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Partially Heterogeneous Tests for Granger Non-causality in Panel Data

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ABSTRACT

The power of Granger non-causality tests in panel data depends on the type of the alternative hypothesis: feedback from other variables might be homogeneous, homogeneous within groups or heterogeneous across different panel units. Existing tests have power against only one of these alternatives and may fail to reject the null hypothesis if the specified type of alternative is incorrect. This paper proposes a new Union-Intersections (UI) test which has correct size and good power against any type of alternative. The UI test is based on an existing test which is powerful against heterogeneous alternatives and a new Wald-type test which is powerful against homogeneous alternatives. The Wald test is designed to have good size and power properties for moderate to large time series dimensions and is based on a bias-corrected split panel jackknife-type estimator. Evidence from simulations confirm the new UI tests provide power against any direction of the alternative.

Keywords: Panel Data, Granger Causality, VAR.

JEL codes: C13, C33.

1. Introduction

Causality and feedback between variables is one of the main objects of applied time series research. Granger (1969) provided a definition which allows formal statistical testing of the hypothesis that one variable does not "Granger-cause" a second one. Besides time series analysis, this hypothesis is also important in panel data frameworks when examining the relationships between macroeconomic or microeconomic variables, see e.g. Holtz-Eakin et al. (1988). Testing for Granger non-causality in panel data models presents two additional complications: a) whether causation and feedback is homogeneous or heterogeneous across individuals and b) whether the asymptotic distributions used are adequately close to the sampling distributions, a problem which usually arises if the number of cross-section units N is of similar magnitude as the time-series dimension T. Such situations usually appear in macroeconomic panels or at many microeconomic panels with large time dimensions.

Causation may be homogeneous or heterogeneous across individual units or it may even be a mixture of the two, i.e. units may belong to different groups where within these groups there is homogeneity. All these cases describe hypothesis testing alternatives against the null hypothesis of Granger non-causality. Thus the researcher is in the difficult position of conducting a test against an alternative hypothesis which may not be true. This is a serious problem because existing hypothesis tests have very little power in these scenarios. Holtz-Eakin et al. (1988) proposed tests against homogeneous alternatives while Dumitrescu and Hurlin (2012) proposed tests against heterogeneous alternatives. However, neither of these two tests has power against all the possible alternatives; in particular, the tests by Holtz-Eakin et al. (1988) have limited power against heterogeneous alternatives that have mean close to zero while the test by Dumitrescu and Hurlin (2012) has limited power against homogeneous, group and near homogeneous alternatives.

The aim of this paper is to propose new tests of the Granger non-causality hypothesis that have power against every possible type of alternative. Doing so is not straightforward as the aforementioned tests have size and power properties which additionally depend on the length of the time series dimension of the panel. The main problem is that the tests proposed by Holtz-Eakin et al. (1988) are expected not to work well when the number of time series observations is large, see Alvarez and Arellano (2003).

We proceed by first proposing a new test for Granger-non-causality which works well against homogeneous alternatives when the time dimension is moderate to large. The novelty in our approach comes from exploiting the fact that, under the null hypothesis, the individual effects and the autoregressive coefficients are heterogeneous across individuals but the feedback coefficients are all equal to zero and thus homogeneous. We therefore propose the use of a pooled estimator for the feedback coefficients only. Pooling over cross-sections guarantees that the estimator has the faster $\sqrt{NT}$ convergence rate. The pooled estimator suffers from the incidental parameters

problem of Neyman and Scott (1948) due to the presence of the predetermined regressors, see Nickell (1981) and Karavias and Tzavalis (2016). This well known result implies that standard tests for pooled estimators do not control size asymptotically, unless $N << T$. To overcome this problem we use the idea of Split Panel Jackknife (SPJ) of Dhaene and Jochmans (2015), to construct an estimator which is free from the "Nickell bias". This type of bias correction works very well under circumstances that are empirically relevant: moderate time dimension, heterogeneous nuisance parameters and high persistence, as argued by Dhaene and Jochmans (2015), Fernández-Val and Lee (2013) and Chambers (2013), respectively. Furthermore, Chudik et al. (2018) argue that SPJ procedures are suitable for panels as long as N/T^3 is small. We test the null hypothesis of Granger non-causality by using a Wald test based on our bias-corrected estimator.

Next, we devise a test which is powerful against all types of alternatives. This test is using the union-intersection approach by combining the test of Dumitrescu and Hurlin (2012), which is powerful against heterogeneous alternatives and our new Wald test, which is powerful against homogeneous or near homogeneous alternatives. This test employs the Bonferroni correction and thus rejects the null hypothesis if either of the Dumitrescu and Hurlin (2012) test or our new Wald test reject at the $a/2$ level, where a is the significance level of the overall test.

Given that both of these tests have good finite sample properties for moderate to large time dimensions the new Bonferroni-type test has power against all types of alternatives and good finite sample properties. Monte Carlo simulations show that the Bonferroni corrected test has excellent size and power properties and does not suffer under any form of alternative.

The paper is organized as follows: Section 2 sets up the general model and the hypothesis of interest. It also provides the SPJ estimator and the new Wald test. Section 3 introduces the union-intersection test. Section 2.2 briefly presents the alternative methods in the literature. Section 4 contains the results of the Monte Carlo experiment and Section 5 concludes the paper.

2. The testing framework

2.1. The model

For the purpose of this paper we consider a simple linear dynamic panel data model with one additional regressor x_t:

$$y_{i,t} = \phi_{0,i} + \sum_{p=1}^{P} \phi_{p,i} y_{i,t-p} + \sum_{p=1}^{P} \beta_{p,i} x_{i,t-p} + \varepsilon_{i,t}, \tag{2.1}$$

for $i = 1, \dots, N$ and $t = 1, \dots, T$. Thus we assume that y_t follows an ARDL(P,P) process,[1] or more generally can be seen as one of the equations of the joint VAR(p) model for $(y_t, x_t)'$. The

[1] It can be extended to ARDL(P,Q) with $P \neq Q$ without affecting the main conclusions of this paper.

bivariate system $(y_{i,t}, x_{i,t})'$ is considered for simplicity of presentation as our results are easily extended to multivariate systems.[2] The $\phi_{0,i}$ are the fixed effects, the $\varepsilon_{i,t}$ are the innovations, the $\phi_{p,i}$ are the heterogeneous autoregressive coefficients and the $\beta_{p,i}$ are the heterogeneous feedback coefficients.

