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2\right)}. \tag{18}$$

If $\rho = 0$, it is easy to see that $U_K(M)$ decreases with M, and converges to zero as M goes to infinity, as stated in the proposition. If $\rho > 0$, $U_K(M)$ is represented by

$$U_K(M) = \frac{\sqrt{\sigma_\omega^2}\sigma_0^2\left(\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}\right)}{g(M)},$$

where $g(M)$ is defined as

$$g(M) = \sqrt{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\epsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}}\left\{2\left(\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2\right)\right\}.$$

By noting that

$$\begin{aligned}\frac{dg(M)}{dM} &= \frac{1}{2}\frac{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\epsilon^2-\rho(2M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2}{\sqrt{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\epsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}}}\left\{2\left(\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2\right)\right\}\\ &\quad+\sqrt{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\epsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}}\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2\right),\end{aligned}$$

we also define $h(M)$ so that it holds $\frac{dg(M)}{dM} = \frac{h(M)}{\sqrt{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\epsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}}}$ as follows:

$$\begin{aligned}h(M) &= \frac{1}{2}\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\epsilon^2-\rho(2M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}\left\{2\left(\sigma_0^2+\frac{\sigma_\epsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2\right)\right\}\\ &\quad+M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\epsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2\right).\end{aligned}$$

Note that $h(M)$ is a quadratic equation with respect to M with a negative quadratic coefficient, and that $h(1) > 0$ and $h(M)$ converges to a negative value as $M \to \bar{N}_K$. These imply that $h(M)$ crosses zero once and only once as M increases from one to $\bar{N}_K$. Define $M_K^* \in (1, \bar{N}_K)$ as such M at the crossing point. By definition, $h(M)$ is positive for $M \in (1, M_K^*)$, whereas it is negative for $M \in (M_K^*, \bar{N}_K)$. Given that $\frac{dg(M)}{dM} > 0$ if and only if $h(M) > 0$, the same conclusion on whether it is positive or negative is drawn for $\frac{dg(M)}{dM}$. This implies that $g(M)$ is inverse-U-shaped with peak point at $M = M_K^*$, which in turn implies that $U_K(M)$ is U-shaped, as in the lemma, because $U_K(M)$ is inversely proportional to $g(M)$. Last, by the definition of $g(M)$, we can easily see that $g(M) \to 0$ as $M \to \bar{N}_K$, implying that $U_K(M) \to \infty$ in this limit, as stated in the lemma.

In the case of η-overconfident traders, plugging $\kappa = 1$ and $\eta \in (0,1)$ into Equation (16), we have

$$U_E(M) = \sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2 + \sigma_\epsilon^2 + \rho(M-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\epsilon^2\right\}} \frac{\sigma_0^2\left(\sigma_0^2 + \sigma_\epsilon^2\right)}{2\left(\sigma_0^2 + \sigma_\epsilon^2\right) + (M-1)\left(\sigma_0^2 + \frac{\rho}{\sqrt{\eta}}\sigma_\epsilon^2\right)}}. \tag{19}$$

for every $\rho \in [0,1]$. Then it is easy to see that it decreases with M, and converges to zero as $M \to \infty$, as stated in the lemma.

Finally, we want to prove that $U_R(M) < U_K(M)$ for every $M \geq 1$ regardless of $\rho \in [0,1]$. Given that the benchmark case corresponds to the limit where $\kappa \to 1$ in the case of κ-overconfident traders, it suffices to show that U_K increases with κ. We take the logarithm of U_K and then differentiate it with respect to κ as follows:

$$\begin{aligned}
\frac{\partial \log U_K}{\partial \kappa} &= \frac{-\kappa^{-2}\sigma_\epsilon^2}{\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}} - \frac{1}{2}\frac{M\left\{-2\kappa^{-2}\sigma_\epsilon^2 - \frac{1}{2}(M-1)\rho\sigma_\epsilon^2\kappa^{-\frac{3}{2}}\right\}}{M\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}} \\
&\quad - \frac{-2\sigma_\epsilon^2\kappa^{-2} - \frac{1}{2}(M-1)\rho\sigma_\epsilon^2\kappa^{-\frac{3}{2}}}{2\left(\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\right) + (M-1)\left(\sigma_0^2 + \frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2\right)} \\
&= -\frac{\sigma_\epsilon^2}{\kappa^2\sigma_0^2 + \kappa\sigma_\epsilon^2} + \left\{2\kappa^{-2}\sigma_\epsilon^2 + \frac{1}{2}(M-1)\rho\sigma_\epsilon^2\kappa^{-\frac{3}{2}}\right\}\left(\frac{1}{L_1} + \frac{1}{L_2}\right),
\end{aligned}$$

where

$$\begin{aligned}
L_1 &= 2\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}; \\
L_2 &= 2\left(\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\right) + (M-1)\left(\sigma_0^2 + \frac{\rho}{\sqrt{\kappa}}\sigma_\epsilon^2\right).
\end{aligned}$$

