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Credit constraints, capital portfolios, and measured productivity

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Credit constraints, capital portfolios, and measured productivity [*]

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[*] Views expressed in the paper are those of authors, and do not necessarily reflect the view of their institutions.

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ABSTRACT

We develop a model connecting financial shocks, capital investment decisions by firms, and change in measured aggregate productivity using a dynamic general equilibrium model. Data shows that post the 2008 crisis, firms changed their allocation between assets of varying depreciation rates as credit conditions tightened, which is connected to changes in measured TFP. We propose a model that shows the mechanism of an adverse shock to credit access causing firms to change the balance sheet portfolio composition of productive assets. This reallocation of assets leads to an increase in measured productivity.

Keywords: Financial crisis, measured productivity, collateral, capital assets, credit constraint.

JEL Classification: D5, E13, E22, E32, G01, G11, G23.

1 Introduction

Data on capital asset holdings of the aggregate firm from the U.S. shows that during the financial crisis, when credit conditions deteriorated sharply in 2008, firms reallocated their investments from long term capital assets to short term capital assets. Data also shows that measured aggregate productivity fell sharply during this period. The mechanism seems to work as follows: when credit conditions are favourable, firms invest more in long term assets than in short term assets as financing the more expensive long term assets is easier. They act as better collateral and the leverage facilitates more investment. But when credit conditions tighten (as happened in the 2008 crisis), firms cannot get financing to invest in long term assets and end up reshuffling their portfolio towards more short term assets. The assumption here is that long term, low depreciation assets serve as better collateral than shorter term high depreciation ones, and in 'good times', firms select the long term assets. The short term assets are funded by equity throughout as they do not have much value as collateral. As firms have several different types of assets on their balance sheet with different rates of depreciation, they can reshuffle their portfolio of capital depending on what is optimal given the credit constraint. This reshuffling impacts measured TFP as the production bundle changes.

We model the above mechanism and show that tightening credit conditions lead to an increase in measured aggregate productivity as firms respond by changing the composition of their capital holdings. The portfolio held when credit is free-flowing is heavy in long term assets and as funds are reallocated to short term assets, the output generated is higher compared to the replacement cost of capital, which shows up as an increase in measured TFP. The model is in a general equilibrium setting, where there is a continuum of capital goods and firms face a collateral constraint which limits their access to finance. A negative shock to the LTV can be likened to a loss of confidence in the debt market where banks suddenly demand a higher quality collateral, or are willing to further a loan even lower in amount compared to the collateral.

The point that credit constrained firms might be the reason for amplification of shocks as well as for prolonged recessionary conditions has been studied extensively in the literature, most intuitively by Kiyotaki and Moore (1997) (hereafter KM). In their seminal paper, KM show how the existence of leverage in firms can amplify productivity shocks as well as prolong a recession which otherwise would not have been as deep or as long lasting. Their model generates a tightening of the financial constraint from a fall in productivity and the mechanism of transmitting the productivity shock into prices and, in turn, into reallocation of capital among firms results from the collateral constraints. Bernanke, Gertler, and Gilchrist (1999) use a mechanism similar to KM, but with idiosyncratic shocks and different levels of accumulated wealth for firms. We develop a model which has collateral constraints similar to the KM model, but we do not rely on shocks to technology to generate business cycles. In our model, we show how a tightening of the financial constraint can affect measured productivity. Our model attempts to capture events like the 2008 financial crisis where liquidity dried up and the value of collateral fell sharply due to a spate of repossessions by banks. Similar to KM, our model also relies on some form of reallocation to generate persistence but unlike their model our reallocation happens due to some assets serving as better collateral than others in recessionary conditions whereas theirs is along the extensive margin for the two groups in the model.

Chari, Kehoe, and McGrattan (2007) use U.S. business cycle data and a real business cycle model with time-varying wedges, and find that efficiency wedges and labour wedges explain the long deep downturn of the 1930s and also are largely responsible for fluctuations thereafter over the business cycle. The demonstrate that financial crises are associated with a large fall in measured TFP (which they term the "efficiency wedge"). Compared to Bernanke, Gertler, and Gilchrist (1999), where risk averse consumers end up bearing all the risk from an adverse idiosyncratic shock leading to bankruptcies and a lowering of aggregate net wealth and thus capital accumulation, Chari, Kehoe, and McGrattan (2007) show that this can be modelled in an RBC model as an investment wedge as well as a wedge where the rate of depreciation seems to be time varying[1]. The model we develop also has a 'wedge' which causes depreciation to be time varying, though the channels are very different.

Gorton and Ordonez (2020) note how a positive shock to productivity initially causes lenders to be 'information-insensitive' and to lend to projects without much scrutiny. If, however, average productivity is not sustained and declines over time, lenders scrutinise projects closely, leading to some projects going unfunded; a financial crisis or 'bad boom'. A deep recession results from this crisis as number of firms in the market falls and eventually productivity rises as a result, which leads to another boom, and so forth. In our model, the link is from

[1]Chari, Kehoe, and McGrattan (2007) show that the mapping of

$$c_t + k_t = w_t l_t + (1 - \tau_t^k) r_t k_{t-1} + (1 - \delta(1 - \tau^k)) k_{t-1} + T_t$$

mimics the effect of the suboptimal contracts of Bernanke, Gertler, and Gilchrist (1999).

financial crises to measured TFP, unlike Gorton and Ordonez (2020), and we show that the fall in TFP during a crisis might actually be higher than the measured quantity. In our case the reallocation of investment improves measured TFP, whereas in their case there is a reduction along the extensive margin of the number of firms which lowers measured TFP further.

We introduce heterogeneity in asset types and have a representative aggregate firm. Also, in our model, the shocks actually originate in the financial sector and show up as measured productivity because of the transmission mechanism of the model. These two features set the model apart from the popular approaches of including heterogeneity and having productivity shocks impact financial frictions.

We find that our dynamic model can replicate the stylised facts observed in data, and that allows us to have a theoretical explanation reconciling a shock in the financial sector causing changes in the capital portfolio held by the aggregate firm and a change in measured TFP. The main messages can be summarised as:

1. Financially constrained firms reallocate funds to short term assets after a financial shock
2. There is an increase in measured TFP from this reallocation as the initial holdings are skewed towards long term assets (as they offer better collateral). The firm's bundle is producing more output relative to the replacement cost of that capital, which shows up in higher measured efficiency

We discuss some related literature in Section 2, present stylised facts from data in the Section 3, and set up and explain the model in Section 4. Section 5 presents the results and Section 6 concludes.

2 Related Literature

Most existing literature introduces heterogeneity of firms and idiosyncratic shocks or firm specific taxes on factors to generate misallocation. There is also literature which links financial friction to misallocation through the lack of access to funds forcing firms to have lower factors of production. A change in individual productivity affects access to finance, which causes misallocation of factors, and hence aggregate productivity is affected. The changes in individual productivity are exacerbated by the financial friction, which reduces measured aggregate productivity.

Midrigan and D. Y. Xu (2014) model heterogeneous agents with idiosyncratic productivity and borrowing constraints where net worth depends on productivity and acts as an entry barrier which lowers aggregate productivity from misallocation of resources. Buera, Kaboski, and Shin (2011) model differential fixed costs as well as differential access to capital which distorts allocation and find, like Midrigan and D. Y. Xu (2014), that in addition to misallocation, the distortion is also a function of the number of production units in manufacturing being too low. Our model presents an alternative channel through which capital accumulation is affected by the financial friction, in that, the representative firm chooses from a continuum of asset classes driven by limited access to finance which impacts measured TFP.

Moll (2014) studies the transition dynamics for an economy with financial friction in the form of collateral constraints. He finds that the persistence of the productivity shock is important in determining the magnitude of productivity losses as well as the duration of convergence to steady state. If shocks are persistent, it gives agents an opportunity to save and overcome the financial friction, but the convergence to steady state is slower, whereas transitory shocks lead to quick convergence but larger productivity losses in the long term. Methodologically, they aggregate over all agents with their wealth shares as weights and arrive at a representation of aggregate TFP which is endogenous. We also derive a comparable representation of endogenous TFP which is dependent on the financial friction directly, whereas in Moll (2014), it depends indirectly on the friction, in that, each agents response to the degree of persistence of the productivity shock *given the friction* is the basis of their result. Their results also touch upon the issue of size and productivity, mentioned by Bartelsman, Haltiwanger, and Scarpetta (2013) and Guner, Ventura, and Y. Xu (2008), that bigger but less numerous firms increase aggregate productivity, but here that outcome is not a result of any tax wedge and depends on the quality of credit markets.

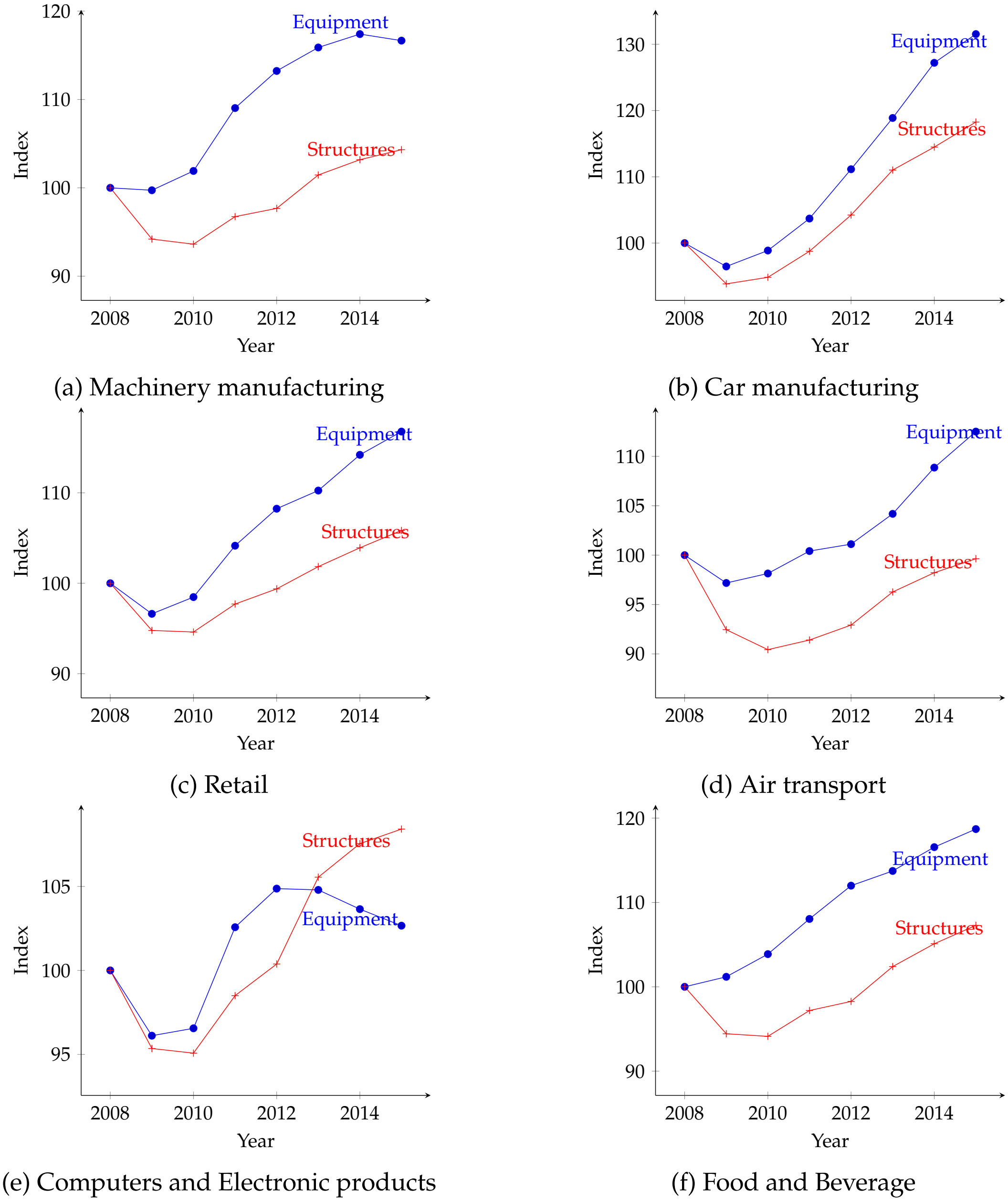
Khan and Thomas (2013) add another friction in the form of investment irreversibility to a dynamic general equilibrium model of heterogeneous agents with collateral constraints to study how capital reallocation is distorted and lowers aggregate productivity. In the presence of such additional real friction, they find that a financial shock can produce a deep and prolonged recession in their model which is qualitatively similar to the 2008 U.S. recession, more so than what a negative shock to technology can produce. Our base model will not have any real frictions, but we also demonstrate how a financial shock can produce a deep recession through reallocation of the aggregate capital portfolio.

