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THE BEHAVIORAL ECONOMICS OF CURRENCY UNIONS: ECONOMIC INTEGRATION AND MONETARY POLICY[*]

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Abstract

Currency unions are often modeled as homogeneous economies, although they are fundamentally different. The expectations that impact macroeconomic behavior in any given country are not the expectations of variables at the currency-union level but at the country level. We model these expectations with a behavioral reinforcement learning model. In our model, economic integration is of particular importance in determining whether economic behavior in a currency union is stable. Monetary policy alone is insufficient to guarantee stable economic behavior, as the central bank cannot conduct different monetary policies in different countries. These results are easily overlooked when modeling expectations as rational.

JEL classification: E03, F45, E52, D84
Keywords: Behavioral Macroeconomics; Monetary Unions; Reinforcement Learning; Expectation Formation

1 Introduction

Economists often resort to modeling currency unions as homogeneous economies. In the presence of a single currency, common monetary policy, and free trade, it appears that the currency union is best treated as one economy. Such modeling is problematic, however, as we demonstrate in this paper. If a currency union is modeled as a homogeneous economy, this means that only expectations of variables at the currency-union level matter for people's decisions, in particular the expectations of output and inflation. Things change fundamentally if the expectations that matter for a country in the currency union are not the expectations of currency-union aggregates but the expectations of variables in this country (e.g., if what matters for wage setting in France are the expectations of output and inflation in France rather than the expectations of these variables in the whole euro area). Then the countries need to be modeled separately, while being linked through monetary policy and other economic connections (e.g., because of trade).

In macroeconomics, the standard assumption is that expectations are formed rationally. This means that all agents in the economy not only know the exact theoretical relationships governing macroeconomic variables, but also that all agents are capable of performing the mathematically involved computations to arrive at the rational expectation solution. This assumption contradicts common sense, as well as a great deal of economic and psychological evidence that humans are not fully rational (accumulated at least since Tversky and Kahneman, 1974, and Grether and Plott, 1979). Instead of assuming rationality, we model expectations with a behavioral reinforcement learning model, which was developed over a long series of research projects (e.g., Hommes et al., 2005b; Anufriev and Hommes, 2012; we analyze the version of the model with rational expectations as a comparison). The behavioral model of expectation formation assumes that agents use different heuristics, such as a rule extrapolating trends or an anchoring-and-adjustment rule, assuming that trends are important in the short run while variables return to an anchor in the long run. Agents rely on different heuristics at different moments in time; agents switch between these heuristics depending on how well the heuristics have predicted economic variables in the recent past.

We present several findings. First, according to our model, economic behavior in a currency union can be very different from what a corresponding model of a homogeneous economy would suggest. In particular, economic behavior can be much less stable. This means that, for a variety of parameter combinations, we can observe large and persistent deviations of countries' output gap and inflation from the steady state. Second, the stability of economic behavior in a currency union depends crucially on the level of economic integration. More economic integration leads to more stable economic behavior. Third, if economic behavior in a currency union is unstable, the central bank cannot stabilize it with its interest rate decisions. This is so because the central bank can only react to currency union aggregates; it cannot conduct different monetary policies in different countries. The interest rate it sets will, thus, always be too high for some countries and too low for others. Fourth, none of these findings pertain when modeling the currency union with rational expectations. Rational expectations lead to extreme stability (by assumption), so that large shocks are needed in order to observe sizable deviations from the steady states. However, even large shocks are not enough to cause serious economic trouble, as economic variables revert quickly to their steady states.

While our results are not obtained under rational expectations, they are robust to using a variety of other behavioral specifications of expectation formation. These include simpler heuristic switching models and homogeneous adaptive expectations. The fact that the findings persist under a variety of expectation formation mechanisms, including both sophisticated models based on evolutionary learning and simple rules, such as adaptive expectations, is a strong reason to question the outcomes of the rational model rather than those of the behavioral model. In addition to our main model, we consider another macroeconomic model, which relies less on expectations and more on realizations of economic variables. Our findings persist.