The null hypothesis that the time series $x_{i,t}$ does not Granger-cause (linearly) the time series $Y_{i,t}$ can be formulated as a set of linear restrictions on the parameters in (2.1):

$$H_0: \quad \beta_{p,i} = 0, \quad \text{for all } i \text{ and } p, \tag{2.2}$$

and will be tested against the alternative that

$$H_1: \quad \beta_{p,i} \neq 0 \quad \text{for some } i \text{ and } p. \tag{2.3}$$

The model and the null and alternative hypotheses here are as in Dumitrescu and Hurlin (2012). As in the case of panel unit root testing, the rejection of null hypothesis should be interpreted as existence of sufficient cross-sectional units i in which the null-hypothesis is violated.

2.2. Available tests

In this section we provide a brief overview of alternative testing procedures. The seminal paper of Holtz-Eakin et al. (1988) provides one of the early contributions to the literature on Granger non-causality testing in panels. Using Anderson and Hsiao (1982) type moment conditions they provide a GMM framework to test for Granger non-causality in short T panels with homogeneous coefficients. While this approach is designed for setups with T short, it becomes less appealing when T is sizeable. On the other hand, other fixed T methods, e.g. Binder et al. (2005), Karavias and Tzavalis (2017), Juodis (2013), Arellano (2016), and Juodis (2018) are also applicable when T is large. All these methods have one main drawback, as they are explicitly designed to estimate panels with homogeneous slope parameters.

For above reasons, by far the most popular approach among practitioners is the one of Dumitrescu and Hurlin (2012) that can accommodate heterogeneous slopes both under null and alternative hypothesis. Their approach uses individual specific Wald statistics W_i, obtained using individual specific estimates $\hat{\boldsymbol{\beta}}_i$. The corresponding test statistics are then pooled using simple sample average to construct a standardized test statistic which has asymptotic normal limit as $T \to \infty$ followed by $N \to \infty$:

$$W_{DH} = \sqrt{N} \frac{\sum_{i=1}^{N} W_i - P}{\sqrt{2P}}. \tag{2.4}$$

This approach is reminiscent of the Im et al. (2003) (IPS) panel unit root test for heterogeneous panels. The approach of Dumitrescu and Hurlin (2012) does not implicitly or explicitly account

[2] Also, to save space, we do not provide an exposition for how to test bi-directional causality, which happens in the same way.

for the "Nickell" bias. As a result their method can be theoretically justified only for sequences with $N/T^2 \to 0$ as with standard Mean-group type of approaches. Instead, assuming that $\varepsilon_{i,t}$ is normally distributed, they propose to center the test statistic using the moments of an appropriate F distribution rather than that of $\chi^2(P)$:

$$\tilde{W}_{DH} = \sqrt{\frac{N}{2P}\frac{T-2P-5}{T-P-3}}\left(\left(\frac{T-2P-3}{T-2P-1}\right)\frac{1}{N}\sum_{i=1}^{N}W_i - P\right). \tag{2.5}$$

This statistic was found to provide better size control in finite samples. However, despite the claim of fixed T results in that paper (see e.g. their Proposition 3), the modified statistic of $\tilde{W}_{DH}$ is not standard normal for T fixed even under the assumption of normal error terms, as their fixed T approximation assumes that regressors are strictly exogenous.[3]

2.3. A partially heterogenous test

Expressing the setup in (2.1) using the vector notation

$$y_{i,t} = \boldsymbol{z}_{i,t}'\boldsymbol{\phi}_i + \boldsymbol{x}_{i,t}'\boldsymbol{\beta}_i + \varepsilon_{i,t}, \tag{2.6}$$

where $\boldsymbol{z}_{i,t} = (1, y_{i,t-1}, \ldots, y_{i,t-p})'$, $\boldsymbol{x}_{i,t} = (x_{i,t-1}, \ldots, x_{i,t-p})'$, $\boldsymbol{\phi}_i = (\phi_{0,i}, \ldots, \phi_{p,i})'$ and $\boldsymbol{\beta}_i = (\beta_{1,i}, \ldots, \beta_{p,i})'$. Furthermore, let $\boldsymbol{y}_i = (y_{i,1}, \ldots, y_{i,T})'$, $\boldsymbol{Z}_i = (\boldsymbol{z}_{i,1}, \ldots, \boldsymbol{z}_{i,T})'$, $\boldsymbol{X}_i = (\boldsymbol{x}_{i,1}, \ldots, \boldsymbol{x}_{i,T})'$ and $\boldsymbol{\varepsilon}_i = (\varepsilon_{i,1}, \ldots, \varepsilon_{i,T})'$, then we can write (2.6) as

$$\boldsymbol{y}_i = \boldsymbol{Z}_i\boldsymbol{\phi}_i + \boldsymbol{X}_i\boldsymbol{\beta}_i + \boldsymbol{\varepsilon}_i. \tag{2.7}$$

We propose estimating model (2.7) using a pooled estimator (over i) for $\boldsymbol{\beta}$ where we assume that $\boldsymbol{\beta}_i = \boldsymbol{\beta}$ for all $i = 1, \ldots, N$. Observe that under the null hypothesis of Granger non-causality the true parameter vector $\boldsymbol{\beta}_0 = \mathbf{0}_P$. Assuming homogeneity in $\boldsymbol{\beta}_i$ (2.7) becomes

$$\boldsymbol{y}_i = \boldsymbol{Z}_i\boldsymbol{\phi}_i + \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{\varepsilon}_i. \tag{2.8}$$

In what follows we use the above model specification to estimate common parameter $\boldsymbol{\beta}$.

Given the statistical model in (2.7), which holds for each i, the OLS estimator of $\boldsymbol{\beta}$ (or the FE-type estimator, or LSDV-type estimator) is simply given by

$$\hat{\boldsymbol{\beta}} = \left(\sum_{i=1}^{N}\boldsymbol{X}_i'\boldsymbol{M}_{\boldsymbol{Z}_i}\boldsymbol{X}_i\right)^{-1}\left(\sum_{i=1}^{N}\boldsymbol{X}_i'\boldsymbol{M}_{\boldsymbol{Z}_i}\boldsymbol{y}_i\right). \tag{2.9}$$

This estimator generalizes the standard FE estimator where all slope coefficients are homogeneous, see e.g. Hahn and Kuersteiner (2002). Note that for this estimator to be well defined, a

[3] In this respect the comparison made by the authors with the panel unit root test of Im et al. (2003) is inappropriate.

sufficient number of $\boldsymbol{M}_{\boldsymbol{Z}_i}$ matrices should be non-zero. As in this paper we limit our attention to balanced panels, the necessary condition for that is that $T > 1+P$; which is enough to ensure that the underlying coefficient $\boldsymbol{\phi}_i$ is estimable.