Matsuyama (2007) introduces heterogeneity among investment projects (instead of among firms) to a standard neoclassical setting, where the most pledgeable investments are the least productive ones. Investments are dependent on access to financing through collateralisation, and hence a change in credit conditions forces investment into the low productivity projects. Although the focus of the paper is to investigate credit cycles and credit traps, this misallocation would no doubt lower aggregate TFP. Our model also has a representative agent firm like Matsuyama (2007), but our firm chooses assets to invest in and use for production instead of projects, and this choice depends on availability of finance. Also, similar to Matsuyama (2007), our firms invest in low depreciation assets in good times but such assets have lower productivity.

The existing literature models heterogeneity of some form, either firm heterogeneity or heterogeneity of investment projects, and relies on a financial friction to exacerbate misallocation from differential productivity or differential access to finance which, when aggregated, shows up as lower measured aggregate TFP. The link between financial frictions and lower aggregate TFP hinges on heterogeneity of firms in some form. Some notable contributions would be Buera and Moll (2015), A. Banerjee and Munshi (2004), A. V. Banerjee and Moll (2010), López (2017), Hsieh and Klenow (2009), Gilchrist, Sim, and Zakrajšek (2013), among others. We, however, model a representative firm with a borrowing constraint where the amount of credit depends on the composition of the balance sheet portfolio. This allows us to draw a clear link from a financial shock into portfolio reallocation and to measured TFP. Our model only relies on longer term assets serving as better collateral, which is a fairly realistic assumption. A financial shock then causes portfolio reallocation which maps into measured productivity.

3 Stylised facts

We present some facts from the data regarding the reallocation by firms between assets with different depreciation rates after the financial crisis.



(a) Machinery manufacturing
(b) Car manufacturing
(c) Retail
(d) Air transport
(e) Computers and Electronic products
(f) Food and Beverage

Figure 1: Post crisis Asset deviations

Source: BEA

Figure 1 presents the change in asset positions for short term assets, represented by equipment, versus long term assets, represented by structures[2] for nonresidential fixed asset estimates. We present the data for 6 different industries: Manufacturing of machinery, Car manufacturing, Retail, Air transport and related equipment manufacturing, Computers and electronic equipment manufacturing, and Food and beverages. The amounts are current cost, indexed to 2008.

Figure 1 shows that across the industries, low depreciation assets either fall by less, or in

[2]The components of equipments vs structures are outlined in Appendix E.2.

some cases even increase, after the financial shock whereas long term assets always show a drop.

We also study asset reallocation post a financial shock using data from the Bureau of Economic Analysis (BEA) for 100 different asset classes, ranging from the lowest to highest depreciation rates. For Figure 2, we divide the data into two time periods: 2001-2008 and 2009-2018 (as the pre and post crisis periods respectively)[3]. We use the data on age of each asset class, calculated by the BEA using the permanent inventory method (PIM) which calculates age for any asset class at the end of the year based on how much was added in the current year, and how much depreciation was charged on that class[4]. Here we have used age calculated on the historical value of assets. We take the average of the age for each asset class from 1987 to arrive at an approximation of mean age over time for that asset class. Then we take the average age for the pre and post crisis periods and compute the deviations for each asset class in each time period compared to the overall average for each asset class. We then weigh these deviations by the respective asset class in the balance sheet, and plot the weighted deviations for each asset class in each time period in Figure 2.

[3]The choice of the post crisis period being till 2018 does not imply that we think the recession lasted till 2018, or that credit conditions were tight till 2018. It is a choice made to be consistent with the number of years in the pre crisis period used. Despite the credit conditions being fairly loose after 2010, it is quite revealing to see the impact of the crisis still comes through.

[4]Further information can be found at the BEA information page

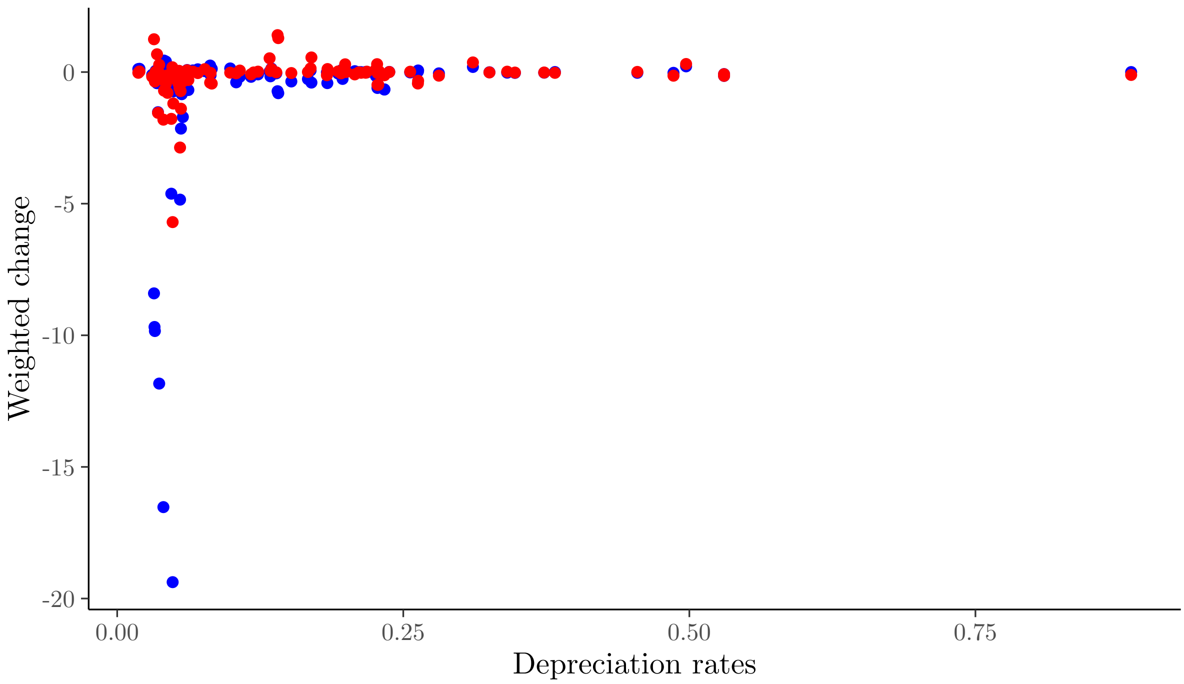


Figure 2: Pre vs post 2008 crisis

A negative observation shows that the age of the asset class has increased, meaning that the investment in that asset class has fallen in the given time period. The red dots represent the 'control' group, i.e. the pre crisis period of 2001-2008, and the blue dots represent the 'treatment' group for the crisis period 2009-2018.

In Figure 2, we plot the weighted change in asset holdings measured by changes in average age of the asset class according annual depreciation rates. A negative observation shows that the age of the asset class has increased, meaning that the investment in that asset class has fallen in the given time period. The red dots and line is the 'control' group representing the pre crisis period of 2001-2008 and the blue dots and line represent the 'treatment' group for the crisis period 2009-2018. We can again see that the drop in asset holdings is higher at the lower end of the depreciation scale for the period of the crisis.

Another observation from the data is regarding the average rate of depreciation, defined as the current cost depreciation charge over the current cost net stock of fixed assets.

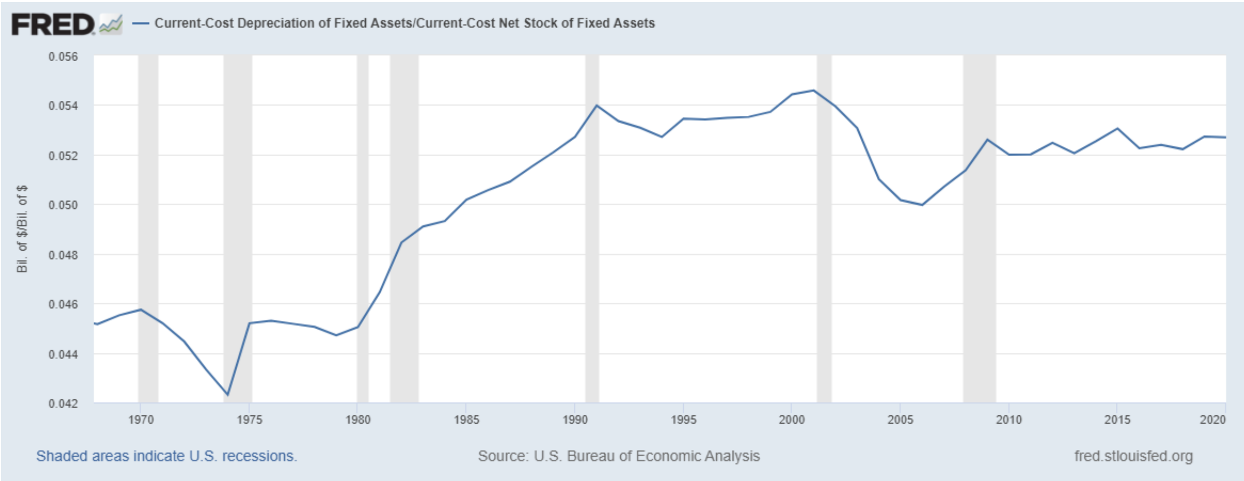


Figure 3: Average rate of depreciation (data)

As can be seen from Figure 3, the average rate of depreciation increased during the 2008 financial crisis, which is consistent with facts presented previously regarding the weight of short term assets increasing in the balance sheet.

Broadly, we can summarise the data work into following stylised facts:

Stylised Fact 1. *A financial shock causes reshuffling of the portfolio as credit conditions worsen*

Stylised Fact 2. *The reshuffling increases the weight of short term assets in the balance sheet compared to long term assets*

Stylised Fact 3. *The overall balance sheet shrinks*

Our model reconciles these data facts by showing how a shock originating in the financial sector (as happened in 2008) causes firms to reshuffle their capital asset portfolio, in turn showing up as a lowering of measured productivity.

4 Model

The model consists of two sectors; production firms and households.

4.1 Production firms

Following the results from Altug and Labadie (2008), and the specification of Gertler, Kiyotaki, and Prestipino (2016), we define the individual firm's dynamic optimisation problem as under;

$$V_t = \max_{k,k(i)_{+1},n,\ell_{+1}} \mathbb{E}_t \left[\sum_{j=0}^{\infty} (1-\theta)\theta^{t+j} m_{t+j} \Pi_{t+j} \right] \tag{1}$$

where $m_{t+j} := \beta^j \cdot \frac{\mathbb{E}_t \lambda_{t+j}}{\lambda_t}$ is the stochastic discount factor, and the cash flow is

$$\Pi_{t+j} = k_{t+j}^{\alpha} n_{t+j}^{1-\alpha} - W_{t+j} n_{t+j} - \int k_{t+j+1}(i)\, \mathrm{d}i + \int (1-\delta(i)) k_{t+j}(i)\, \mathrm{d}i + \ell_{t+j+1} - R_{t+j}\ell_{t+j} - q_t$$
$$\forall j = \{0, 1, 2, ..\}.$$

We also introduce survival probability for individual firms, represented by θ. For each individual firm, the probability of existing at date $t + j$ is given by $(1-\theta)\theta^{t+j}$. The firm maximises its discounted cash flows Π, which, as shown by Altug and Labadie (2008), is equivalent to maximising end of period value. The inflows for the firm consist of sales, loans, and the market value of nondepreciated capital stock, while the outflows are wages, new capital investment, and repayment of debt from previous period. We have a continuum of capital assets with corresponding, constant rates of depreciation. The loans carry the risk free rate of interest. Lower case letters k, n, ℓ represent individual firm capital, labour, loans whereas upper cases indicate aggregates. The adjustment cost q is a distance function which depends on each individual asset type's deviation from it's respective steady state value;

$$q_t := \int (\ln(k_{t+1}(i)) - \ln(k_{ss}(i)))^2\, \mathrm{d}i. \tag{2}$$

In steady state (2) is 0, and as each capital asset type deviates from it's steady state value on impact of a shock, the cost q rises.