This paper has two main policy implications. First, as economic integration is of crucial importance for the functioning of a currency union, policy makers should undertake reforms that strengthen economic ties between countries. However, such reforms may only bear fruit in the long run. As stabilization is also important in the short and medium run, we arrive at the second implication: Monetary policy should not be the sole stabilization tool. An additional policy tool is required, since monetary policy is insufficient to stabilize the currency union (insufficient does not mean unimportant as badly conducted monetary policy remains a potential source of problems). The most natural, if not the only, candidate for this stabilizing tool in the short and medium run is fiscal policy. Thus, fiscal policy should be made available for stabilization in a currency union and not rendered inflexible by constitutional arrangements or multilateral agreements.

This paper is organized as follows. Section 2 shows the underlying models of the macroeconomy and expectation formation. Section 3 contains the results, their interpretation, and a discussion of the robustness to alternative forms of behavioral expectations. Section 4 analyzes the robustness of the findings to using a different macroeconomic model. Section 5 concludes.

2 Macroeconomic Models and Expectation Formation

Before showing the economic model to analyze economic behavior in a currency union, we briefly describe the model for a homogeneous closed economy. We only present aggregate equations here. Microfoundations under behavioral expectations can be found in Hommes et al. (2017, Appendix A) for a homogeneous economy. We derive microfoundations for a currency union under behavioral expectations in this paper in Appendix A. Matrix forms of the systems of equations that are shown in this section can be found in Appendix B.

2.1 Homogeneous Economy

A standard description of a homogeneous closed economy is given by the following aggregate New Keynesian model equations:

$$y_t = \tilde{E}_t y_{t+1} - \frac{1}{\sigma}(r_t - \tilde{E}_t \pi_{t+1}) + v_t \tag{1}$$

$$\pi_t = \beta \tilde{E}_t \pi_{t+1} + \kappa y_t + \xi_t \tag{2}$$

$$r_t = max\{\bar{\pi} + \Phi_\pi(\pi_t - \bar{\pi}) + \Phi_y(y_t - \bar{y}),\ 0\}. \tag{3}$$

In these equations, y_t is the output gap, π_t is inflation, and r_t is the nominal interest rate. $\tilde{E}_t y_{t+1}$ and $\tilde{E}_t \pi_{t+1}$ are future expected output gap and inflation, respectively. σ, β, κ, Φ_π, and Φ_y are positive parameters, v_t and ξ_t are random disturbances.[1]

Equation (1) is the dynamic IS equation, where the current output gap depends on the future expected output gap $\tilde{E}_t y_{t+1}$, the real interest rate $r_t - \tilde{E}_t \pi_{t+1}$, and a demand (or technology) shock v_t. Equation (2) is the New Keynesian Phillips curve. Here, inflation depends on expected future inflation $\tilde{E}_t \pi_{t+1}$, the current output gap, and a supply (or cost-push) shock ξ_t. Equation (3) describes the behavior of monetary policy. The central bank sets the interest rate according to a Taylor Rule subject to the zero lower bound. The central bank reacts to deviations of inflation from the inflation target $\bar{\pi}$ and to deviations of the output gap from the steady state $\bar{y} = \frac{(1-\beta)\bar{\pi}}{\kappa}$.

[1]For microfoundations under rational expectations, see, for example, Woodford (2003). For microfoundations under behavioral expectations, see Branch and McGough (2009), Kurz et al. (2013), Massaro (2013), and Hommes et al. (2017).

Output gap and inflation expectations, $\tilde{E}_t y_{t+1}$ and $\tilde{E}_t \pi_{t+1}$, can be formed rationally or according to behavioral models. We assume that these expectations are formed with knowledge of all realizations up to time $t-1$. Details on expectation formation are treated in Section 2.3. As mentioned above, these aggregate equations possess micro-foundations under both rational and behavioral expectations. Equations of this form have been used extensively in the literature assuming both rational and behavioral expectation formation (Evans and Honkapohja, 2006; Branch and McGough, 2010; De Grauwe, 2010, 2011, 2012a,b; Branch and Evans, 2011; Pfajfar and Zakelj, 2014; Busetti et al., 2015).

2.2 Currency Union

We now introduce the currency union model. It is of crucial importance that the expectations that play a role for households and firms' decisions are not expectations of currency union aggregates but expectations of national variables. Countries are interdependent, with variables in one country influencing what happens in other countries. The framework for our analysis is an adaptation of the open economy model from Galí and Monacelli (2005, see also Galì and Monacelli, 2008).