The model in (2.1) belongs to a class of panel data models with nonadditive unobserved heterogeneity studied in Fernández-Val and Lee (2013). In particular, under Conditions 1-2 of that paper which restrict $\boldsymbol{q}_{i,t} = (y_{i,t}, x_{i,t})'$ to be a conditional strong mixing sequence with at least four moments, the asymptotic distribution is readily available. Note that this restriction rules out non-stationary and local-to-unity dynamics in $\boldsymbol{y}_i$.

In order to facilitate further discussion we adapt the conclusions of their Theorem 1 for our setup:

Theorem 2.1. *Under Conditions 1-2 Fernández-Val and Lee (2013) and given* $N/T \to \alpha^2 \in [0;\infty)$ *as* $N, T \to \infty$ *jointly:*

$$\sqrt{NT}\left(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0\right) \xrightarrow{d} \boldsymbol{J}^{-1} N\left(-\alpha \boldsymbol{B}, \boldsymbol{V}\right). \tag{2.10}$$

The Hessian matrix $\boldsymbol{J}$ in our case is given by:

$$\boldsymbol{J} = \operatorname*{plim}_{N,T\to\infty} \frac{1}{NT} \sum_{i=1}^{N} \boldsymbol{X}_i' \boldsymbol{M}_{\boldsymbol{Z}_i} \boldsymbol{X}_i, \tag{2.11}$$

while the exact form of $\boldsymbol{V}$ and $\boldsymbol{B}$ depends on the underlying assumptions of $\varepsilon_{i,t}$ and $\boldsymbol{\psi}_i$. For example, if $\varepsilon_{i,t}$ are independent and identically distributed (iid) over i and t, i.e. $\varepsilon_{i,t} \sim iid(0, \sigma^2)$ then:

$$\boldsymbol{V} = \sigma^2 \boldsymbol{J}. \tag{2.12}$$

The $\boldsymbol{B}$ vector captures the incidental parameters bias of the common parameter estimator, which is induced by estimation of $\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_N$. We will not elaborate on the exact form of this matrix, as for the purpose of this paper it is not needed. For more details on the exact form of all matrices in Theorem 2.1 please refer to Fernández-Val and Lee (2013).

The FE estimator of common parameters, while consistent, has bias in the asymptotic distribution under sequences where N and T grow at the same rate. The presence of bias invalidates any asymptotic inference because the bias is of the same order as the variance, unless $\alpha = 0$. In particular, the use of $\hat{\boldsymbol{\beta}}$ for Granger non-causality testing of $H_0 : \boldsymbol{\beta}_0 = \boldsymbol{0}_P$ will not lead to a test with correct asymptotic size. In particular the Wald test statistic:

$$W = NT\hat{\boldsymbol{\beta}}' \left(\boldsymbol{J}^{-1}\boldsymbol{V}\boldsymbol{J}^{-1}\right)^{-1} \hat{\boldsymbol{\beta}}, \tag{2.13}$$

converges to a non-central $\chi^2(P)$ distribution under the null hypothesis.

The above discussion shows that $\hat{\boldsymbol{\beta}}$ cannot be used in the construction of the Wald test statistic (2.13). Instead, we suggest the use of the same test statistic, but based on an alternative

estimator which is free from asymptotic bias $-\alpha \boldsymbol{B}$. In what follows we focus on the bias-corrected FE estimator which is constructed using the jackknife principle, the Half Panel Jackknife (HPJ) estimator of Dhaene and Jochmans (2015). Given a balanced panel with an even number of time series observations, the HPJ estimator is defined as

$$\tilde{\boldsymbol{\beta}} \equiv 2\hat{\boldsymbol{\beta}} - \frac{1}{2}\left(\hat{\boldsymbol{\beta}}_{1/2} + \hat{\boldsymbol{\beta}}_{2/1}\right), \tag{2.14}$$

where $\hat{\boldsymbol{\beta}}_{1/2}$ and $\hat{\boldsymbol{\beta}}_{2/1}$ the FE estimators of $\boldsymbol{\beta}$ based on the first $T_1 = T/2$ observations, and the second $T_2 = T - T_1$ respectively. The HPJ estimator can be decomposed into a sum of two terms:

$$\tilde{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}} + \left(\hat{\boldsymbol{\beta}} - \frac{1}{2}\left(\hat{\boldsymbol{\beta}}_{1/2} + \hat{\boldsymbol{\beta}}_{2/1}\right)\right) = \hat{\boldsymbol{\beta}} + T^{-1}\hat{\boldsymbol{B}}, \tag{2.15}$$

where the second component implicitly estimates the bias term in (2.10). The use of this estimator can be justified in our setting given that the bias of $\hat{\boldsymbol{\beta}}$ is of order $\mathcal{O}(T^{-1})$ and thus satisfies the expansion requirement of Dhaene and Jochmans (2015). While other ways of splitting the panel in construction of the bias-corrected estimator are possible, as it is shown in Dhaene and Jochmans (2015), the HPJ estimator minimizes the higher order bias in the class of Split Panel Jackknife (SPJ) as long as data are stationary. For this reason we limit our attention to (2.14).