We can cast (1) into a two period optimisation problem and state the Bellman equation as under;

$$V(k_t(i), k_t, \ell_t|\xi_t) = \max_{k(i),k,\ell,n} \left[(1-\theta)\pi_t + \theta \mathbb{E}_t m_{t+1} V(k_{t+1}(i), k_{t+1}, \ell_{t+1}|\xi_{t+1}) \right] \tag{3}$$

where $\pi_t := k_t^{\alpha} n_t^{1-\alpha} - W_t n_t - \int k_{t+1}(i)\mathrm{d}i + \int (1-\delta(i)) k_t(i)\mathrm{d}i + \ell_{t+1} - R_t \ell_t - q_t$.

With probability $(1-\theta)$, the firm cannot survive to the next period, and is given the net liquidation proceeds by the sector. This transfer will be reflected in the aggregate firm flow of funds each period, as we will show later.

Each firm faces a collateral constraint which pins down the amount of borrowing the firm can avail;

$$R_{t+1}\ell_{t+1} = \xi_t \chi \int (1-\delta(i))k_{t+1}(i)\mathrm{d}i \qquad (\phi_t). \tag{4}$$

The amount each firm can borrow depends on the expected future risk free rate R_{+1}, a loan-to-value (LTV) parameter χ, a shock to the LTV ξ, and the non depreciated value of balance sheet capital. The assumption built in is that it takes time to liquidate all capital, and so nondepreciated value of capital is considered. ϕ is the shadow value of the constraint. The longer term assets with low depreciation rates serve as better collateral than the high depreciation assets. The LTV negative shock can be likened to a loss of confidence in the debt market where banks suddenly demand a higher quality collateral, or are willing to further a loan even lower in amount compared to the collateral. This is something that was observed in the financial crisis, where banks were unwilling to lend as house prices collapsed (and no one knew who was holding the bad assets).

Individual capital goods are bundled together into a composite of productive capital goods following the CES aggregator;

$$k_t = \left[\int k_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad (F_{k,t}). \tag{5}$$

The firm chooses assets to form a production bundle. The degree of productive substitutability between assets is given by ε. The derivation of the optimality conditions is outlined in appendix A.5.

4.2 Demand function

The first order condition for capital asset $k(i)$ can be rearranged to obtain the demand function as under;

$$k_{t+1}(i) = k_{t+1} \left(\frac{1-(1-\delta(i))(\theta \mathbb{E}_t[m_{t+1}] + \phi_t \xi_t \chi) + d^{Q}_{k(i),t}}{\theta \mathbb{E}_t[m_{t+1} F_{k,t+1}]} \right)^{-\varepsilon}. \tag{6}$$

The term $d^{Q}_{k(i),t}$ in (6) is the first derivative of the adjustment cost function (2) wrt $k(i)$;

$$d^Q_{k(i),t} = \frac{2(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{k_{t+1}(i)}. \tag{7}$$

(6) implies that if there is a negative shock to LTV, capital accumulation will fall as demand for each type of capital will be lower. This can be seen from:

$$\begin{aligned} \frac{\partial k_{t+1}(i)}{\partial \xi_t} &= \varepsilon \left(\frac{\eta_t(i)}{\theta \mathbb{E}_t m_{t+1} F_{k,t+1}} \right)^{-\varepsilon} \frac{(1-\delta(i))\phi_t \chi}{\eta_t(i)} k_{t+1} \\ &= \varepsilon \cdot k_{t+1}(i) \cdot \frac{(1-\delta(i))\phi_t \chi}{\eta_t(i)} \\ &\geq 0. \end{aligned} \tag{8}$$

If $\delta(i) = 1$, the derivative is zero, and there is no impact of the financial shock on assets with very high depreciation rates. The sign of $d^Q_{k(i),t}$ will depend on the direction of movement of $k_{t+1}(i)$; if asset holding falls, $d^Q_{k(i),t} < 0$, and in that case it will lower asset demand further in second round adjustment.

Proposition 1. *On impact of a financial shock, asset reallocation within the portfolio is such that the share of low depreciation assets falls and that of high depreciation assets rises.*

Proof. We need to check what impact depreciation rates have on asset demand, given a negative LTV shock.

$$\frac{\partial^2 k(i)}{\partial \xi \partial \delta(i)} \gtreqless 0?$$

We know that

$$\begin{aligned} \left.\frac{\partial^2 k_{t+1}(i)}{\partial \xi_t \partial \delta(i)}\right|_{\xi\downarrow} &= -\varepsilon \left(\frac{\eta_t(i)}{\theta \mathbb{E}_t m_{t+1} F_{k,t+1}} \right)^{-\varepsilon} \frac{k_{t+1}\phi_t \chi}{\eta_t(i)} \left(\frac{(1-\delta(i))}{\eta_t(i)} \left[\theta \mathbb{E}_t m_{t+1} + \phi_t \xi_t \chi + \left.\frac{\partial d^Q_{k(i)}}{\partial \delta(i)}\right|_{\xi\downarrow} \right] + 1 \right) \\ &\gtrless 0. \end{aligned} \tag{9}$$

The sign in (9) can become positive based on the response of $\left.\frac{\partial d^Q_{k(i)}}{\partial \delta(i)}\right|_{\xi\downarrow}$. If that response is a large negative, it can flip the sign of the term in parentheses to make it negative and hence the final response is positive. We need to know the sign of $\frac{\partial d^Q_{k(i)}}{\partial \delta(i)}$.

$$\frac{\partial d^Q_{k(i)}}{\partial \delta(i)} = 2\left(\frac{1-(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{(k_{t+1}(i))^2}\right)\frac{\partial k(i)}{\partial \delta} - \frac{2}{k_{t+1}(i)k_{ss}}\frac{\partial k_{ss}(i)}{\partial \delta}. \tag{10}$$

In (10), if we assume $\frac{\partial k(i)}{\partial \delta(i)} = \frac{\partial k_{ss}(i)}{\partial \delta(i)}$, which is < 0 from the data, the sign will depend on :

$$2\left(\frac{1-(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{(k_{t+1}(i))^2} - \frac{1}{k_{t+1}(i)k_{ss}(i)}\right)\Bigg|_{\xi\downarrow}.$$

From (8) we know that $k_{t+1} \downarrow$ when $\xi \downarrow$. So in the above, the first term in parentheses will be positive and greater than one in the numerator. Also, for the denominators, the denominator of the first term will be smaller, and hence that term will be larger. So the net effect is that the whole term is positive. Also, $\frac{\partial k(i)}{\partial \delta(i)} \approx -11$ from the data. So the term $\frac{\partial d^Q_{k(i)}}{\partial \delta(i)}$ is large and negative, and for reasonable parameter values, changes the sign of $\frac{\partial^2 k_{t+1}(i)}{\partial \xi_t \partial \delta(i)}$ to make it positive. Also from (9), we can see that as $\delta(i)$ goes up, the impact becomes smaller[5]. This indicates that the portfolio composition moves from more low depreciation assets to more high depreciation assets.

□

Another way to arrive at that result is by taking the ratio of two capital assets, $k(i)$ and $k(j)$, where $\delta(i) > \delta(j)$;

$$\frac{k(i)}{k(j)} = \left(\frac{\eta(i)}{\eta(j)}\right)^{-\varepsilon}$$

where $\eta(p) = 1 - (1-\delta(p))[\theta \mathbb{E}_t m_{+1} + \phi\xi\chi] + d^Q_{k(p),t}$. Taking the derivative of the above ratio with respect to the shock ξ yields

$$\frac{\partial \frac{k(i)}{k(j)}}{\partial \xi} = \varepsilon\left(\frac{k(i)}{k(j)}\right)\phi\chi\left[\frac{1-\delta(i)}{\eta(i)} - \frac{1-\delta(j)}{\eta(j)}\right].$$

Consider $\delta(i) > \delta(j)$, or for the sake or argument, $\delta(i) = 1$ and $\delta(j) = 0$, so that, from the above, $\frac{\partial \frac{k(i)}{k(j)}}{\partial \xi} < 0$ which implies that when there is a negative shock, the ratio $\frac{k(i)}{k(j)}$ rises, and combining with the fact that $\frac{\partial k(i)}{\partial \xi} > 0 \ \ \forall i \in I$, it appears that the *fall* in demand for $k(i)$ is less than the *fall* in demand for $k(j)$.

[5]For assets with very high rates of depreciation, the imact can also go in the other direction, implying that high depreciation assets will move in the opposite direction of the other low depreciation assets on impact of a financial shock.

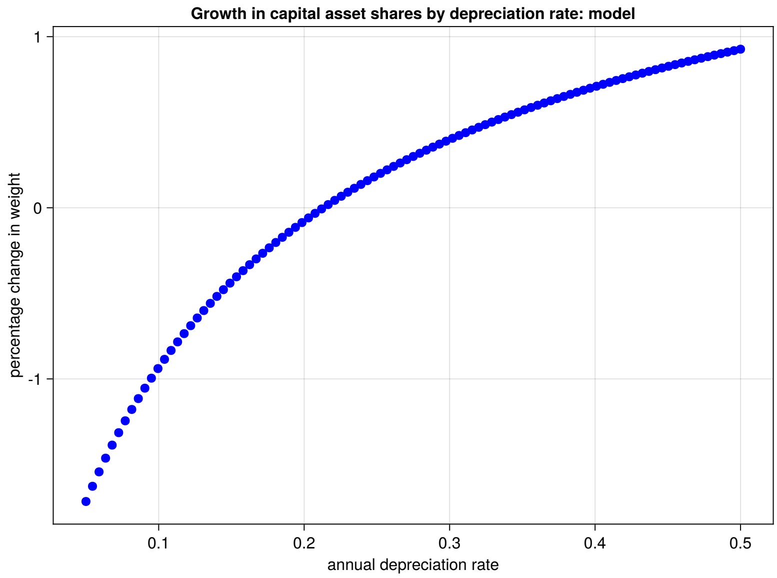


Figure 4: Reallocation after a financial shock

Figure 4 presents the change in the steady distribution implied by the demand function and the dynamics in the first period after a shock. We see that the weights for assets at the lowest end of the X axis scale show the maximum fall and as we go to the right along the X axis, we see that there is a lesser fall in the weight of assets, and as depreciation rate approaches 1, there i actually in increase in asset holding. Overall, the balance sheet is smaller, and more heavy with high depreciation assets. This is the result we have discussed above, and is qualitatively similar to that seen in data, in figure 1 and in figure 2.

4.3 Second order approximations

As in the first chapter, we approximate the integral to the second order around the average rate of depreciation which facilitates solving the model using variance of rates of depreciation. Detailed derivations are available in appendix A.5, while we present the final results below.

The aggregated demand function for entire balance sheet capital is as under;

$$\int k_{t+1}(i)\mathrm{d}i := \overline{k}_{t+1} = k_{t+1}\left(\frac{\eta_t}{\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{-\varepsilon}\cdot(1+\hat{\tau}_t^k) \tag{11}$$

where

$$\eta_t := 1 - (1-\bar{\delta})[\theta \mathbb{E}_t m_{t+1} + \phi_t \xi_t \chi] + d^Q_{k,t} \tag{12}$$

$$d\eta_t := [\theta \mathbb{E}_t m_{t+1} + \phi_t \xi_t \chi]. \tag{13}$$

The price paid today for a marginal unit of capital is $1 + d^Q_{k,t}$ and the benefit gained is the market price of the non depreciated asset tomorrow (given the firm survives) $(1-\bar{\delta})\theta\mathbb{E}_t[m_{t+1}]$, and the amount by which the non depreciated asset relaxes the collateral constraint $(1-\bar{\delta})\phi_t\xi_t\chi$. The lower is the average rate of depreciation $\bar{\delta}$, the larger is the benefit of holding the average asset. $\hat{\tau}^k$ is a measure of the 'wedge' resulting from the approximation [6] which depends on the substitutability between assets ε and the dispersion of the constant rates of depreciation $\delta(i)$.