We consider a currency union of N countries (and, similar to the model of the homogeneous closed economy, we assume that the currency union does not interact with the rest of the world). Here, we again only show aggregate equations, but these equations are fully microfounded under both rational expectations (see Galí and Monacelli, 2005) and under behavioral expectations (we derive this in Appendix A) for a continuum of symmetric small open economies.

Economic behavior in the currency union can be described by the following equations:

$$y_t^i = \tilde{E}_t y_{t+1}^i - \frac{1}{\sigma^i}(r_t - \tilde{E}_t \pi_{t+1}^i) + \gamma^i \tilde{E}_t \Delta s_{t+1}^i + \nu_t^i \tag{4}$$

$$\pi_t^i = \beta^i \tilde{E}_t \pi_{t+1}^i + \kappa^i y_t^i + \xi_t^i \tag{5}$$

$$r_t = max\{\bar{\pi} + \Phi_\pi(\pi_t^{cu} - \bar{\pi}) + \Phi_y(y_t^{cu} - \bar{y}^{cu}),\ 0\}. \tag{6}$$

The superscript i signifies that variables and parameters belong to country i ($i = 1, \ldots, N$). The superscript cu signifies that variables are at the currency union level, so that π_t^{cu} stands for inflation in the currency union, which is a weighted average of inflation at the country level. The weight $w(i)$ of country i represents this country's economic importance, thus $\pi_t^{cu} = \frac{1}{\sum_{k=1}^N w(k)} \sum_{k=1}^N w(k)\pi_t^k$. Similarly, $y_t^{cu} = \frac{1}{\sum_{k=1}^N w(k)} \sum_{k=1}^N w(k) y_t^k$ and $\bar{y}^{cu} = \frac{1}{\sum_{k=1}^N w(k)} \sum_{k=1}^N w(k)\bar{y}^k$. The term $\tilde{E}_t \Delta s_{t+1}^i$ denotes the expected change in the effective terms of trade of country i with the rest of the currency union. As the nominal exchange rate is fixed in a currency union, this only reflects a difference in inflation, $\tilde{E}_t \Delta s^i_{t+1} = \tilde{E}_t \pi^{*i}_{t+1} - \tilde{E}_t \pi^i_{t+1}$.[2] σ^i, β^i, κ^i, γ^i, Φ_π, and Φ_y are positive parameters, which can in general be different for different countries (where indicated by a superscript).

Equation (4) is the dynamic IS equation. It differs from the IS equation of a closed economy by the additional part $\gamma^i \tilde{E}_t \Delta s^i_{t+1}$. This addition signifies that if prices are expected to rise more abroad than in country i, thus improving the competitiveness of country i, the output gap in country i will be higher than it would be if equal rises of prices were expected. Equation (5) is the New Keynesian Phillips curve, which is identical to that of a closed economy. Equation (6) describes the behavior of the central bank, which is now a single central bank for the whole currency union. The central bank reacts with its interest rate decisions only to the inflation and output gap at the currency union level. The interest rate is again subject to the zero lower bound.

In the steady state (not at the zero lower bound), inflation in all countries is equal to the central bank's inflation target, $\bar{\pi}$. The steady state of the output gap in country i is, similar to the homogeneous economy case, $\bar{y}^i = \frac{(1-\beta^i)\bar{\pi}}{\kappa^i}$.

The main difference between modeling a currency union as we do here and modeling it as a homogeneous closed economy is that the expectation terms used here are expectations at the country level. Of course, the interactions of the countries can be important as well. The fact that the parameters and weights can be different across countries may be useful for applications.

2.3 Expectation Formation

Assuming full rationality means assuming that all agents in an economy have a full understanding of how the economy functions (i.e., they know the equations governing the economy including its parametrization) and that all agents have the capacity to perform all necessary calculations to maximize expected discounted lifetime utility. This is unrealistic, and many studies in economics and psychology have documented boundedly rational behavior.[3] Specifically, a great deal of research has documented

[2] π^{*i} is the weighted average of inflation in all countries excluding country i. For general open economies, the expected change in the (log) nominal effective exchange rate $\tilde{E}_t \Delta e^i_{t+1}$ would also play a role, so that $\tilde{E}_t \Delta s^i_{t+1} = \tilde{E}_t \Delta e^i_{t+1} + \tilde{E}_t \pi^{*i}_{t+1} - \tilde{E}_t \pi^i_{t+1}$.