Corollary 2.1. *Under Conditions 1-2 of Fernández-Val and Lee (2013) and given* $N/T \to \alpha^2 \in [0;\infty)$ *as* $N, T \to \infty$ *jointly:*

$$\hat{W}_{HPJ} = NT\tilde{\boldsymbol{\beta}}'\left(\hat{\boldsymbol{J}}^{-1}\hat{\boldsymbol{V}}\hat{\boldsymbol{J}}^{-1}\right)^{-1}\tilde{\boldsymbol{\beta}} \xrightarrow{d} \chi^2(P), \tag{2.16}$$

where assuming $\varepsilon_{i,t} \sim iid(0, \sigma^2)$

$$\begin{aligned}
\hat{\boldsymbol{J}} &= \frac{1}{NT}\sum_{i=1}^{N} \boldsymbol{X}_i' \boldsymbol{M}_{\boldsymbol{Z}_i} \boldsymbol{X}_i \\
\hat{\boldsymbol{V}} &= \hat{\sigma}^2 \hat{\boldsymbol{J}} \\
\hat{\sigma}^2 &= \frac{1}{N(T-1-P)-P}\sum_{i=1}^{N}\left(\boldsymbol{y}_i - \boldsymbol{X}_i\hat{\boldsymbol{\beta}}\right)' \boldsymbol{M}_{\boldsymbol{Z}_i}\left(\boldsymbol{y}_i - \boldsymbol{X}_i\hat{\boldsymbol{\beta}}\right).
\end{aligned}$$

The proof of this corollary follows from the corresponding results in Fernández-Val and Lee (2013) and Dhaene and Jochmans (2015). The formula for $\hat{\boldsymbol{V}}$ can be easily modified to accommodate heteroscedasticity in both cross-sectional and time-series dimensions. Given the recent results in Chudik et al. (2018) we conjecture that for SPJ approach to work it is only necessary to assume $N/T^3 \to 0$.

Jackknife is by no means the only approach that corrects the incidental parameters bias of the FE estimator. Alternatively one can consider analytical bias-correction as in Hahn and Kuersteiner (2002) and Fernández-Val and Lee (2013). However, the analytical approach has several practical limitations such as the need to specify a kernel function and the corresponding

bandwidth. Furthermore, analytical corrections are only applicable for stationary data, and can be severely biased if the underlying process is persistent, see e.g. Hahn and Kuersteiner (2002). In this respect the HPJ approach of Dhaene and Jochmans (2015) has some clear advantages.

3. Union of intersections

The null hypothesis (2.2) is rejected when either the feedback coefficients are equal to each other and different from zero or they are simply different from each other. The Wald test proposed in Section 2.3 is powerful against the first type of alternatives while the Dumitrescu and Hurlin (2012) test is powerful against the second type of alternatives. Here we propose a union-intersection test which is powerful against any alternative. Following Harvey et al. (2009):

$$UI := \hat{W}_{HPJ} I(\hat{W}_{HPJ} > c_{1-a/2,\chi^2(P)}) + \tilde{W}_{DH} I(\hat{W}_{HPJ} \leq c_{1-a/2,\chi^2(P)}), \tag{3.1}$$

where $\tilde{W}_{DH}$ is the Dumitrescu and Hurlin (2012) test statistic, defined in (2.5). If $UI = \hat{W}_{HPJ}$ we reject the null hypothesis at level a when $UI = \hat{W}_{HPJ} > c_{1-a/2,\chi^2(P)}$, where $c_{1-a/2,\chi^2(P)}$ is the level $1 - a/2$ critical value coming from the $\chi^2(P)$ distribution. Otherwise, if $UI = \hat{W}_{DH}$ we reject the null hypothesis when $|UI| \geq c_{1-a/4,N(0,1)}$, where $c_{1-a/4,N(0,1)}$ is the level $1 - a/4$ critical value from the the standard normal distribution.[4] That is, if either test rejects, then the null hypothesis is rejected.

4. Monte Carlo simulation

4.1. The setup

To illustrate the performance of the new testing procedure we adapt the Monte Carlo setup of Binder et al. (2005) and Juodis (2018). In particular, we assume that the bivariate vector $\boldsymbol{y}_{i,t} = (y_{i,t}, x_{i,t})'$ follows a VAR(1) process:

$$\boldsymbol{y}_{i,t} = \boldsymbol{\Phi}_i \boldsymbol{y}_{i,t-1} + \boldsymbol{\varepsilon}_{i,t}, \quad \boldsymbol{\varepsilon}_{i,t} \sim N(\mathbf{0}_2, \boldsymbol{\Sigma}) \tag{4.1}$$

for all $i = 1, \ldots, N$, and $t = 1, \ldots, T$. The vector $\boldsymbol{y}_{i,t}$ is assumed to be initialized in a distant past, in particular we set $\boldsymbol{y}_{i,-50} = \mathbf{0}_2$ and discard the first 50 observations in estimation. Note that our setup excludes individual specific slope coefficients $\phi_{0,i}$ as we limit our attention to FE-estimators that are invariant to $\phi_{0,i}$ under stationarity.

In order to simplify the setup we assume that some of the design matrices are common for all i. In particular, we adopt Design 2 of Juodis (2018) for the error variance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} 0.07 & 0.05 \\ 0.05 & 0.07 \end{pmatrix}. \tag{4.2}$$

[4] In unreported simulations we have also considered the alternative methods which appear in Harvey et al. (2009) but found them generally inferior to the union-intersection test.

For the purpose of our research question we slightly modify the autoregressive matrix in Juodis (2018) to:

$$\boldsymbol{\Phi}_i = \begin{pmatrix} 0.4 & \kappa_i \\ -0.1 & \rho \end{pmatrix}, \tag{4.3}$$

where we consider $\rho = \{0.4; 0.9\}$, respectively. The ρ parameter controls the degree of persistence in $\boldsymbol{y}_{i,t}$, which can be either moderate $\rho = 0.4$ or high $\rho = 0.9$.

The main parameter of interest is κ_i. For $\kappa_i = 0$, the $\boldsymbol{\Phi}_i$ matrix is lower triangular so that $x_{i,t}$ does not Granger-cause $y_{i,t}$. In this case we will investigate the size of the test. On the other hand if $\kappa_i \neq 0$ for sufficiently large fraction of cross-sectional units, then we consider power. In order to cover a broad range of possible alternative hypothesis we consider three distinct schemes for this purpose:

1. (Homogeneous). $\kappa_i = \kappa$ for all i. $\kappa = \{0.00; 0.01; 0.03; 0.05\}$.
2. (Heterogeneous). $\kappa_i = \kappa + U[-\nu; \nu]$. κ as in homogeneous case. $\nu = \{0.1; 0.3; 0.5\}$.
3. (Group structure). $\kappa_i = \tilde{\kappa}_{g_i}$ where $g_i \in \{1; 2\}$. For $\tilde{\kappa}_1 = -0.01$ and $\tilde{\kappa}_2 = 0.03$. N_2 denotes the fraction of cross-sectional units with $\tilde{\kappa}_2$. We set $N_2 = \{0.1; 0.3; 0.5\}$.