We also approximate the time varying effective rate of depreciation from $(1 - D_t)\overline{k}_t = \int(1-\delta(i))k_t(i)\mathrm{d}i$;

$$D_t = 1 - (1-\bar{\delta}) \cdot \frac{1+\hat{\tau}^d_{t-1}}{1+\hat{\tau}^k_{t-1}}. \tag{14}$$

The capital Euler which results from approximating the production bundle construction (5) constraint is as under;

$$\lambda_t = \frac{\theta\beta\mathbb{E}_t\lambda_{t+1}\left[F_{k,t+1} + (1-\bar{\delta})(1+\hat{\tau}^\gamma_t)^{\frac{1}{1-\varepsilon}}\right]}{(1-(1-\bar{\delta})\phi_t\xi_t\chi + d^Q_{k,t})(1+\hat{\tau}^\gamma_t)^{\frac{1}{1-\varepsilon}}} \tag{15}$$

where $\hat{\tau}^d$ and $\hat{\tau}^\gamma$ are also measures of the 'wedges' generated by asset substitutability.

(15) is different from a standard capital Euler in two respects: $\hat{\tau}^\gamma$ and the term in the denominator. τ^γ is the 'wedge' which results from approximating around the mean of the depreciation rates, and goes away if there is only one rate of depreciation, implying $\sigma^2_\delta = 0$. In the standard RBC case, we also have that $\chi = 0$ when loans are not present, which, if imposed, reduces (15) to a standard capital Euler[7]. However, the presence of the denominator influences the model dynamics and transmission mechanism significantly, as we will discuss in the dynamic model results.

[6] Further details and derivation in appendix.

[7] If $\varepsilon = \chi = 0$, (15) becomes: $\lambda_t = \theta\mathbb{E}_t\lambda_{t+1}(F_{k,t+1} + (1-\delta))$. This is identical to a standard RBC model with investment adjustment costs (assuming d^Q_k to be ~ 0), except, in that case the rental rate appears in place of the marginal product of capital. But in equilibrium under perfect competition in factor markets, the rental rate is, in fact, the marginal product of capital.

We also need an approximation[8] for the adjustment cost (2):

$$q_t \approx q_t|_{\delta=\bar{\delta}} + \frac{1}{2}\left[d^Q_{\bar{\delta}} \cdot \frac{\partial \bar{k}}{\partial \delta} + d^Q_{\bar{k}} \frac{\partial^2 \bar{k}}{\partial \delta^2} + d^Q_{\bar{\delta}_{ss}} \cdot \frac{\partial \bar{k}_{ss}}{\partial \delta} + d^Q_{\bar{k}_{ss}} \frac{\partial^2 \bar{k}_{ss}}{\partial \delta^2}\right] \sigma^2_{\delta} \tag{16}$$

Also,

$$q_t|_{\delta=\bar{\delta}} = \left(\ln(\bar{k}_{t+1}) - \ln(\bar{k}_{ss})\right)^2 . \tag{17}$$

In the steady state, $q|_{\delta=\bar{\delta}} = d^Q_{\bar{k}} = d^Q_{\bar{\delta}} = d^Q_{\bar{k}_{ss}} = d^Q_{\bar{\delta}_{ss}} = 0$.

4.4 Households

Households are standard utility maximising agents who solve the following optimisation problem;

$$\max_{C_{t+j}, N_{t+j}, L_{t+j}} \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\sigma} - 1}{1-\sigma} - \nu \frac{N_{t+j}^{1+\psi}}{1+\psi}\right)\right] \tag{18}$$

subject to the period budget constraint

$$C_t + L_{t+1} = W_t N_t + R_t L_t + T_t \qquad \forall t \qquad (\lambda_t). \tag{19}$$

Above are all aggregate variables, because the households are identical and each represents the aggregate. T are net transfers received from the aggregate firm, discussed in detail in the next sub section.

The resulting optimality conditions are the standard ones, and are stated in the appendix.

4.5 Market clearing

The way we have set up the production sector, with firms exiting each period and being replaced by new firms, requires that each period firms that close be paid off their liquidation value and also seed capital for new firms be received from households. Unless the exiting firms are returned their net worth, the aggregate firm will accumulate capital indefinitely, and there will be no need for loans to finance further capital acquisition. This will make the collateral

[8] Refer Appendix A.5 for further details

constraint redundant, as well as causing issues with market clearing and the National Income Accounting (NIA) identity, which will not hold.

The aggregate flow of funds constraint is as under;

$$K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - \int K_{t+1}(i)\mathrm{d}i - R_t L_t + \int (1-\delta(i))K_t(i)\mathrm{d}i + L_{t+1} - Q_t - T_t = 0. \tag{20}$$

Similar to the individual firms (represented by variables in lower cases)[9], inflows are from aggregate sales, loans, and market value of undepreciated capital, whereas outflows are in the form of wages, new investment, loan repayments, and net transfers T. The transfers are arrived at as under;

$$T_t = \zeta \overline{K}_{ss} - (1-\theta)(\overline{K}_{t+1} - L_{t+1}). \tag{21}$$

The proportion of steady state balance sheet portfolio $\bar{K}$ which the households provide to the new firm as seed capital is ζ. The amount returned to all firms that close down is their capital assets net of loan obligations, $(\bar{K}_{t+1} - L_{t+1})$, and such proportion of firms are $(1-\theta)$ of the total number of firms by the law of large numbers considering that the probability of exiting is $(1-\theta)$ each period.

Combining (20) and (19) yields the NIA;

$$Y_t = C_t + \overline{K}_{t+1} - (1 - D_t)\overline{K}_t + Q_t \tag{22}$$

where Y results from the production function;

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{23}$$

Next we discuss how misallocation is relevant in our set up and what causes the efficiency wedge.

4.6 Depreciation rates

There are three different measures of depreciation mentioned so far. The first, $\delta(i)$, is the individual rate of depreciation for asset (i). This rate is considered constant and set by taxation

[9]The upper case variables at the aggregate firm level are just firm level variables aggregated over a continuum of j identical firms.

schedules. The theoretical minimum value is 0 for near permanent assets like land, and theoretical maximum is 1 for assets which can be fully depreciated in about a year.

The second measure of depreciation is the average of all of the above constant rates, $\bar{\delta} := \int_0^1 \delta(i)\mathrm{d}i$. This is the rate of depreciation around which our second order approximations are made, and the mean rate around which the variance σ_δ^2 is calculated.

The final measure, the time varying rate of depreciation D, is our contribution from this exercise, and it depends on how the portfolio composition changes on impact of a technology shock.

4.7 Reallocation and measured TFP

As discussed previously, conventionally most misallocation literature takes as its base heterogeneous firms, and then either some kind of firm specific wedge is introduced in the form of taxes, or there are idiosyncratic productivity shocks which cause misallocation of factors between firms. Some papers also assume two types of agents, borrowers and lenders, and show how a heterogeneous agent model can generate misallocation , and how firms can overcome that over time with savings. They find that most times the shock to productivity needs to be large for misallocation to be persistent. In our model, however, we do not have a productivity shock. Instead, we have a financial shock which mimics a productivity shock. Also, we have heterogeneity along the asset types dimension, not along the firm dimension.

A shock originating in the financial sector changes the balance sheet portfolio composition of the aggregate firm by making the high depreciation assets easier to access than the low depreciation, longer term assets. As the balance sheet portfolio changes, the production bundle also is forced to change and these choices, which are driven by the fact that firms are constrained for loans, lead to changes in measured productivity. The mechanism is explained in the previous section, but the intuition for the change in measured productivity needs more discussion. In the steady state of the model, firm's portfolio is heavy in low depreciation assets. These assets serve as better collateral for loans, but the holding them in such a high proportion leads to a portfolio which is not the most productive in terms of output. When there is reallocation after a financial shock, the portfolio composition changes and the new capital bundles formed from this portfolio produce more output relative to the replacement cost of that portfolio (given that marginal product of higher depreciation assets is greater than that of lower depreciation assets, as shown in Appendix C), and this is what shows up as an increase in measured TFP. This response is also largely dependent on the calibration of the elasticity of substitution between capital assets, ε. If assets were perfect complements (substitutes), the impact of a balance sheet reallocation on the production bundle would be larger (almost none), and the response of measured TFP would change accordingly.

The *efficiency wedge*, or the measure of productivity, takes the following form[10];

$$\tau_t = \left(\frac{K_t}{\overline{K}_t}\right)^{\alpha}. \tag{24}$$

As (24) shows, the wedge today reflects past investment decisions. This specification results from replacing K with $\overline{K}$ in the production function (23);

$$Y_t = \tau_t A_t \overline{K}_t^{\alpha} N_t^{1-\alpha}. \tag{25}$$

The product τA is the Solow residual, or the measure of total factor productivity productivity, used in the literature. The movement in τ results from optimising behaviour in response to a financial shock by the aggregate firm, and not from exogenous disturbances themselves as is the case in some of the literature discussed earlier.

4.8 Competitive equilibrium

For the competitive equilibrium, the states $\boldsymbol{s}$ are portfolio capital $\overline{K}$, production capital K, and the level of the financial and technology processes ξ and A.

Definition 1. *Recursive Equilibrium: A recursive competitive equilibrium is defined as a set of functions for (1) households' policies $C^h(\boldsymbol{s})$, $N^h(\boldsymbol{s})$, and $L^l(\boldsymbol{s})$; (2) production firms' policies $K(\boldsymbol{s})$, $\overline{K}(\boldsymbol{s})$, $L(\boldsymbol{s})$, and $N(\boldsymbol{s})$; (3) aggregate prices $W(\boldsymbol{s})$ and $R(\boldsymbol{s})$; (4) law of motion for aggregate states $\boldsymbol{s'} = \boldsymbol{\Phi}(\boldsymbol{s})$; such that (i) households policies satisfy its first order conditions; (ii) firms' policies are optimal and $V(\boldsymbol{s})$ satisfies the Bellman equation; (iii) wage and interest rates clear the labour and capital markets, and $m(\boldsymbol{s}) = \beta U_c(C',N')/U_c(C,N)$; (iv) the law of motion for $\boldsymbol{\Phi}(\boldsymbol{s})$ is consistent with individual decisions and the stochastic process for ξ.*

The full system of equations that describes the entire model is presented in appendix B.

[10]Fluctuations in the efficiency wedge does not necessarily imply departure from constrained efficiency in the model. We leave welfare analysis to future work.

The system is linearised and solved using perturbation methods around a deterministic steady state.

4.9 Calibration

The parameters values used are as under;

Parameter Values		
Parameter	Value	Description
β	0.995	Discount factor
α	1/3	Capital share
σ	1.01	Household intertemporal substitutability
ψ	0.25	Frisch elasticity parameter
ν	1	Disutility from labour
ε	0.3	Substitutability of capital assets
ζ	0.11†	Transfers parameter
χ	0.5	LTV parameter
θ	0.9	Probability of firm survival
$\bar{\delta}$	0.05◇	Average rate of depreciation
σ_{δ}^2	0.01	Variance of depreciation rates
ρ_{ξ}	0.9	Shock persistence
ρ_A	0.95	Technology shock persistence
σ_{ξ}^2	0.01	Variance of financial shock
σ_A^2	0.01	Variance of technology shock
$\partial\bar{k}_{t+1}/\partial\delta$	-11.1*	Slope of capital
$\partial\bar{k}_{ss}/\partial\delta$	-11.1*	Slope of capital (Steady State)
$\partial^2\bar{k}_{t+1}/\partial\delta^2$	0‡	Change in slope of capital
$\partial^2\bar{k}_{ss}/\partial\delta^2$	0‡	Change in slope of capital (Steady State)

†Calibrated to clear markets in steady state
◇Chosen such that steady state rate of time-varying depreciation, $D \approx 2.6\%$
*Obtained from data
‡Assumed to be linear based on above

Table 1: Parameters

The degree of asset substitutability $\varepsilon = 0.3$ implies that the capital assets are complements. This value is chosen as it gives good steady state results and also sensible dynamics for the model. In fact, $\varepsilon \in [0, 1)$ gives sensible results. If $\varepsilon > 1$, the steady state values seem implausible, likely from the fact that the higher is the substitutability between assets, the less reason there is for firms to hold shorter term assets. In the approximations, ε appears in the variables τ^k, τ^d, and τ^γ which are important in pinning down $\overline{K}$, D, and λ respectively. In the first two, ε appears as $(\varepsilon + 1)$, so the fact whether it is greater and or smaller than 1 does not matter greatly.