[3] Since the early contributions of Tversky and Kahneman (1974) and Grether and Plott (1979), the literature has exploded and covers most aspects of economic life, from saving and consumption decisions (Thaler and Benartzi, 2004) over reactions to taxes (Weber and Schram, 2017) and perceptions of political institutions (Weber, 2017) to health-conducive and hazardous lifestyle choices (Richman, 2005).

that expectations in macroeconomics and finance are not formed rationally (Carroll, 2003; Branch, 2004; Blanchflower and MacCoille, 2009; Pfajfar and Santoro, 2010; Cornea et al., 2017; Malmendier and Nagel, 2016). Instead, people use relatively simple heuristics to form expectations and forecast future economic variables. This does not mean that agents are "stupid"; using such heuristics can be considered a clever way of dealing with cognitive limitations (see Gigerenzer and Todd, 1999, or Gigerenzer and Selten, 2002).

We now briefly discuss a model of expectation formation that was developed over a long series of research projects (starting with Brock and Hommes, 1997, and Brock and Hommes, 1998; see Hommes, 2011, for a more recent survey). The model is a heuristic switching model; agents have a set of heuristics available and switch between them depending on how well these heuristics have performed in the recent past. The description here is similar to the description in Hommes et al. (2017).

In order to forecast variable x, agents can make use of H different heuristics. The set of available heuristics is denoted by $\mathscr{H}$. A forecasting heuristic $h \in \mathscr{H}$ can be described as

$$x^e_{h,t+1} = f_h(x_{t-1}, x_{t-2} \ldots ; x^e_{h,t}, x^e_{h,t-1} \ldots). \tag{7}$$

In this paper, the variable x represents either inflation π or the output gap y. Note that the information available for forecasts of x in period $t+1$ consists of the variable up to period $t-1$ (this is the structure required by the rest of the paper).

This framework allows for all sorts of heuristics, from extremely simple rules of thumb to complicated formulas. However, while agents may use relatively simple forecasting heuristics, the model assumes that agents learn from past mistakes. In short, agents switch between heuristics according to how well these have done in the (recent) past. Such reinforcement learning precludes overly irrational behavior. The model thus makes use of a selection mechanism that steers which heuristics are chosen each period through an evolutionary fitness criterion. The fitness of forecasting heuristic h is denoted by U_h and defined as

$$U_{h,t-1} = F(x^e_{h,t-1} - x_{t-1}) + \eta U_{h,t-2}, \tag{8}$$

where F is a function of the distance of heuristic h's forecast from the actual realization, which we assume to be the squared error. $0 \leq \eta \leq 1$ measures the relative weight of errors in the more distant past and is called a memory parameter. If $\eta = 0$, the fitness measure solely depends on the most recent observation. If $0 < \eta < 1$, the fitness measure depends on all past prediction errors with exponentially declining weights.

If $\eta = 1$, the fitness measure weights all past observations equally. Assuming that all agents update the heuristic they use each period, one can describe the probability that an agent uses heuristic h in period t (or similarly the fraction of agents using heuristic h in period t) as follows:

$$n_{h,t} = \frac{\exp\left(\mu U_{h,t-1}\right)}{\sum_{h=1}^{H} \exp\left(\mu U_{h,t-1}\right)} \,. \tag{9}$$

One can derive the multinomial logit expression described in Equation (9) from a random utility model (see Brock and Hommes, 1997). $\mu \geq 0$ determines how likely it is that agents choose the optimal forecasting heuristic according to the fitness measure U_h. This parameter is called intensity of choice. If $\mu = 0$, the fraction of agents choosing heuristic h in period t, denoted by $n_{h,t}$, is constant, which means that agents do not react to past performance. If $\mu = \infty$, all agents always switch to the optimal forecasting heuristic according to the fitness measure.

The model described by Equation (9) is extended in Hommes et al. (2005a) and Diks and van der Weide (2005). The extension allows for asynchronous updating, which means that it is possible that not all agents update the forecasting heuristic they use each period (this is consistent with empirical evidence; see Hommes et al., 2005b, and Anufriev and Hommes, 2012). One then arrives at a more general version of Equation (9):

$$n_{h,t} = \delta n_{h,t-1} + (1-\delta)\frac{\exp\left(\mu U_{h,t-1}\right)}{\sum_{h=1}^{H} \exp\left(\mu U_{h,t-1}\right)} \,. \tag{10}$$

$0 \leq \delta \leq 1$ is the probability that an agent does not update the forecasting heuristic in a given period or, similarly, the average fraction of individuals not updating their forecasting heuristic.