Let us briefly motivate the choice of these distinct setups. The homogeneous design covers the classical pooled setup of Holtz-Eakin et al. (1988). On the other hand, heterogeneity introduced in the second design is qualitatively closer to Dumitrescu and Hurlin (2012). Note how in heterogeneous case $\mathrm{E}[\kappa_i] = \kappa$. Finally, the last design with group-specific (clustered) coefficients is included to better reflect recent interest in panel data literature towards partially heterogeneous models, see e.g. Bonhomme and Manresa (2015) and Su et al. (2016).

Given that the procedure of Dumitrescu and Hurlin (2012) is primarily used in medium-size macro-panels, we also limit our attention to combination of (N, T) that better reflect these applications. In particular we limit our attention to the following 9 combinations:

$$N = \{20; 50; 100\}, \quad T = \{20; 50; 100\}. \tag{4.4}$$

We focus our attention on four different testing procedures:

- "DH" - the Dumitrescu and Hurlin (2012) Wald test statistic in eq. (2.4).
- "DHT" - the Dumitrescu and Hurlin (2012) "fixed-T" Wald test statistic in eq. (2.5).
- "HPJ" - the proposed pooled Wald test statistic in eq. (2.16).
- "UI" - test statistic (3.1), which rejects if either "DHT" or "HPJ" rejects with Bonferroni correction. We prefer to base the UI test on the "DHT" test statistic because the latter provides better size control than DH.

All tests are conducted at the 5% nominal size, while the total number of Monte Carlo replications is set to $M = 10000$. In order to better emphasize nominal differences in rejections under the null hypothesis we do not report size-adjusted power.

4.2. The results

In this section we briefly describe simulation results. We mainly focus our attention to the most representative case of $N = 100$. Non size-adjusted results for $\rho = 0.4$ and $\rho = 0.9$ are presented in Tables A.3 and A.6.[5]

- Only the test statistic based on the HPJ estimator has a substantial power against homogeneous alternatives. Both DH and DHT approaches lack power under such circumstances. The UI approach in this respect provides a useful refinement, as long as $T > 50$. The reverse holds once zero-mean heterogeneous alternatives ($\kappa = 0$) are considered. For example, for $T = 100$ and $\nu = 0.1$, the power of the DH test is 0.592, while the corresponding value for HPJ is only 0.099. Sizeable power for HPJ starts to show up only once ν sufficiently increases. The UI approach again provides a useful refinement, as it has power which only marginally smaller than that of DH, e.g. 0.524.

- For the simple setup with heterogeneous grouped alternatives we consider, the HPJ approach dominates. However, in this case the power is low in general.

- By combining HPJ with either DH (or DHT) substantial power gains can be achieved in both homogeneous and heterogeneous alternatives cases. This is our preferred approach as long as T is sufficiently large. For small values of T, e.g. $T = 20$ we suggest that one uses simple the HPJ procedure which better control size, even at the expense of power.

- We can observe that for $T = 20$ all tests over-reject under null hypothesis; this is especially visible in the case of DH, DHT and the combination UI test. This effect slowly disappears as T increases, but can be still visible even for $T = 100$.

The results for $\rho = 0.9$ are qualitatively similar to those for $\rho = 0.4$. However, one can observe a clear deterioration of all test statistics under the null hypothesis. This can be explained by that fact that the bias of the FE-type estimators grows as the persistence of the overall process increases, e.g. Hahn and Kuersteiner (2002). Furthermore, $\rho = 0.9$ at $T = 20$ can be effectively classified as local-to-unity situation, which is ruled out by our assumptions. For example, for $T = 20$ all tests have rejection frequencies well above the nominal 5% level. Both the DH and the DHT test are effectively useless in this case. The situation somewhat improves for $T = 50$, but in that case only the HPJ approach can be useful as a testing tool. This confirms the usability

[5]For comparison purposes we also include size-adjusted results in A.9 and A.12.

of the HPJ approach even if the process is close to the boundary of the stationarity region. We conjecture that over-rejections documented for the HPJ statistic can be mitigated using the bootstrap critical values as discussed in Dhaene and Jochmans (2015) and Gonçalves and Kaffo (2015).

These results demonstrate that using DH and DHT against homogeneous alternatives and HPJ against heterogeneous alternatives leads to heavy loss of power. The UI however safeguards against these situations. The UI test does not dominate any of the DH, DHT or HPJ tests but it is, overall, the test with the minimum risk for the researcher.

5. Conclusions

In this paper we consider the problem of Granger non-causality testing in heterogeneous panels. While this testing problem has long standing traditions in panel data econometrics, little was known about the power properties when some of the parameters are heterogeneous. In this paper, we propose a new test statistic based on the Half Panel Jackknife bias-correction procedure of Dhaene and Jochmans (2015). Using simulated data we document superior size properties of the new statistic for shorter time-series dimensions. For situations where alternatives are expected to be sufficiently heterogeneous over cross-sectional units, we propose a union-intersection statistic which combines the jackknife based statistic and the one previously suggested in Dumitrescu and Hurlin (2012). This approach can be effectively used for larger values of T, irrespective if the suspected alternative is homogeneous or heterogeneous.

The statistical model considered in this paper is rather restrictive as it rules out any forms of the cross-sectional dependence in $\varepsilon_{i,t}$. This restriction can be easily relaxed if one is willing to assume that cross-sectional dependence is strong and is either generated by additive time effects τ_t, or via the factor structure $\boldsymbol{\lambda}_i'\boldsymbol{f}_t$. In the former case all procedures discussed in this paper can be used after a trivial modification, while in the latter case either the Common Correlated Effects (CCE) approach of Pesaran (2006)/ Chudik and Pesaran (2015) can be used, or the PC estimator of Bai (2009)/Ando and Bai (2016). In those setups the HPJ based statistic provides a natural starting point, as Dumitrescu and Hurlin (2012) finite T corrections are not feasible.