However, for τ^y (from (15)), it appears as $(\varepsilon - 1)$, so if it is greater than (less than) 1 it will have a positive (negative) effect on the value of the variable and $\varepsilon \lesseqgtr 1 \Leftrightarrow \tau^y \downarrow\uparrow$. Moreover, it is raised to the power $\frac{1}{\varepsilon-1}$, so either it is raised to a high positive power if $\varepsilon > 1$ or to a high negative power if $\varepsilon < 1$. So, if $\varepsilon > 1$ the net effect is positive, whereas if $\varepsilon < 1$ the net effect is negative on λ_t. We find, however, that this does not change much as regards response of λ_t as the movement in other variables seems to dominate the changes in τ^y.

The transfers parameter ζ is arrived at by finding the transfers from the firm to households in steady state that clear the market, and then backing out the implied parameter from (21) in the steady state. This will be explained further in the section on steady state analysis.

Survival probability θ for individual firms is set to 90%. An increase in survival probability reduces transfers from aggregate firm to households each period and if survival rate is 100%, then firms again over accumulate capital with households providing seed capital for new firms each period, and that makes the collateral constraint irrelevant. A survival rate of 90% implies average lifespan for a firm of around 5.5 years, or 22 quarters.

The Frisch elasticity parameter ψ is calibrated such that the elasticity of labour supply to change in wage rate, $\frac{\partial N/N}{\partial W/W} = \frac{1}{\psi} = 4$, so an increase in ψ actually reduces the slope of the labour supply curve, and hence the impact of a change in supply on wage rate. The value chosen here is relatively high, and as in Gertler and Kiyotaki (2010), is chosen to compensate for the absence of frictions such as nominal wage and price rigidities.

The variance of depreciation rates σ_δ^2 is set at 0.01, although it might be argued that it can be anything up to 0.08 or thereabout. However, the role of this parameter is only to change the magnitude of response in the dynamics, and to some extent, change slightly the values in steady state, so it does not seem to be critical to the extent of the value of ε.

The average rate of depreciation $\bar{\delta}$ has been chosen such that the time varying rate of depreciation D in the steady state is around $\sim 2.6\%$ which is the standard rate of depreciation used in an RBC model.

The derivative of the capital with respect to depreciation rate has been obtained from data using a simple linear regression with the specification:

$$\ln k(i) = a + b \cdot \delta(i) \tag{26}$$

where we create bins for depreciation rates and also for $k(i)$ accordingly. The value of b

is $\partial k(i)/\partial\delta(i)$, and we assume that this slope is constant in and out of steady state. Also, given the linear form of the relationship we have assumed between assets and depreciation rates, the term $\partial^2 k(i)/\partial\delta(i)^2 = 0$.

The discount factor β, capital share α, intertemporal substitutability of households σ, the disutility of labour ν, and the persistence of the shock ρ have been calibrated to the standard RBC values. A discount factor of $\beta = 0.995$ implies an annual risk free rate of around 2%.

Next we analyse the steady state and how it is solved.

4.10 Steady state analysis

Some variables are pinned down in the steady state. Among them, $R = \frac{1}{\beta}$, $m = \beta$ help pin down ϕ. This pins down η and $d\eta$, which in turn pins down τ^k, τ^d, τ^y, D, and F_k.

This reduces the system in steady state to the following;

$$C^{-\sigma} = \lambda \tag{27}$$

$$\nu N^{\psi} = \lambda W \tag{28}$$

$$W = (1-\alpha)AK^{\alpha}N^{-\alpha} \tag{29}$$

$$F_k = \alpha A K^{\alpha-1}N^{1-\alpha} \tag{30}$$

$$\overline{K} = K\left(\frac{\eta}{\theta\beta F_k}\right)^{-\varepsilon}(1+\hat{\tau}^k) \tag{31}$$

$$0 = C - WN - T + L(1-R) \tag{32}$$

$$Y = C + D\overline{K} \tag{33}$$

$$Y = AK^{\alpha}N^{1-\alpha} \tag{34}$$

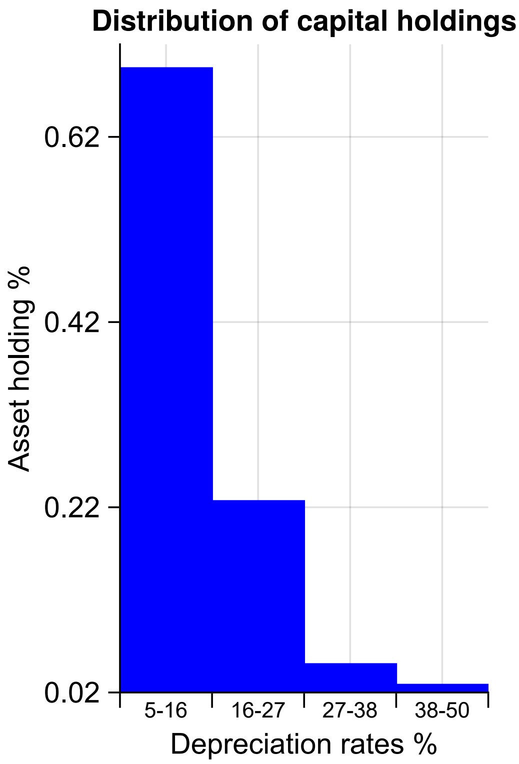
$$RL = \chi(1-D)\overline{K} \tag{35}$$

The 9 variables are

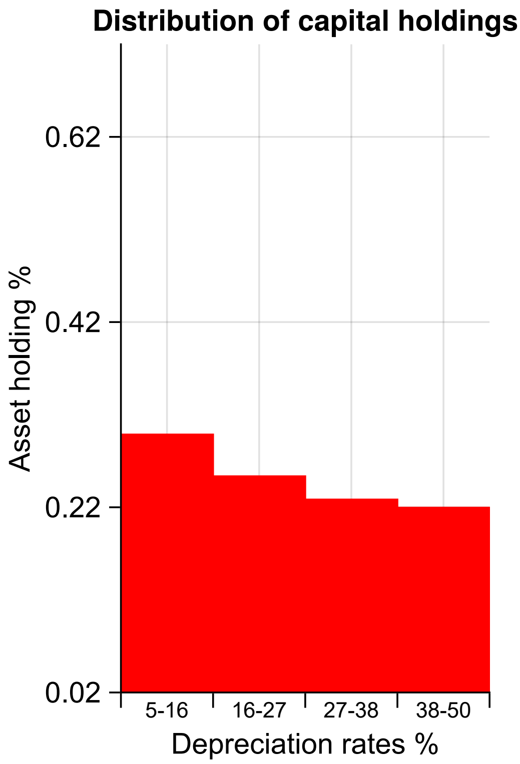
$$C \;\; \lambda \;\; N \;\; W \;\; \overline{K} \;\; Y \;\; K \;\; T \;\; L$$

We pin down the transfers in steady state using the aggregate firm flow of funds and the national income accounting identity. The NIA must hold in each period including in the equilibrium, and we use this to back out transfers as the balance from aggregate firm flow of funds. This numerical amount obtained for T is then used in (21) on the left hand side, along with the steady state values of $\overline{K}$ and L to back out the value of parameter ζ.

In the steady state we can compare the asset distribution generated by the model versus that observed in the data.



(a) Data



(b) Model

Figure 5: Asset distribution: Data vs. Model

The left panel shows asset distribution in data for the year 2007, the year before the crisis, divided into four large bins. The balance sheet is heavy in low depreciation assets, and has few high depreciation assets. The right panel shows the distribution of assets generated by the model in the same number of bins. The model also generates a balance sheet heavy in low depreciation assets, but not to the extent of the data.

The numerical steady state solution is as under;

Steady State Values		
Variable	Description	Model
C/Y	Consumption-Output ratio	0.90
$\overline{K}/Y$	Portfolio Capital-Output ratio	3.75
K/Y	Production Capital-Output ratio	3.35
N	Labour	0.78
D	Time-varying aggregate depreciation	0.026
τ	Efficiency wedge	0.96

Table 2: Steady State Values

Table 2 presents the steady state ratios generated by the model with capital portfolio and individual capital adjustment costs. The amount of capital is lower than a standard RBC for similar calibration because of the higher average rate of depreciation δ in our model, as well as the adjustment costs.

5 Results

We present results from the dynamic model in this section. First we show the impulse response functions from a shock to the LTV, and then discuss them in detail.

5.1 Impulse response functions

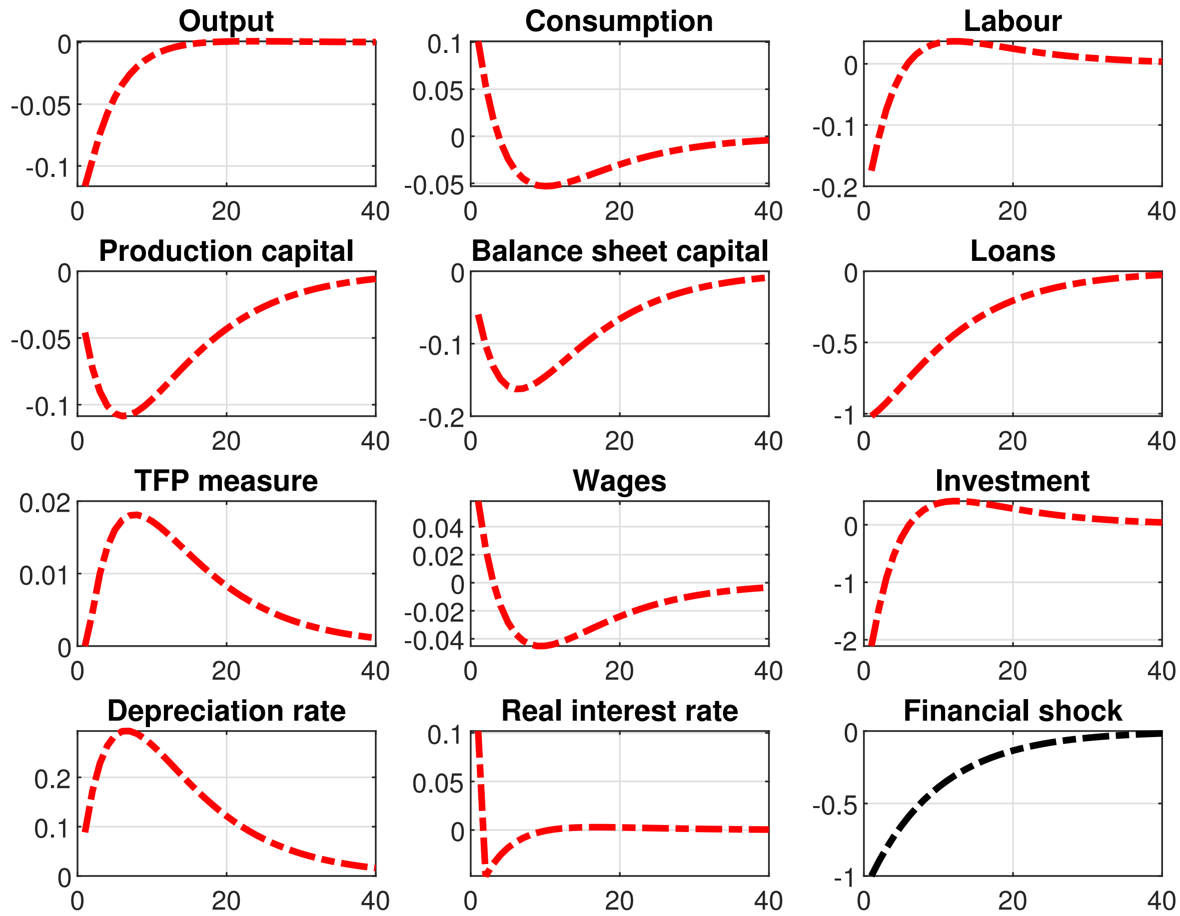


Figure 6: Impulse Response functions for a financial shock

Responses have been scaled by $1e + 2$ for clarity

5.2 Discussion

The response of variables in the portfolio model depends upon how the shock is transmitted through the economy, and it is different from a technology shock in the canonical RBC case. Here, as the LTV falls suddenly, we have seen how demand for capital assets falls which causes the balance sheet portfolio to shrink and be reallocated as well, with low depreciation assets holding a proportionally smaller share, and other assets being reallocated as seen previously. As the portfolio shrinks, loans fall. With a different balance sheet portfolio to choose from, the production bundle shrinks as well. The assets are now in a very different proportion, and the fall in production bundle is lower than the fall in balance sheet portfolio. The existing capital seems excessive for the new environment, which causes marginal product of capital to fall as well. With a fall in capital accumulation, labour supply and demand falls.