In general, the set $\mathscr{H}$ can contain any number of heuristics. However, when applying the model, one set of forecasting heuristics needs to be specified. The set of heuristics that we use is described in Table 1 (our results are not peculiar to this specific set of heuristics; we show results with other ways of forming expectations in Section 3.5).

Table 1: Set of heuristics

ADA	adaptive expectations	$x^e_{1,t+1} = 0.65x_{t-1} + 0.35x^e_{1,t}$
WTR	weak trend-following	$x^e_{2,t+1} = x_{t-1} + 0.4(x_{t-1} - x_{t-2})$
STR	strong trend-following	$x^e_{3,t+1} = x_{t-1} + 1.3(x_{t-1} - x_{t-2})$
LAA	learning, anchoring, and adjustment	$x^e_{4,t+1} = 0.5(x^{av}_{t-1} + x_{t-1}) + (x_{t-1} - x_{t-2})$

Notes: x^{av}_{t-1} denotes the average of all observations up to time $t-1$.

The switching model with these heuristics describes individual forecasting behavior well in a variety of situations (e.g., Hommes et al., 2005b; Hommes et al., 2008; Anufriev and Hommes, 2012). In Assenza et al. (2014) and Hommes et al. (2017) the model explains output gap and inflation forecasts well in learning-to-forecast experiments based upon model equations (1)-(3). Following these papers, we use the calibration $\mu = 0.4$, $\delta = 0.9$, and $\eta = 0.7$. We initialize the model by starting out in the steady state and with an equal fraction of agents using each of the four heuristics.

3 Results

We now present the main results of the paper. They can be summarized as follows:

1. Economic behavior in currency unions can be very different from economic behavior in homogeneous economies. Specifically, in currency unions deviations from the steady states can be much more persistent and economic behavior can be considerably less stable.
2. Economic integration plays a crucial rule in determining the stability of a currency union. More economic integration makes the currency union more stable.
3. Monetary policy alone cannot ensure the stability of a currency union.
4. If currency unions are modeled with rational expectations, the problems of currency unions (Results 1-3) are overlooked.

We use three countries with equal weights and equal macroeconomic parameters, throughout ($\beta^i = \beta$, $\kappa^i = \kappa$, $\sigma^i = \sigma$, and $\gamma^i = \gamma$ for all i; γ is of course not defined for a homogeneous economy). We use three countries and a symmetric set-up to have a simple illustration while avoiding the all-too-classic two-country case (the results are not restricted to the three-country case). The error terms are independent and distributed according to a normal distribution with mean zero and standard deviation 0.25 for each of the three countries in the currency-union case. For the homogeneous economy we adjust the standard errors, so that the shocks correspond to the aggregated shock in the currency union.[4] We analyze parameter changes below, but in many cases we resort to a default calibration. This calibration (used unless otherwise stated) is $\beta = 0.99$, $\kappa = 0.15$, $\sigma = 1$, and $\gamma = 0.66$.[5] The coefficients of the Taylor Rule are $\Phi_\pi = 1.5$ and

[4] The sum of the shocks of the currency-union countries (weighted by the relative size of these countries) is again distributed according to a normal distribution with mean zero. In the case of three symmetric countries, the standard deviation for the homogeneous economy is then $0.25 * \sqrt{3}$. The adjustments of the standard deviations of the shocks are not crucial for our results.

[5] The periods of the model can be interpreted as quarters. For easier interpretation, the parameters are annualized values.

$\Phi_y = 0.5$. The inflation target is $\bar{\pi} = 2$. Our parameterization is in line with standard calibrations in the macro literature.

In this paper, we use the term "stability" in a very pragmatic way. Thus, we consider behavior as stable if it does not deviate too much from the steady state. This is not equivalent to the mathematical stability of a steady state of a dynamical system. While such mathematical stability may be of theoretical interest, it is for our purpose more useful to define stability as behavior that does not deviate too much from the steady state. If the steady states of output gap and inflation are mathematically unstable, but the oscillatory or even chaotic behavior stays extremely close to the steady state, an economist or policy maker concerned with economic stability should be satisfied. If, however, a steady state is mathematically stable, but convergence to this steady state only occurs within a narrow band around this steady state, an economy or currency union is in trouble, from the perspective of a practitioner. At the end of this section, we briefly discuss the mathematical stability of the steady states.