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Table A.1: Empirical rejection rates for various Granger non-causality tests. $N = 20$ and $\rho = 0.4$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel A. Null hypothesis														
0	0	-	0.067	0.055	0.055	0.058	0.091	0.063	0.068	0.080	0.202	0.090	0.107	0.125
Panel B: homogeneous alternatives														
0.01	0	-	0.070	0.059	0.077	0.076	0.092	0.065	0.069	0.074	0.204	0.089	0.103	0.123
0.03	0	-	0.087	0.075	0.209	0.171	0.105	0.073	0.135	0.129	0.210	0.097	0.129	0.138
0.05	0	-	0.154	0.136	0.464	0.382	0.132	0.096	0.269	0.230	0.220	0.099	0.169	0.171
Panel C: heterogeneous alternatives														
0	0.1	-	0.228	0.199	0.089	0.182	0.169	0.131	0.077	0.130	0.238	0.110	0.106	0.135
0	0.2	-	0.806	0.785	0.196	0.750	0.524	0.455	0.132	0.415	0.400	0.223	0.126	0.227
0	0.3	-	0.992	0.991	0.373	0.988	0.893	0.858	0.250	0.833	0.631	0.442	0.166	0.435
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.072	0.059	0.064	0.069	0.098	0.070	0.070	0.081	0.217	0.097	0.099	0.119
-0.01 (0.03)	-	0.3	0.076	0.064	0.058	0.072	0.100	0.069	0.063	0.074	0.207	0.091	0.104	0.122
-0.01 (0.03)	-	0.5	0.081	0.070	0.074	0.082	0.103	0.073	0.073	0.086	0.208	0.096	0.112	0.129

Notes. *DH and DHT denote the large T and the fixed T versions from Dumitrescu and Hurlin (2012); HPJ is the Wald statistic based on the bias-corrected FE estimator; UI is the test statistic which rejects whenever DHT or HPJ reject. All tests have a nominal size of* 5%. *Power is not size-adjusted.*

Table A.2: Empirical rejection rates for various Granger non-causality tests. $N = 50$ and $\rho = 0.4$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel A. Null hypothesis														
0	0	-	0.070	0.053	0.056	0.059	0.117	0.074	0.074	0.082	0.318	0.121	0.099	0.137
Panel B: homogeneous alternatives														
0.01	0	-	0.075	0.059	0.092	0.086	0.114	0.071	0.086	0.093	0.317	0.118	0.113	0.139
0.03	0	-	0.126	0.100	0.439	0.364	0.146	0.094	0.266	0.233	0.340	0.123	0.173	0.190
0.05	0	-	0.238	0.199	0.830	0.766	0.199	0.132	0.530	0.456	0.372	0.147	0.281	0.283
Panel C: heterogeneous alternatives														
0	0.1	-	0.382	0.336	0.086	0.293	0.274	0.191	0.082	0.171	0.407	0.172	0.106	0.176
0	0.2	-	0.987	0.982	0.243	0.971	0.820	0.752	0.159	0.703	0.667	0.381	0.134	0.358
0	0.3	-	1.000	1.000	0.489	1.000	0.996	0.992	0.328	0.991	0.912	0.739	0.215	0.710
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.093	0.074	0.072	0.077	0.133	0.084	0.071	0.084	0.338	0.130	0.106	0.146
-0.01 (0.03)	-	0.3	0.092	0.073	0.057	0.069	0.143	0.094	0.067	0.091	0.330	0.119	0.104	0.131
-0.01 (0.03)	-	0.5	0.102	0.083	0.094	0.103	0.136	0.085	0.086	0.101	0.329	0.119	0.110	0.138

Notes. See Table A.1.

Table A.3: Empirical rejection rates for various Granger non-causality tests. $N = 100$ and $\rho = 0.4$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel A. Null hypothesis														
0	0	-	0.084	0.062	0.054	0.064	0.162	0.085	0.066	0.083	0.503	0.165	0.103	0.173
Panel B: homogeneous alternatives														
0.01	0	-	0.081	0.063	0.139	0.128	0.144	0.079	0.108	0.117	0.502	0.161	0.124	0.179
0.03	0	-	0.162	0.119	0.718	0.614	0.196	0.118	0.424	0.360	0.509	0.172	0.238	0.263
0.05	0	-	0.373	0.310	0.983	0.969	0.301	0.191	0.815	0.741	0.558	0.202	0.444	0.436
Panel C: heterogeneous alternatives														
0	0.1	-	0.592	0.522	0.099	0.460	0.450	0.329	0.082	0.270	0.622	0.254	0.109	0.247
0	0.2	-	1.000	0.999	0.323	0.999	0.973	0.946	0.192	0.919	0.879	0.584	0.146	0.537
0	0.3	-	1.000	1.000	0.657	1.000	1.000	1.000	0.469	1.000	0.994	0.941	0.266	0.920
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.110	0.083	0.090	0.094	0.168	0.100	0.079	0.101	0.512	0.173	0.113	0.181
-0.01 (0.03)	-	0.3	0.120	0.091	0.062	0.082	0.179	0.103	0.063	0.088	0.517	0.173	0.103	0.180
-0.01 (0.03)	-	0.5	0.130	0.099	0.134	0.132	0.181	0.103	0.115	0.119	0.516	0.178	0.115	0.195

Notes. See Table A.1.

Table A.4: Empirical rejection rates for various Granger non-causality tests. $N = 20$ and $\rho = 0.9$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel A. Null hypothesis														
0	0	-	0.081	0.067	0.070	0.081	0.136	0.100	0.102	0.122	0.351	0.189	0.214	0.262
Panel B: homogeneous alternatives														
0.01	0	-	0.126	0.108	0.108	0.117	0.168	0.126	0.125	0.146	0.380	0.205	0.241	0.283
0.03	0	-	0.280	0.250	0.390	0.356	0.282	0.222	0.279	0.291	0.413	0.228	0.312	0.350
0.05	0	-	0.544	0.506	0.745	0.711	0.445	0.375	0.511	0.514	0.499	0.306	0.416	0.464
Panel C: heterogeneous alternatives														
0	0.1	-	0.405	0.371	0.127	0.345	0.297	0.233	0.122	0.236	0.400	0.228	0.224	0.297
0	0.2	-	0.961	0.955	0.277	0.952	0.748	0.687	0.206	0.660	0.557	0.360	0.243	0.393
0	0.3	-	1.000	1.000	0.414	1.000	0.977	0.966	0.299	0.964	0.758	0.589	0.285	0.597
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.074	0.063	0.091	0.087	0.127	0.093	0.098	0.115	0.336	0.169	0.205	0.242
-0.01 (0.03)	-	0.3	0.122	0.105	0.074	0.107	0.161	0.119	0.108	0.136	0.364	0.193	0.221	0.270
-0.01 (0.03)	-	0.5	0.159	0.138	0.108	0.136	0.196	0.147	0.129	0.163	0.393	0.204	0.231	0.282

Notes. See Table A.1.