Consumption rises slightly on impact as the households have more disposable income due to a fall in borrowing by firms. Wages rise slightly on impact as the fall in demand for labour is greater than the fall in supply of labour due to the income effect of a perceived increase in lifetime wealth.

τ rises as the composition of the portfolio changes and more short term assets are included. As seen in Appendix C, the marginal product of short term assets is higher than that of long term assets. As the proportion of these assets increases in the production bundle, output produced *relative to the replacement cost* is higher. This causes measured TFP τ to rise[11]. Response of TFP is also conditional on the calibration of the substitution elasticity between capital assets[12], ε. The higher is ε, the higher is the measure τ, and in the limit as assets become perfect complements, the measure is 1, which is the standard RBC case.

The response of variables to a financial shock is closer to what is observed in the data for the 2008 crisis, than the response to a technology shock.

5.3 IRFs for a Technology shock

For comparison, impulse response functions for a technology shock are presented below:

[11] Analytically, as $\overline{K}$ falls (and its composition changes), K also falls, but by less as it is constructed using a CES aggregator where elasticities are taken into account. Thus the ratio $\frac{K}{\overline{K}}$ rises and measured TFP τ rises.

[12] It is easy to verify that :

$$\frac{\partial \tau_t}{\partial \varepsilon} = \alpha(1+\hat{\tau}_t^k)^{-\alpha}\left(\frac{\eta_t}{\theta \mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{\varepsilon\alpha}\left[\ln\left(\frac{\eta_t}{\theta \mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right) - \frac{\frac{\partial \hat{\tau}_t^k}{\partial \varepsilon}}{1+\hat{\tau}_t^k}\right] < 0.$$

as $\frac{\partial \tau_t^k}{\partial \varepsilon} > 0$, also easily verifiable analytically. The term in square brackets is negative as the ln value is almost 0 or even negative.

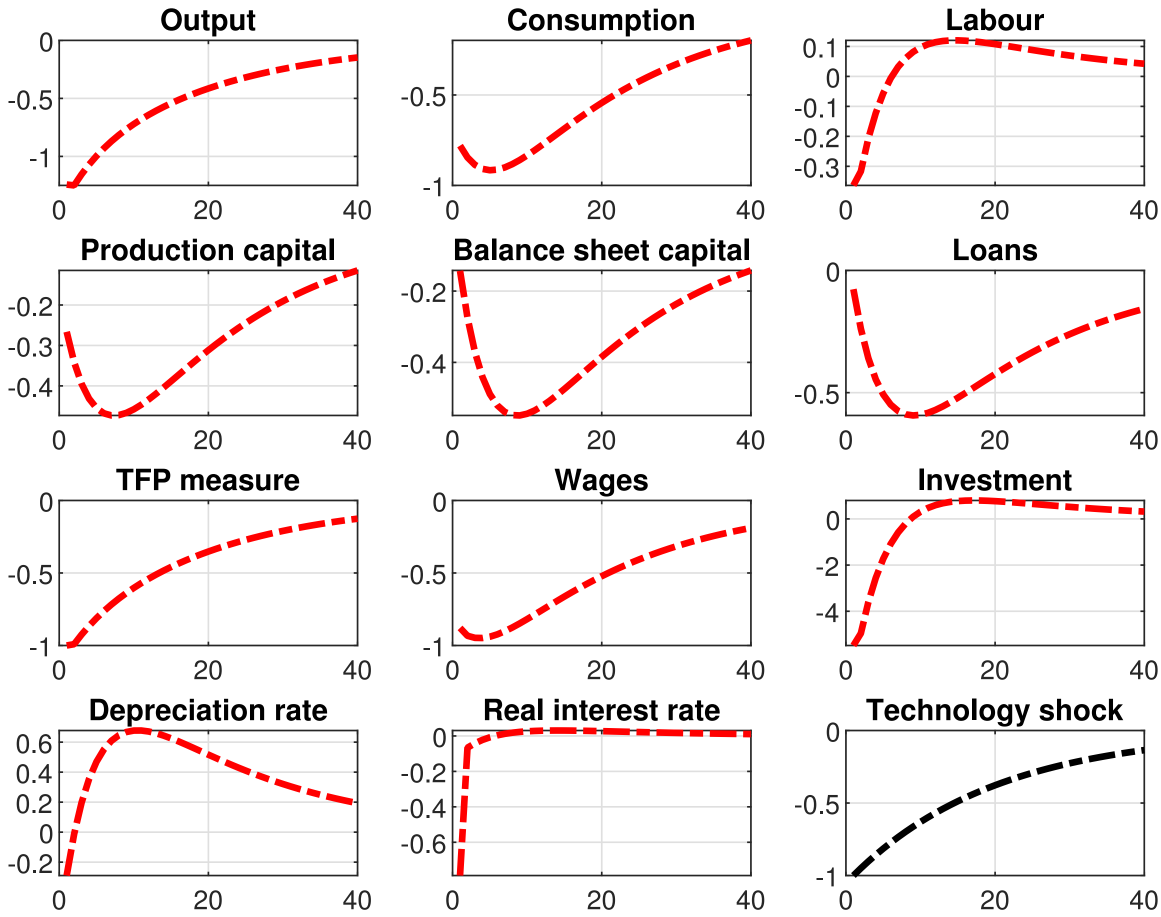


Figure 7: Impulse Response functions for a technology shock

Responses have been scaled by $1e + 2$ for clarity

The response to a technology shock is very similar to that for a standard RBC model. A technology shock generates a business cycle with output, consumption, labour, capital, and investment falling on impact. The measured TFP also falls on impact. However, the depreciation rate falls on impact, which indicates that initially there is a movement towards long term assets on impact of shock. This is contrary to what is observed in the data. Also, for the 2008 crisis, data shows that the aggregate rate of depreciation (computed as total current cost depreciation charge over total current cost net fixed assets) has risen, as seen in Figure 3. These two facts seem to be more in line with a financial shock, rather than a technology shock.

5.4 Second moments

We compute the second moments for variables in our model and compare them with moments from the data and from a standard RBC model. All data is from the BEA and is expressed as real per capita quarterly figures. Output (Y) is the GDP series starting from 1947 Q1 till 2020

Q3, Investment (I) is the sum of private fixed investment and durables consumption starting 2002 Q1 till 2020 Q3, Consumption (C) is the nondurables and services consumption starting 1947 Q1 till 2020 Q3, Labour hours (N) are the recorded hours in the nonfarm sector starting 1948 Q1 till 2020 Q3 and Wages (W) is the nonfarm compensation starting 1947 Q1 till 2020 Q3. All data are in logs and filtered using the HP filter. This methodology is the same as adopted by King and Rebelo (1999).

As for the RBC model and our model, we solve both models using parameter values stated previously. We then simulate each variable series for 1000 periods and discard the first 100 periods. We then HP filter the data and compute moments based on the cyclical deviations.

Moments for model with collateral constraint

	σ_X/σ_Y			$ACF(1)$			$corr(X,Y)$		
Variable	Data	RBC	Model	Data	RBC	Model	Data	RBC	Model
Y	1.0	1.0	1.0	0.78	0.67	0.72	1.0	1.0	1.0
I	2.4	2.4	4.8	0.88	0.65	0.67	0.75	0.99	0.95
C	0.6	0.5	0.75	0.63	0.71	0.80	0.79	0.97	0.93
N	1.1	0.6	0.4	0.81	0.65	0.67	0.89	0.99	0.78
W	0.6	0.5	0.75	0.65	0.67	0.78	0.02	0.99	0.96

Table 3: Moments for model with collateral constraint

The relative standard deviations for investment and consumption are higher in the portfolio model as compared to the data or RBC model. The RBC model underestimates the relative standard deviation of consumption to output, while the portfolio model overestimates it. Investment in the portfolio model is more volatile than the RBC model and the data. However, the portfolio model does a much better job than the RBC model at matching autocorrelation coefficients. The autocorrelations for output, investment, and labour supply are also closer to data for the portfolio model compared to RBC. Also, the cross correlations among variables are marginally better in the portfolio model than the RBC model; all variables in the RBC model are driven uniformly by the technology shock and hence the cross correlations are almost one, whereas in the portfolio model the shock passes through the model very differently, as discussed earlier, and generates more realistic cross correlations. Overall, the moments from the portfolio model seem more realistic and closer to the data as compared to moments from the RBC model, except that investment in the portfolio model is much more volatile. Perhaps higher costs of adjusting investment can help match the data better.

6 Conclusion

We build a general equilibrium model with financial shocks to connect stylised facts from data, namely; a shock originating in the financial sector changes balance sheet portfolio of the aggregate firm and impacts measured productivity. We distinguish between the balance sheet portfolio and production bundle deployed by the firm and use a continuum of capital assets which the firm can choose from, depending on the credit constraint. The longer term assets serve as better collateral, but firms are unable to access these assets when the credit market conditions deteriorate, causing the balance sheet portfolio and production bundle to be reshuffled[13]. Given the degree of substitutability between different assets types, the distinction between balance sheet and production capital shows up as a 'wedge' when measuring productivity. A negative financial shock causes this wedge to move counter-cyclically and thus raises measured productivity. The intuitive explanation is as follows: when financing becomes harder, firms choose the cheaper, higher depreciation assets. Although this results from optimal choice in a deteriorating environment, there is an increase in measured TFP due to the firms having a more balanced portfolio which is not as skewed towards long term assets. This causes output of the portfolio to exceed its replacement cost, showing up as an increase in measured TFP.

Our model connects financial shocks to measured productivity without relying on productivity shocks, or any other kind of heterogeneity among firms as regards credit access or individual productivity, as is the case in most of the existing literature. The next step would be to add nominal rigidities and generate policy implications in this set up.

[13]This is not to mean, however, that in the absence of credit constraints, firms would hold more low depreciation assets. In fact, as shown in Figure D1 in Appendix D, firms hold less of long term assets in the absence of credit constraints. This implies that the only purpose of holding long term assets is their value as collateral, and to leverage them to obtain loans.

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Appendix A Second order approximations

A.1 Approximation technique

Consider the following Taylor series expansion;

$$
\begin{aligned}
y &= \left[\int_0^1 x_i^{\frac{1}{p}}\, \mathrm{d}i\right]^p \\
&\approx \left[\int_0^1 \left(x_0^{\frac{1}{p}} + \frac{1}{p}x_0^{\frac{1-p}{p}}(x_i - x_0) + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}(x_i - x_0)^2\right)\mathrm{d}i\right]^p \\
&\approx \left(x_0^{\frac{1}{p}} + \frac{1}{p}x_0^{\frac{1-p}{p}}\left(\int x_i\, \mathrm{d}i - x_0\right) + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}\left(\int x_i^2\, \mathrm{d}i - 2\int x_i\, \mathrm{d}i\; x_0 + x_0^2\right)\right)^p \\
&\approx \left(x_0^{\frac{1}{p}} + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}\operatorname{var}_i[x_i]\right)^p \qquad \left(\text{Assuming} \quad x_0 = \int x_i\, \mathrm{d}i\right) \\
&\approx \left(x_0^{\frac{1}{p}} + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}\operatorname{var}_i[x_i]\right)^p \frac{x_0}{x_0} \\
&\approx \left[\frac{1}{x_0^{\frac{1}{p}}}\left(x_0^{\frac{1}{p}} + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}\operatorname{var}_i[x_i]\right)\right]^p x_0 \\
&\approx x_0\left(1 + \frac{(1-p)}{2p^2}\frac{\operatorname{var}_i[x_i]}{x_0^2}\right)^p
\end{aligned}
\tag{A1}
$$

We will use the general result in (A1) in Appendix B to derive the system equations.