The above derivative is positive if and only if

$$\frac{\sigma_\epsilon^2}{\kappa^2\sigma_0^2 + \kappa\sigma_\epsilon^2} L_1 L_2 < \left\{\frac{2\sigma_\epsilon^2}{\kappa^2} + \rho(M-1)\sigma_\epsilon^2\kappa^{-\frac{3}{2}}\right\}(L_1 + L_2),$$

which is equivalent to

$$L_1 L_2 < \left(\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\right)\left\{2 + \rho(M-1)\sqrt{\kappa}\right\}(L_1 + L_2). \tag{20}$$

Claim 2. $L_1 < \left(\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\right)\left\{2 + \rho(M-1)\sqrt{\kappa}\right\}$ holds true.

Proof. Noting that $L_1 = 2\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - \rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2\right\}$ by definition, the claim is equivalent to

$$2\left(\frac{2}{\kappa} - 1\right)\sigma_\epsilon^2 - 2\rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2 < \sqrt{\kappa}\rho(M-1)\sigma_0^2 + \frac{\sigma_\epsilon^2}{\kappa}\left\{2 + \rho(M-1)\sqrt{\kappa}\right\},$$

which is in turn equivalent to

$$\frac{2}{\kappa}\sigma_\epsilon^2 - 2\sigma_\epsilon^2 < \sqrt{\kappa}\rho(M-1)\sigma_0^2 + 2\rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\epsilon^2 + \frac{\rho}{\sqrt{\kappa}}(M-1)\sigma_\epsilon^2.$$

It is easy to see that the above inequality holds true, since the left-hand side is negative but the right-hand side is positive. ☐

By Claim 2 and $L_2 < L_1 + L_2$, it immediately follows that Equation (20) holds true, completing the proof that U_K increases with κ for every $\rho \in [0,1]$ and $M \geq 1$, which in turn establishes the last statement of the proposition.

Proof of Proposition 4

Recall from Lemma 1 that U_R decreases with M and converges to zero as M goes to infinity. If $c \in (0, U_R(1))$, this implies that there exists a unique crossing point $N_R^*(c)$ where $U_R(N_R^*(c)) = c$ by the Intermediate Value Theorem.

If $N \in [1, N_R^*(c))$, it holds that $U_R(M) > c$ for every $M \in [1, N]$ by the definition of $N_R^*(c)$. This implies that all traders choose to be informed. The resulting subgame is equivalent to the basic model with N strategic traders by Lemma 1.

If $N > N_R^*(c)$, it holds that $U_R(M) > c$ for $M \in [1, N_R^*(c))$, whereas $U_R(M) < c$ for $M \in (N_R^*(c), N]$. This implies that only $N_R^*(c)$ traders choose to be informed in equilibrium. The resulting subgame with $N_R^*(c)$ informed traders is equivalent to the basic model with $N_R^*(c)$ strategic traders by Lemma 1.

Finally, if $c > U_R(1))$, then it holds that $c > U_R(M)$ for every $M \geq 1$. This implies that all traders choose not to be informed.

Proof of Proposition 5

Recalling from Lemma 1 that $U_K(M)$ is U-shaped in M, denote its minimum by $M_K^* \in (1, \bar{N}_K)$.

If $c \in (0, U_K(M_K^*))$, then it holds that $c < U_K(M)$ for every $M \in [1, \bar{N}_K)$. This implies that all traders choose to be informed.

Now consider $c \in (U_K(M_K^*), U_K(1))$, there exists two crossing points $N_K^*(c) \in (1, M_K^*)$ and $N_K^{**}(c) \in (M_K^*, \bar{N}_K)$ such that $U_K(N_K^*) = U_K(N_K^{**}) = c$ by applying the Intermediate Value Theorem separately to $U_K(M)$ defined on $(1, M_K^*)$ and $(M_K^*, \bar{N}_K)$. The existence of the crossing point for the former interval is guaranteed by $c < U_K(1)$, and that for the latter interval is guaranteed by $U_K(M) \to \infty$ as $M \to \bar{N}_K$ by Lemma 1.

If $N \in [1, N_K^*(c))$, it holds that $U_K(M) > c$ for every $M \in [1, N]$ by the definition of $N_K^*(c)$. Thus, all traders choose to be informed. The resulting subgame is equivalent to the basic model with N strategic traders by Lemma 1.

If $N \in (N_K^*(c), N_K^{**}(c))$, it holds that $U_K(M) > c$ for $M \in [1, N_K^*(c))$, whereas $U_K(M) < c$ for $M \in (N_K^*(c), N_K^{**}(c))$ by the definitions of $N_K^*(c)$ and $N_K^{**}(c)$. Thus, only $N_K^*(c)$ traders choose to be informed in equilibrium. The resulting subgame with $N_K^*(c)$ informed traders is equivalent to the basic model with $N_K^*(c)$ strategic traders by Lemma 1.