Table A.5: Empirical rejection rates for various Granger non-causality tests. $N = 50$ and $\rho = 0.9$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel A. Null hypothesis														
0	0	-	0.101	0.084	0.070	0.088	0.212	0.144	0.094	0.140	0.602	0.316	0.244	0.365
Panel B: homogeneous alternatives														
0.01	0	-	0.175	0.141	0.150	0.163	0.274	0.192	0.166	0.212	0.628	0.333	0.302	0.412
0.03	0	-	0.473	0.422	0.707	0.661	0.484	0.379	0.486	0.504	0.695	0.417	0.473	0.563
0.05	0	-	0.845	0.818	0.983	0.974	0.744	0.648	0.834	0.837	0.795	0.525	0.649	0.711
Panel C: heterogeneous alternatives														
0	0.1	-	0.676	0.630	0.128	0.581	0.509	0.408	0.130	0.376	0.669	0.387	0.253	0.415
0	0.2	-	0.999	0.999	0.267	1.000	0.965	0.944	0.200	0.929	0.854	0.633	0.280	0.616
0	0.3	-	1.000	1.000	0.435	1.000	1.000	1.000	0.291	1.000	0.973	0.899	0.330	0.886
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.100	0.077	0.098	0.105	0.193	0.128	0.100	0.132	0.566	0.293	0.220	0.335
-0.01 (0.03)	-	0.3	0.159	0.129	0.074	0.119	0.257	0.174	0.111	0.167	0.601	0.316	0.252	0.368
-0.01 (0.03)	-	0.5	0.235	0.195	0.160	0.211	0.315	0.219	0.174	0.243	0.647	0.361	0.311	0.426

Notes. See Table A.1.

Table A.6: Empirical rejection rates for various Granger non-causality tests. $N = 100$ and $\rho = 0.9$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel A. Null hypothesis														
0	0	-	0.134	0.101	0.068	0.095	0.320	0.205	0.099	0.186	0.825	0.482	0.298	0.506
Panel B: homogeneous alternatives														
0.01	0	-	0.242	0.187	0.234	0.243	0.422	0.288	0.231	0.327	0.863	0.533	0.410	0.587
0.03	0	-	0.718	0.653	0.940	0.920	0.728	0.604	0.746	0.763	0.916	0.664	0.677	0.791
0.05	0	-	0.984	0.976	1.000	1.000	0.934	0.880	0.981	0.983	0.959	0.784	0.872	0.918
Panel C: heterogeneous alternatives														
0	0.1	-	0.907	0.879	0.127	0.833	0.737	0.611	0.129	0.566	0.895	0.623	0.313	0.623
0	0.2	-	1.000	1.000	0.272	1.000	0.999	0.997	0.201	0.999	0.979	0.870	0.340	0.846
0	0.3	-	1.000	1.000	0.423	1.000	1.000	1.000	0.300	1.000	0.999	0.988	0.406	0.985
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.116	0.087	0.121	0.123	0.283	0.180	0.108	0.176	0.813	0.460	0.243	0.461
-0.01 (0.03)	-	0.3	0.231	0.177	0.071	0.159	0.395	0.271	0.121	0.239	0.838	0.505	0.317	0.537
-0.01 (0.03)	-	0.5	0.368	0.303	0.242	0.311	0.494	0.345	0.222	0.352	0.857	0.554	0.412	0.619

Notes. See Table A.1.

Table A.7: Empirical rejection rates (size-adjusted) for various Granger non-causality tests. $N = 20$ and $\rho = 0.4$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel B: homogeneous alternatives														
0.01	0	-	0.049	0.049	0.076	0.061	0.047	0.047	0.066	0.051	0.047	0.047	0.053	0.049
0.03	0	-	0.057	0.057	0.224	0.158	0.053	0.053	0.123	0.089	0.051	0.051	0.071	0.060
0.05	0	-	0.087	0.087	0.454	0.347	0.059	0.059	0.239	0.164	0.047	0.047	0.105	0.074
Panel C: heterogeneous alternatives														
0	0.1	-	0.123	0.123	0.087	0.114	0.083	0.083	0.060	0.072	0.053	0.053	0.060	0.052
0	0.2	-	0.691	0.691	0.195	0.636	0.306	0.306	0.118	0.255	0.098	0.098	0.067	0.082
0	0.3	-	0.984	0.984	0.370	0.978	0.770	0.770	0.230	0.722	0.226	0.226	0.105	0.197
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.056	0.056	0.061	0.055	0.050	0.050	0.056	0.045	0.051	0.051	0.055	0.052
-0.01 (0.03)	-	0.3	0.049	0.049	0.051	0.043	0.051	0.051	0.055	0.049	0.050	0.050	0.054	0.049
-0.01 (0.03)	-	0.5	0.050	0.050	0.072	0.061	0.052	0.052	0.059	0.048	0.049	0.049	0.049	0.048

Notes. See Table A.1.

Table A.8: Empirical rejection rates (size-adjusted) for various Granger non-causality tests. $N = 50$ and $\rho = 0.4$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel B: homogeneous alternatives														
0.01	0	-	0.050	0.050	0.094	0.070	0.052	0.052	0.074	0.055	0.043	0.043	0.052	0.046
0.03	0	-	0.067	0.067	0.436	0.341	0.052	0.052	0.229	0.162	0.050	0.050	0.099	0.074
0.05	0	-	0.135	0.135	0.819	0.743	0.067	0.067	0.514	0.398	0.049	0.049	0.189	0.131
Panel C: heterogeneous alternatives														
0	0.1	-	0.245	0.245	0.086	0.209	0.103	0.103	0.063	0.086	0.057	0.057	0.055	0.057
0	0.2	-	0.967	0.967	0.229	0.948	0.593	0.593	0.134	0.508	0.140	0.140	0.077	0.125
0	0.3	-	1.000	1.000	0.476	1.000	0.983	0.983	0.310	0.974	0.448	0.448	0.130	0.388
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.049	0.049	0.058	0.055	0.048	0.048	0.062	0.054	0.050	0.050	0.055	0.051
-0.01 (0.03)	-	0.3	0.056	0.056	0.050	0.051	0.052	0.052	0.059	0.048	0.048	0.048	0.057	0.052
-0.01 (0.03)	-	0.5	0.059	0.059	0.095	0.082	0.050	0.050	0.077	0.061	0.048	0.048	0.057	0.049

Notes. See Table A.1.