A.2 Demand function approximation

The demand function from (6) is as under;

$$
k_{t+1}(i) = k_{t+1}\left(\frac{\overbrace{1 - (1-\delta(i))(\theta\mathbb{E}_t[m_{t+1}Q_{t+1}] + \phi_t\xi_t\chi) + d_{k,t}^{Q}}^{\eta_t(i)}}{\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{-\varepsilon}
$$

We start by approximating $\eta_t(i)^{-\varepsilon}$ around $\bar{\delta} = \int \delta(i)\mathrm{d}i$, the average rate of depreciation. We have the following additional variables in this specification;

$$
\begin{aligned}
d\eta_t &= \theta \mathbb{E}_t m_{t+1} Q_{t+1} + \phi_t \xi_t \chi && \text{(A2)}\\
d_{\bar{k}}^{Q} &= 2 \cdot \frac{(\ln(\bar{k}_{t+1}) - \ln(\bar{k}_{ss}))}{\bar{k}_{t+1}} && \text{(A3)}\\
d_{\delta}^{Q} &= 2\left(\frac{1 - (\ln(\bar{k}_{t+1}) - \ln(\bar{k}_{ss}))}{(\bar{k}_{t+1})^2}\right)\frac{\partial \bar{k}}{\partial \delta} - \frac{2}{\bar{k}_{t+1}\bar{k}_{ss}}\frac{\partial \bar{k}_{ss}}{\partial \delta} && \text{(A4)}\\
d_{\bar{k}_{ss}}^{Q} &= -2 \cdot \frac{(\ln(\bar{k}_{t+1}) - \ln(\bar{k}_{ss}))}{\bar{k}_{ss}} && \text{(A5)}\\
d_{\delta_{ss}}^{Q} &= 2\left(\frac{1 + (\ln(\bar{k}_{t+1}) - \ln(\bar{k}_{ss}))}{(\bar{k}_{ss})^2}\right)\frac{\partial \bar{k}_{ss}}{\partial \delta} - \frac{2}{\bar{k}_{t+1}\bar{k}_{ss}}\frac{\partial \bar{k}}{\partial \delta} && \text{(A6)}
\end{aligned}
$$

where $\frac{\partial \bar{k}}{\partial \delta}$ and $\frac{\partial \bar{k}_{ss}}{\partial \delta}$ are parameters obtained from data.

The approximation is as under:

$$
\begin{aligned}
\eta_t(i)^{-\varepsilon} &\approx \eta_t^{-\varepsilon} + \left[-\varepsilon \eta_t^{-\varepsilon-1}(d\eta + d_{\delta}^{Q})\right](\delta(i) - \bar{\delta})\\
&+ \frac{1}{2}\left[\varepsilon(\varepsilon+1)\eta_t^{-\varepsilon-2}((d\eta_t)^2 + d\eta_t d_{\delta}^{Q}) - \varepsilon \eta_t^{-\varepsilon-1} d_{\delta}^{2Q}\right](\delta(i) - \bar{\delta})^2 + ||O||^3\\
\Rightarrow \int \eta_t(i)^{-\varepsilon}\mathrm{d}i &\approx \eta_t^{-\varepsilon}\left\{1 + \underbrace{\frac{1}{2}\varepsilon\sigma_{\delta}^2\left[(\varepsilon+1)\left(\frac{(d\eta_t)^2 + d_{\delta,t}^{Q}}{(\eta_t)^2}\right) - \frac{d_{\delta,t}^{2Q}}{\eta_t}\right]}_{\hat{\tau}_t^k}\right\}
\end{aligned}
\tag{A7}
$$

Substituting (A7) into (6) gives (11);

$$
\int k_{t+1}(i)\mathrm{d}i := \bar{k}_{t+1} = k_{t+1}\left(\frac{\eta_t}{\theta \mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{-\varepsilon} \cdot \{1 + \hat{\tau}_t^k\}
$$

A.3 Effective rate of depreciation approximation

We start with the identity

$$
\begin{aligned}
(1 - D_t)\bar{k}_t &\equiv \int (1 - \delta(i))k_t(i)\mathrm{d}i\\
&= \int (1 - \delta(i))\eta_{t-1}(i)^{-\varepsilon}\mathrm{d}i \; k_t(\theta \mathbb{E}_{t-1}[m_t F_{k,t}])^{\varepsilon}
\end{aligned}
\tag{A8}
$$

We now approximate $(1-\delta(i))\eta_{t-1}(i)^{-\varepsilon}\mathrm{d}i$;

$$
\begin{aligned}
(1-\delta(i))\eta_{t-1}(i)^{-\varepsilon} \approx\ & (1-\delta)\eta_{t-1}^{-\varepsilon} + \left[-\varepsilon(1-\bar{\delta})\eta_{t-1}^{-\varepsilon-1}(d\eta_{t-1}+d_{\delta,t-1}^{Q}) - \eta_{t-1}^{-\varepsilon}\right](\delta(i)-\bar{\delta}) + \\
& \frac{1}{2}\Big[\varepsilon\eta_{t-1}^{-\varepsilon-1}(d\eta_{t-1}+d_{\delta,t-1}^{Q}) + \varepsilon(\varepsilon+1)(1-\bar{\delta})\eta_{t-1}^{-\varepsilon-2}((d\eta_{t-1})^2 + d\eta_{t-1}d_{\delta,t-1}^{Q}) \\
& - \varepsilon(1-\bar{\delta})\eta_{t-1}^{-\varepsilon-1}d_{\delta,t-1}^{2Q} + \varepsilon\eta_{t-1}^{-\varepsilon-1}(d\eta_{t-1}+d_{\delta,t-1}^{Q})\Big](\delta(i)-\bar{\delta})^2 + ||O||^3 \\
\Rightarrow \int(1-\delta(i))\eta_{t-1}(i)^{-\varepsilon}\mathrm{d}i \approx\ & (1-\bar{\delta})\eta_{t-1}^{-\varepsilon}\Bigg\{1 + \frac{1}{2}\frac{\varepsilon\sigma_{\delta}^{2}}{\eta_{t-1}}\Bigg(\frac{2}{1-\delta}\cdot(d\eta_{t-1}+d_{\delta,t-1}^{Q}) + \\
& (\varepsilon+1)\left(\frac{(d\eta_{t-1})^2 + d\eta_{t-1}d_{\delta,t-1}^{Q}}{\eta_{t-1}}\right) - d_{\delta,t-1}^{2Q}\Bigg)\Bigg\} \\
\approx\ & (1-\bar{\delta})\eta_{t-1}^{-\varepsilon}\cdot(1+\hat{\tau}_{t-1}^{d})
\end{aligned}
\tag{A9}
$$

Combining (A7), (A8), and (A9) yields the following;

$$
D_t = 1-(1-\bar{\delta})\cdot\left\{\frac{\left\{1+\hat{\tau}_{t-1}^{d}\right\}}{\left\{1+\hat{\tau}_{t-1}^{k}\right\}}\right\} \tag{A10}
$$

A.4 Capital Euler approximation

We start with the production bundle construction (5)

$$
k_t = \left[\int k_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad (F_{k,t})
$$

Moving it forward one period, and substituting for $k(i)$ gives;

$$
1 = \left[\int \eta_t(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}(\theta\mathbb{E}_t m_{t+1}F_{k,t+1})^{\varepsilon} \tag{A11}
$$

We approximate $\int\eta_t(i)^{1-\varepsilon}\mathrm{d}i$ around $\int\delta(i)\mathrm{d}i = \bar{\delta}$:

$$\eta_t(i)^{1-\varepsilon} \approx \eta_t^{1-\varepsilon} + (1-\varepsilon)\eta_t^{-\varepsilon}(d\eta_t + d^Q_{\delta,t})(\delta(i) - \bar{\delta}) + \frac{1}{2}\left[-\varepsilon(1-\varepsilon)\eta_t^{-\varepsilon-1}\left((d\eta_t)^2 + d\eta_t d^Q_{\delta,t} - \frac{\eta_t}{\varepsilon}d^{2Q}_{\delta,t}\right)\right](\delta(i)-\bar{\delta})^2 + ||O||^3$$

$$\Rightarrow \int \eta_t(i)^{1-\varepsilon}\mathrm{d}i \approx \eta_t^{1-\varepsilon}\left\{\underbrace{1 - \frac{1}{2}\varepsilon(1-\varepsilon)\frac{\sigma^2_\delta}{\eta_t^2}\left((d\eta_t)^2 + d\eta_t d^Q_{\delta,t} - \frac{\eta_t}{\varepsilon}d^{2Q}_{\delta,t}\right)}_{\hat{\tau}^\gamma}\right\} \tag{A12}$$

Substituting (A12) into (A11) and rearranging gives:

$$1 = \frac{\eta_t(\hat{\tau}_t^\gamma)^{\frac{1}{1-\varepsilon}}}{\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]} \tag{A13}$$

A more intuitive representation of (A13) would be the below;

$$\lambda_t = \frac{\theta\beta\mathbb{E}_t\lambda_{t+1}\left[F_{k,t+1} + (1-\bar{\delta}(\hat{\tau}_t^\gamma)^{\frac{1}{1-\varepsilon}}\right]}{(1-(1-\bar{\delta})\phi_t\xi_t\chi)(\hat{\tau}_t^\gamma)^{\frac{1}{1-\varepsilon}}} \tag{A14}$$

A.5 Approximating adjustment cost

Finally, we also need to have an approximation for the adjustment cost. The general form is;

$$q_t \approx q_t|_{\delta=\bar{\delta}} + \left[d^Q_{k(i)}\frac{\partial k(i)}{\partial\delta} + \frac{\partial q}{\partial k_{ss}}\frac{\partial k_{ss}}{\partial\delta}\right]\left(\int\delta(i)\mathrm{d}i - \bar{\delta}\right) + \frac{1}{2}\left[d^Q_\delta\cdot\frac{\partial k(i)}{\partial\delta} + d^Q_{k(i)}\frac{\partial^2 k(i)}{\partial\delta^2} + d^Q_{\delta_{ss}}\cdot\frac{\partial k_{ss}}{\partial\delta} + d^Q_{k_{ss}}\frac{\partial^2 k_{ss}}{\partial\delta^2}\right]\left(\int\delta(i)\mathrm{d}i - \bar{\delta}\right)^2 + ||O||^3 \tag{A15}$$

Since $\int\delta(i)\mathrm{d}i = \bar{\delta}$,

$$q_t \approx q_t|_{\delta=\bar{\delta}} + \frac{1}{2}\left[d^Q_\delta\cdot\frac{\partial k(i)}{\partial\delta} + d^Q_{k(i)}\frac{\partial^2 k(i)}{\partial\delta^2} + d^Q_{\delta_{ss}}\cdot\frac{\partial k_{ss}}{\partial\delta} + d^Q_{k_{ss}}\frac{\partial^2 k_{ss}}{\partial\delta^2}\right]\sigma^2_\delta \tag{A16}$$

Here, given our functional form;

$$d^Q_{k(i)} = \frac{2(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{k_{t+1}(i)} \tag{A17}$$

$$d^Q_{\delta} = 2\left(\frac{1-(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{(k_{t+1}(i))^2}\right)\frac{\partial k(i)}{\partial \delta} - \frac{2}{k_{t+1}(i)k_{ss}}\frac{\partial k_{ss}}{\partial \delta} \tag{A18}$$

$$d^Q_{k_{ss}} = -\frac{2(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{k_{ss}} \tag{A19}$$

$$d^Q_{\delta_{ss}} = 2\left(\frac{1+(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{(k_{ss})^2}\right)\frac{\partial k_{ss}}{\partial \delta} - \frac{2}{k_{t+1}(i)k_{ss}}\frac{\partial k(i)}{\partial \delta} \tag{A20}$$

As expected, in the steady state, the approximation for q yields,

$$q|_{\delta=\bar{\delta}} = 0$$
$$d^Q_{k(i)} = d^Q_{\delta} = d^Q_{k_{ss}} = d^Q_{\delta_{ss}} = 0$$

Appendix B Full system equations

$$C_t^{-\sigma} = \lambda_t \tag{B1}$$

Marginal Utlity of Consumption

$$\nu N_t^{\psi} = \lambda_t W_t \tag{B2}$$

Labour Supply

$$\lambda_t = \beta \mathbb{E}_t[\lambda_{t+1} R_{t+1}] \tag{B3}$$

Loans Euler

$$W_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{B4}$$

Wages

$$F_{k,t} = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{B5}$$

Marginal Product of Capital

$$Y_t = \tau_t A_t \overline{K}_t^{\alpha} N_t^{1-\alpha} \tag{B6}$$

Production Function

$$Y_t = C_t + \overline{K}_{t+1} - (1 - D_t)\overline{K}_t \tag{B7}$$

National Income Accounting Identity

$$\lambda_t = \frac{\theta \beta \mathbb{E}_t \lambda_{t+1} \left[F_{k,t+1} + (1-\bar{\delta}) \cdot (\hat{\tau}_t^{\gamma})^{\frac{1}{1-\varepsilon}} \right]}{(1 - (1-\bar{\delta}) \phi_t \xi_t \chi) \cdot (\hat{\tau}_t^{\gamma})^{\frac{1}{1-\varepsilon}}} \tag{B8}$$