If $N \in (N_K^{**}(c), \bar{N}_K)$, it holds that $U_K(M) > c$ for $M \in [1, N_K^*(c))$, $U_K(M) < c$ for $M \in (N_K^*(c), N_K^{**}(c))$, and $U_K(M) > c$ for $M \in (N_K^{**}(c), \bar{N}_K)$. As a result, there are two equilibria at $M = N_K^*(c)$ and $M = N$, respectively. The former equilibrium results in a subgame with $N_K^*(c)$ informed traders, which is equivalent to the basic model with $N_K^*(c)$ strategic traders by Lemma 1. On the other hand, the latter equilibrium results in a subgame with N informed traders, which is equivalent to the basic model with N strategic traders.

Finally, we want to prove the last statement that $N_K^*(c) > N_R^*(c)$ for every $c \in (U_K(M_K^*), U_K(1))$. Note that two functions $U_R(M)$ and $U_K(M)$ defined on $[1, M_K^*)$ are decreasing in M, and it holds that $U_R(M) < U_K(M)$. Given that $N_R^*(c)$ and $N_K^*(c)$ are their crossing points with c as defined above, and that $N_K^*(c) > 0$ by $c < U_K(1)$, we have $N_K^*(c) > N_R^*(c)$.

The above inequality implies that κ-overconfidence weakly increases the number of informed traders even in the equilibrium with a lower number of informed traders. Denote by M_R and M_K the number of informed traders in the benchmark case and the equilibrium with a lower number of traders under κ-overconfidence, respectively. The observation that $M_K \geq M_R$ appears to be intuitive by comparing the curves of $U_K(M)$ and $U_R(M)$ on the (M, U)-space and then noting that M_R and M_K are determined at these curves' crossing points with $U = c$ subject to $M_R \leq N$ and $M_K \leq N$. We formally prove this by considering two cases: (1) $U_R(1) > U_K(M_K^*)$ and (2) $U_R(1) < U_K(M_K^*)$. In the first case, the range of $c \in (0, U_K(1))$ is divided into three subcases: (1a) $c \in (0, U_K(M_K^*))$, (1b) $c \in (U_K(M_K^*), U_R(1))$, and (1c) $c \in (U_R(1), U_K(1))$. Similarly, in the second case, the range of $c \in (0, U_K(1))$ is divided into three subcases: (2a) $c \in (0, U_R(1))$, (2b) $c \in (U_K(M_K^*), U_K(M_K^*))$, and (2c) $c \in (U_K(M_K^*), U_K(1))$.

The following table describes M in equilibrium in all these possible subcases to verify that M weakly increases with κ-overconfidence (i.e., $M_K > M_R$):

(1) $U_R(1) > U_K(M_K^*)$	Benchmark (M_R)	κ-overconfidence (M_K)
(1a) $c \in (0, U_K(M_K^*))$	$\max\{N_R^*(c), N\}$	N
(1b) $c \in (U_K(M_K^*), U_R(1))$	$\max\{N_R^*(c), N\}$	$\max\{N_K^*(c), N\}$
(1c) $c \in (U_R(1), U_K(1))$	0	$\max\{N_K^*(c), N\}$
(2) $U_R(1) < U_K(M_K^*)$	Benchmark (M_R)	κ-overconfidence (M_K)
(2a) $c \in (0, U_R(1))$	$\max\{N_R^*(c), N\}$	N
(2b) $c \in (U_K(M_K^*), U_K(M_K^*))$	0	$\max\{N_K^*(c), N\}$
(2c) $c \in (U_K(M_K^*), U_K(1))$	0	$\max\{N_K^*(c), N\}$

Proof of Corollary 3

As in the proof of Proposition 5, denote by M_R and M_K the number of informed traders in the benchmark case and the equilibrium with a lower number of traders under κ-overconfidence, respectively.

In the benchmark case, we have $M_R = \max\{N_R^*(c), N\}$, respectively. Note that where $N_R^*(c)$ takes zero for $c \in (U_R(1), U_K(1))$ by Proposition 4. For $N < N_R^*(c)$, M_R increases with N, and thus, trading volume and price informativeness also increase with N as well. For $N > N_R^*(c)$, M_R stays constant with respect to N, and thus, trading volume and price informativeness also stay constant as well.

In the case of κ-overconfident traders, we have two possible equilibria by Proposition 5.

- In the equilibrium with a lower number of informed traders, recall that $M_K = \max\{N_K^*(c), N\}$. For $N < N_K^*(c)$, M_K increases with N, and thus, trading volume and price informativeness also increase with N as well. For $N > N_K^*(c)$, M_K stays constant with respect to N, and thus, trading volume and price informativeness also stay constant as well.

- In the other equilibrium that exists for $N \in (N_K^*, \bar{N}_K)$, note first that all strategic traders choose to be informed. Thus, trading volume and price informativeness increase with N and, as $N \to \bar{N}_K$, trading volume goes to infinity and price informativeness goes to PI^* by Proposition 3 and Corollary 2.

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