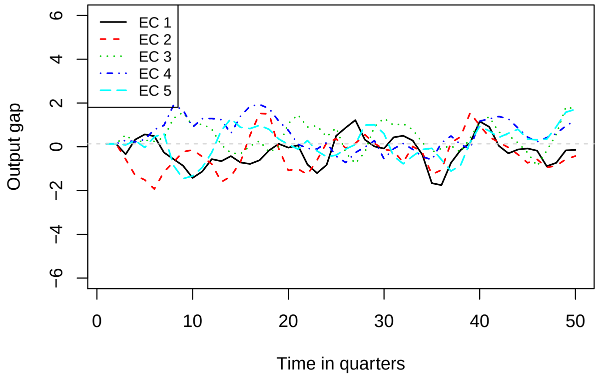
Concerning the interpretation of the time series of economic variables, it ought to be kept in mind that the models shown here are linearized. This means that these models are useful in interpreting behavior relatively close to the steady states. The models can also be used to predict when large deviations from the steady states first arise. However, the behavior of the model should not be considered meaningful from an economic point of view once economic variables are far from the steady state. This implies that the models are not useful in predicting when economic variables return to the steady state after large deviations from it occur.[6]

3.1 Economic Behavior and Stability

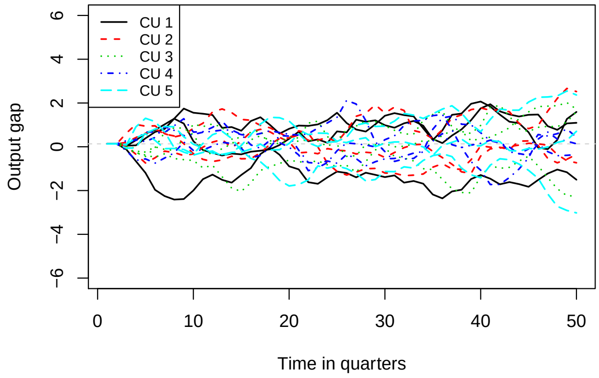
We now compare economic behavior in currency unions with economic behavior in homogeneous economies. To begin, Figure 1 shows examples of output gap, inflation, and interest rate in five homogeneous economies and in five currency unions. In the graphs for the homogeneous economy, each line corresponds to one economy; in the graphs for the currency union, three identical line types represent the three countries of one currency union.

As one can see, over a time frame of 50 periods (corresponding to 12.5 years), economic behavior in the homogeneous economy is very stable. The output gaps of all five economies stay within a narrow band around the steady state. The same holds

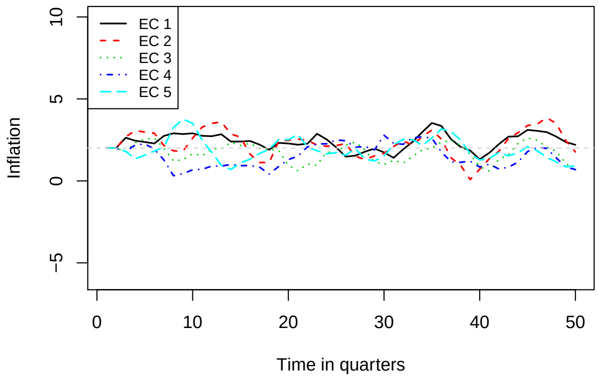
[6] Explosive behavior, then, except for the initial phase of the“explosion”, has no economic meaning (the linearization can, for example, lead to numbers of inflation below -100%).