Table A.9: Empirical rejection rates (size-adjusted) for various Granger non-causality tests. $N = 100$ and $\rho = 0.4$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel B: homogeneous alternatives														
0.01	0	-	0.048	0.048	0.135	0.100	0.049	0.049	0.091	0.069	0.052	0.052	0.058	0.054
0.03	0	-	0.062	0.062	0.692	0.590	0.054	0.054	0.390	0.292	0.054	0.054	0.144	0.107
0.05	0	-	0.195	0.195	0.981	0.964	0.079	0.079	0.779	0.680	0.051	0.051	0.313	0.226
Panel C: heterogeneous alternatives														
0	0.1	-	0.384	0.384	0.092	0.317	0.139	0.139	0.069	0.118	0.064	0.064	0.052	0.064
0	0.2	-	0.999	0.999	0.303	0.999	0.853	0.853	0.167	0.799	0.244	0.244	0.079	0.208
0	0.3	-	1.000	1.000	0.634	1.000	1.000	1.000	0.424	1.000	0.718	0.718	0.184	0.676
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.052	0.052	0.085	0.071	0.047	0.047	0.065	0.050	0.052	0.052	0.053	0.055
-0.01 (0.03)	-	0.3	0.052	0.052	0.054	0.052	0.045	0.045	0.050	0.046	0.054	0.054	0.047	0.048
-0.01 (0.03)	-	0.5	0.050	0.050	0.118	0.086	0.052	0.052	0.086	0.070	0.050	0.050	0.062	0.060

Notes. See Table A.1.

Table A.10: Empirical rejection rates (size-adjusted) for various Granger non-causality tests. $N = 20$ and $\rho = 0.9$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel B: homogeneous alternatives														
0.01	0	-	0.053	0.053	0.083	0.064	0.053	0.053	0.070	0.059	0.051	0.051	0.064	0.057
0.03	0	-	0.132	0.132	0.332	0.263	0.093	0.093	0.177	0.136	0.062	0.062	0.072	0.065
0.05	0	-	0.344	0.344	0.712	0.631	0.169	0.169	0.380	0.305	0.077	0.077	0.114	0.102
Panel C: heterogeneous alternatives														
0	0.1	-	0.235	0.235	0.105	0.195	0.101	0.101	0.071	0.087	0.058	0.058	0.057	0.053
0	0.2	-	0.908	0.908	0.245	0.878	0.467	0.467	0.139	0.403	0.099	0.099	0.080	0.096
0	0.3	-	0.998	0.998	0.386	0.998	0.900	0.900	0.228	0.871	0.243	0.243	0.098	0.209
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.045	0.045	0.063	0.053	0.050	0.050	0.060	0.055	0.056	0.056	0.054	0.054
-0.01 (0.03)	-	0.3	0.052	0.052	0.054	0.049	0.056	0.056	0.059	0.057	0.054	0.054	0.055	0.053
-0.01 (0.03)	-	0.5	0.068	0.068	0.086	0.075	0.059	0.059	0.074	0.065	0.055	0.055	0.056	0.051

Notes. See Table A.1.

Table A.11: Empirical rejection rates (size-adjusted) for various Granger non-causality tests. $N = 50$ and $\rho = 0.9$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel B: homogeneous alternatives														
0.01	0	-	0.065	0.065	0.135	0.104	0.060	0.060	0.090	0.078	0.053	0.053	0.067	0.060
0.03	0	-	0.246	0.246	0.683	0.588	0.129	0.129	0.345	0.273	0.059	0.059	0.120	0.088
0.05	0	-	0.670	0.670	0.983	0.966	0.320	0.320	0.730	0.646	0.092	0.092	0.234	0.179
Panel C: heterogeneous alternatives														
0	0.1	-	0.472	0.472	0.114	0.407	0.143	0.143	0.074	0.125	0.060	0.060	0.056	0.054
0	0.2	-	0.999	0.999	0.252	0.998	0.788	0.788	0.139	0.737	0.135	0.135	0.078	0.118
0	0.3	-	1.000	1.000	0.377	1.000	0.998	0.998	0.230	0.998	0.450	0.450	0.113	0.386
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.045	0.045	0.071	0.062	0.055	0.055	0.058	0.057	0.044	0.044	0.054	0.049
-0.01 (0.03)	-	0.3	0.072	0.072	0.056	0.061	0.049	0.049	0.056	0.051	0.046	0.046	0.050	0.043
-0.01 (0.03)	-	0.5	0.086	0.086	0.138	0.116	0.061	0.061	0.083	0.076	0.050	0.050	0.062	0.055

Notes. See Table A.1.

Table A.12: Empirical rejection rates (size-adjusted) for various Granger non-causality tests. $N = 100$ and $\rho = 0.9$.

			$T = 100$				$T = 50$				$T = 20$			
κ	ν	N_2	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI	DH	DHT	HPJ	UI
Panel B: homogeneous alternatives														
0.01	0	-	0.063	0.063	0.225	0.150	0.064	0.064	0.130	0.097	0.054	0.054	0.072	0.059
0.03	0	-	0.408	0.408	0.933	0.884	0.210	0.210	0.586	0.494	0.073	0.073	0.177	0.137
0.05	0	-	0.904	0.904	1.000	0.999	0.537	0.537	0.952	0.915	0.137	0.137	0.396	0.320
Panel C: heterogeneous alternatives														
0	0.1	-	0.702	0.702	0.115	0.616	0.223	0.223	0.078	0.174	0.061	0.061	0.059	0.058
0	0.2	-	1.000	1.000	0.269	1.000	0.964	0.964	0.131	0.946	0.225	0.225	0.077	0.174
0	0.3	-	1.000	1.000	0.412	1.000	1.000	1.000	0.233	1.000	0.709	0.709	0.102	0.616
Panel D: Grouped alternatives														
-0.01 (0.03)	-	0.1	0.050	0.050	0.112	0.079	0.053	0.053	0.071	0.058	0.050	0.050	0.057	0.052
-0.01 (0.03)	-	0.3	0.070	0.070	0.065	0.065	0.059	0.059	0.054	0.055	0.050	0.050	0.052	0.048
-0.01 (0.03)	-	0.5	0.112	0.112	0.221	0.174	0.078	0.078	0.115	0.096	0.057	0.057	0.063	0.058

Notes. See Table A.1.