Capital Euler

$$\hat{\tau}_t^{\gamma} := 1 - \frac{1}{2} \varepsilon (1-\varepsilon) \frac{\sigma_{\delta}^2}{\eta_t^2} \left((d\eta_t)^2 + d\eta_t d_{\delta,t}^{Q} - \frac{\eta_t}{\varepsilon} d_{\delta,t}^{2Q} \right) \tag{B9}$$

Euler wedge

$$\overline{K}_{t+1} = K_{t+1}\left(\frac{\eta_t}{\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{-\varepsilon}\cdot(1+\hat{\tau}_t^k) \tag{B10}$$

Portfolio & B/S Capital

$$\eta_t := 1-(1-\bar{\delta})[\theta\mathbb{E}_t[m_{t+1}]+\phi_t\xi_t\chi]+d_{k,t}^{Q} \tag{B11}$$

Intermediate variable

$$d\eta_t := \theta\mathbb{E}_t[m_{t+1}Q_{t+1}]+\phi_t\xi_t\chi \tag{B12}$$

$\frac{\partial\eta}{\partial\bar{\delta}}$

$$\hat{\tau}_t^k := \frac{1}{2}\varepsilon\sigma_\delta^2\left[(\varepsilon+1)\left(\frac{(d\eta_t)^2+d_{\delta,t}^{Q}}{(\eta_t)^2}\right)-\frac{d_{\delta,t}^{2Q}}{\eta_t}\right] \tag{B13}$$

Capital wedge

$$D_t = 1-(1-\bar{\delta})\cdot\frac{1+\hat{\tau}_{t-1}^k}{1+\hat{\tau}_{t-1}^d} \tag{B14}$$

Time varying depreciation rate

$$\hat{\tau}_t^d = \left\{1+\frac{1}{2}\varepsilon\sigma_\delta^2\left(\frac{d\eta_t}{\eta_t}\right)\left[\frac{2}{1-\bar{\delta}}+(\varepsilon+1)\frac{d\eta_t}{\eta_t}\right]\right\} \tag{B15}$$

Depreciation wedge

$$\phi_t = \frac{1}{\mathbb{E}_t R_{t+1}}-\theta\mathbb{E}_t m_{t+1} \tag{B16}$$

Loans foc

$$R_{t+1}L_{t+1} = \chi\xi_t(1-D_{t+1})\overline{K}_{t+1} \tag{B17}$$

Collateral constraint

$$\mathbb{E}_t m_{t+1} = \beta \left(\frac{\mathbb{E}_t \lambda_{t+1}}{\lambda_t} \right) \tag{B18}$$

Stochastic discount factor

$$T_t = \zeta \overline{K}_{ss} - (1-\theta)(\overline{K}_{t+1} - L_{t+1}) \tag{B19}$$

Transfers

$$\tau_t = \left(\frac{K_t}{\overline{K}_t} \right)^{\alpha} \tag{B20}$$

New TFP Measure

$$q_t \approx q_t|_{\delta=\bar{\delta}} + \frac{1}{2}\left[d^Q_{\delta} \cdot \frac{\partial k(i)}{\partial \delta} + d^Q_{k(i)} \frac{\partial^2 k(i)}{\partial \delta^2} + d^Q_{\delta_{ss}} \cdot \frac{\partial k_{ss}}{\partial \delta} + d^Q_{k_{ss}} \frac{\partial^2 k_{ss}}{\partial \delta^2} \right] \sigma^2_{\delta} \tag{B21}$$

Adjustment cost

$$d^Q_{k(i)} = \frac{2(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{k_{t+1}(i)} \tag{B22}$$

$$d^Q_{\delta} = 2\left(\frac{1 - (\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{(k_{t+1}(i))^2} \right) \frac{\partial k(i)}{\partial \delta} - \frac{2}{k_{t+1}(i) k_{ss}} \frac{\partial k_{ss}}{\partial \delta} \tag{B23}$$

$$d^Q_{k_{ss}} = -\frac{2(\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{k_{ss}} \tag{B24}$$

$$d^Q_{\delta_{ss}} = 2\left(\frac{1 + (\ln(k_{t+1}(i)) - \ln(k_{ss}(i))}{(k_{ss})^2} \right) \frac{\partial k_{ss}}{\partial \delta} - \frac{2}{k_{t+1}(i) k_{ss}} \frac{\partial k(i)}{\partial \delta} \tag{B25}$$

$$d^{2Q}_{\delta,t} = (1 - (\ln \bar{k}_t - \ln \bar{k}_{ss})) \cdot \left(\frac{2}{\bar{k}_t^2} \frac{\partial^2 k}{\partial \delta^2} - \frac{4}{\bar{k}_t^3} \left(\frac{\partial k}{\partial \delta} \right)^2 \right) + \frac{2}{\bar{k}_t \bar{k}_{ss}} \left(\frac{1}{\bar{k}_t} \frac{\partial k}{\partial \delta} \frac{\partial k_{ss}}{\partial \delta} + \frac{1}{\bar{k}_{ss}} \left(\frac{\partial k_{ss}}{\partial \delta} \right)^2 - \frac{\partial^2 k_{ss}}{\partial \delta^2} \right) \tag{B26}$$

$$\xi_t = \xi_{t-1}^{\rho_{\xi}} e^{v_t} \tag{B27}$$

LTV shock

$$A_t = A_{t-1}^{\rho_A} e^{u_t} \tag{B28}$$

Technology shock

The above forms a system of 28 equations in the following 28 variables;

$$C \;\; \lambda \;\; N \;\; W \;\; M \;\; R \;\; L \;\; D \;\; \overline{K} \;\; K \;\; \phi \;\; Y \;\; F_k \;\; \eta \;\; d\eta \;\; \hat{\tau}^k \;\; \hat{\tau}^d \;\; \hat{\tau}^{\gamma} \;\; \tau \;\; T \;\; q \;\; d^Q_k \;\; d^Q_{\delta} \;\; d^Q_{k_{ss}} \;\; d^Q_{\delta_{ss}} \;\; d^{2Q}_{\delta} \;\; \xi \;\; A$$

Appendix C Marginal products of individual assets

Replacing the CES aggregator into the production function;

$$Y_t = A_t \left[\int k_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\alpha\varepsilon}{\varepsilon-1}} N_t^{1-\alpha}$$

$$\Rightarrow MPK_t(i) = \alpha A_t k_t^{\frac{\varepsilon(\alpha-1)+1}{\varepsilon}} N_t^{1-\alpha} k_t(i)^{-\frac{1}{\varepsilon}}$$

Using demand function;

$$MPK_t(i)\Big|_{\delta(i)=1} = \frac{\alpha A_t k_t^{\frac{\varepsilon(\alpha-1)+1}{\varepsilon}} N_t^{1-\alpha}}{\theta \mathbb{E}_t[m_{t+1}F_{k,t+1}]}(1 + d^Q_{k(i)}|_{\delta(i)=1})$$

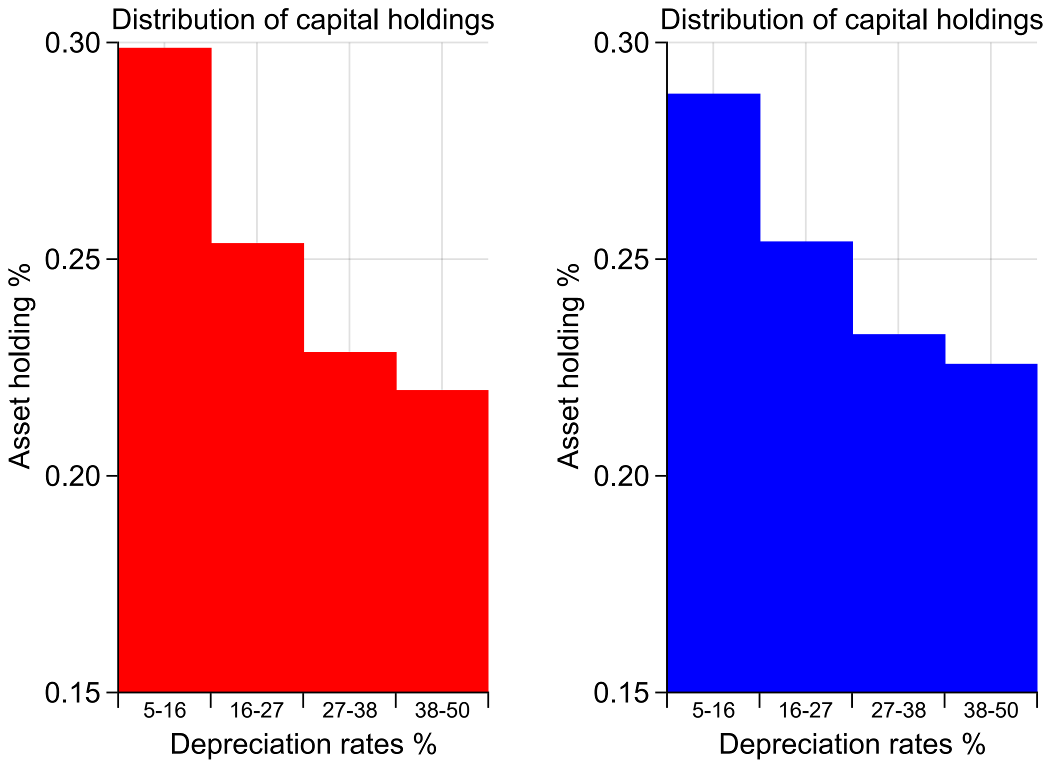
$$MPK_t(i)\Big|_{\delta(i)=0} = \frac{\alpha A_t k_t^{\frac{\varepsilon(\alpha-1)+1}{\varepsilon}} N_t^{1-\alpha}}{\theta \mathbb{E}_t[m_{t+1}F_{k,t+1}]}(1 - [\theta \mathbb{E}_t m_{t+1} + \phi_t \xi_t \chi] + d^Q_{k(i)}|_{\delta(i)=0})$$

$$\Rightarrow MPK_t(i)|_{\delta(i)=1} > MPK_t(i)|_{\delta(i)=0}$$

even though $d^Q_{k(i)}|_{\delta(i)=0} > d^Q_{k(i)}|_{\delta(i)=1}$, due to the log specification.

Appendix D Comparing asset distributions

Comparison of model with and without collateral constraints:



(a) Constrained steady state (b) Unconstrained steady state

Figure D1: Asset distribution: Constrained vs. Unconstrained

Figure D1 shows that if the firms are not constrained, they choose a lower proportion of long term assets than in the constrained state. Thus, it seems like the only purpose of loading up on long term assets is their better value as collateral.

Appendix E Components of Equipments and Structures

E.1 Equipment

Mainframes, PCs, DASDs, Printers, Terminals, Tape drives, Storage devices, System integrators, Communications, Nonelectro medical instruments, Electro medical instruments, Nonmedical instruments, Photocopy and related equipment, Office and accounting equipment, Nuclear fuel, Other fabricated metals, Steam engines, Internal combustion engines, Metalworking machinery, Special industrial machinery, General industrial equipment, Electric transmission and distribution, Light trucks (including utility vehicles), Other trucks, buses and truck trailers, Autos, Aircraft, Ships and boats, Railroad equipment, Household furniture, Other furniture, Other agricultural machinery, Farm tractors, Other construction machinery, Construction tractors, Mining and oilfield machinery, Service industry machinery, Household appliances, Other electrical, Other.

E.2 Structures

Office, Hospitals, Special care, Medical buildings, Multimerchandise shopping, Food and beverage establishments, Warehouses, Mobile structures, Other commercial, Manufacturing, Electric, Wind and solar, Gas, Petroleum pipelines, Communication, Petroleum and natural gas, Mining, Religious, Educational and vocational, Lodging, Amusement and recreation, Air transportation, Other transportation, Other railroad, Track replacement, Local transit structures, Other land transportation, Farm, Water supply, Sewage and waste disposal, Public safety, Highway and conservation and development.