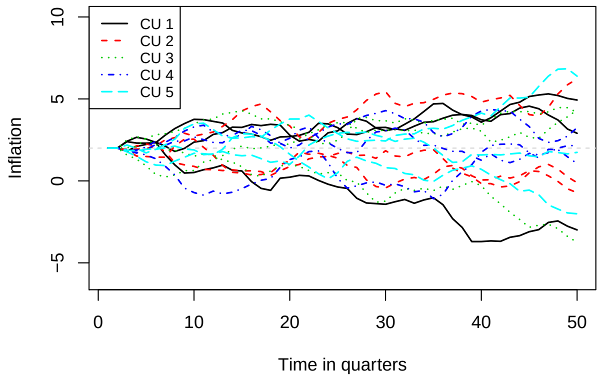
(a) Output gap, homogeneous economies



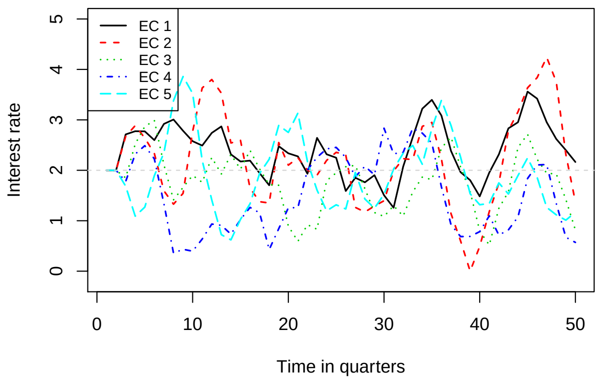
(b) Output gap, currency unions



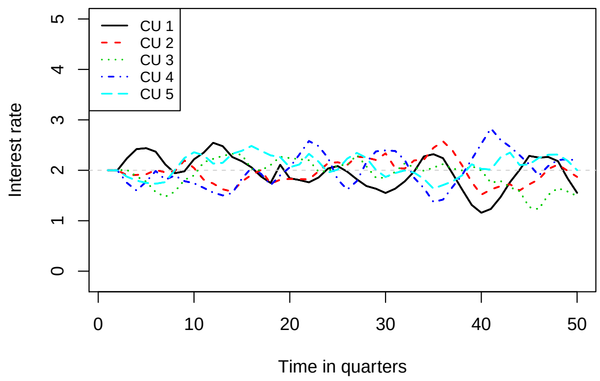
(c) Inflation, homogeneous economies



(d) Inflation, currency unions



(e) Interest rate, homogeneous economies



(f) Interest rate, currency unions

Figure 1: Economic Behavior in Homogeneous Economies and Currency Unions

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25 (for CU), sd=$0.25\sqrt{3}$ (for HE).

for inflation. The interest rate set by the central bank to stabilize economic behavior fluctuates a little bit more but still within moderate bands. The behavior of the currency unions is more interesting. Note that the deviations of output gap and inflation in the different countries is very persistent. Countries in a currency union can have above steady-state levels of output gap and inflation for many periods without reverting back to the steady-state level. Changes from above the steady state to below or vice versa can also be observed in some cases. The interest rate stays relatively stable here, however. Figure 15 in Appendix C.1 shows the same graphs for 100 economies and currency unions.[7] The general picture remains the same for simulations with 100 economies; persistent deviations from the steady state can be observed, as well as divergent behavior one the one hand, and changes from above the steady state to below and vice versa.[8]

Figure 2 shows the stability of economic behavior for different values of κ and σ (in a range of reasonable values according to standard calibrations). The other parameters are fixed at their default calibrations. Red represents homogeneous economies and blue currency unions. Each mark in the graph stems from 1000 simulations. The marks and lines shown correspond to two measures of instability. One measure is output gap instability. Each mark in the homogeneous economy case shows the fraction of simulations with the output gap deviating more than ten percent from the steady state at least once. Each mark in the currency union case similarly shows the fraction of simulations with the average of absolute deviations from the steady state in the currency union being at least ten percent (choosing different numbers does not alter our conclusions). The second measure similarly shows the instability of inflation.

The left panel of Figure 2 shows that, in a currency union, a higher κ leads to less stable economic behavior. This means that a larger feedback from the output gap on inflation in the New Keynesian Phillips Curve is destabilizing. This can be explained as follows. Via κ, deviations of the output gap from its steady state lead to movements of inflation (which in turn influence the output gap). If these movements, for example after a demand shock, are reinforced (via trend-following behavior in the expectation formation), this can lead to even greater fluctuations in inflation and output gap alike. Economic behavior in a homogeneous economy is overall much more stable. However,

[7]In the appendix, we only show graphs with 100 simulations corresponding to Figure 1, but whenever we show graphs of five economies, the general picture is the same for 100 economies.

[8]In general, depending on the draws of the random shocks and the parameter values, very different behavior can be observed in our currency union model. It is possible to observe low but very persistent deviations from the steady state, stable economic behavior with only extremely small deviations from the steady state, relatively stable oscillatory behavior around the steady state, or even explosive behavior and unstable oscillations increasing in amplitude (see, for example, Figure 17 in Appendix C).

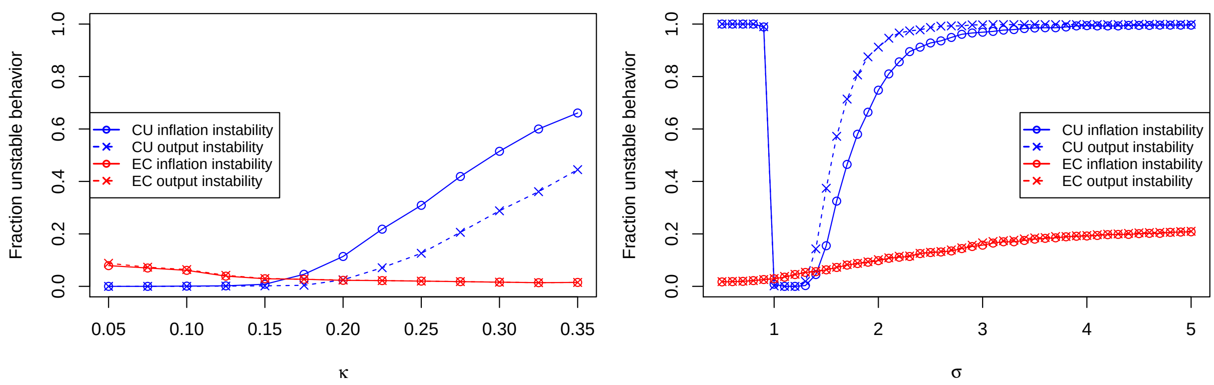


Figure 2: Instability Depending on κ and σ

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25 (for CU), sd=$0.25\sqrt{3}$ (for HE).

the comparably few simulations that do show some unstable behavior of inflation in the homogeneous economy reveal that here a lower κ can lead to relatively more instability. A lower κ can lead to higher instability if supply shocks (or their reinforcement via trend-following expectations) play an important role. In that case, output gap and inflation move in opposite directions and a higher value of κ is stabilizing, because it represents a force pulling inflation in the same direction as the output gap. The right panel of Figure 2 investigates the role of the economies' sensitivity to the interest rate for economic stability. Greater values of σ represent less reaction of the output gap to changes in the real interest rate. Interestingly, when looking at the results from the currency unions, one can observe a U-shaped pattern. Both, very strong reactions to the interest rate and very weak reactions to the interest rate lead to unstable behavior, while economic behavior is stable for intermediate values of σ. It is not surprising that a strong reaction to the interest rate (low σ) leads to unstable behavior. As the interest rate only reacts to currency union level aggregates, it is in general too low or too high for most, if not for all, countries. If these countries react strongly to this "mis-specification", instability may result. It may be more surprising that a very low sensitivity to the interest rate can also be destabilizing (i.e., high values of σ). This can be the case, because monetary policy is still a stabilizing force at the currency union level. If σ is high, economies hardly react to the interest rate decisions of the central bank, which in turn has less influence on economic behavior. For homogeneous economies, economic behavior is again much more stable. Inflation is a bit more likely to be unstable for higher values of σ (low sensitivity to the interest rate). This means that economic behavior tends to become less stable when the central bank has less influence on economic behavior with its interest rate decisions, which is very intuitive.

Figure 3 shows a similar graph for changes in β and in the standard deviation (again with the default calibration otherwise). In the left panel, we can see that in a currency union a lower β tends to lead to more stable economic behavior (although the effects observed here are rather small). This means that the less expected inflation translates into actual inflation (in the New Keynesian Phillips Curve) the more stable is economic behavior. This is intuitive if one thinks of trend-following behavior in expectation formation. Such destabilizing behavior is less important with lower values of β. The homogeneous economies are very stable and we can hardly observe an effect of β. In the right panel of Figure 3 one can see the effect of the size of the shocks. In a currency union, we can observe that a higher standard deviation of the shocks leads to less stable economic behavior, which is not surprising. The same holds for homogeneous economies (the size of the effect is here even a bit larger).

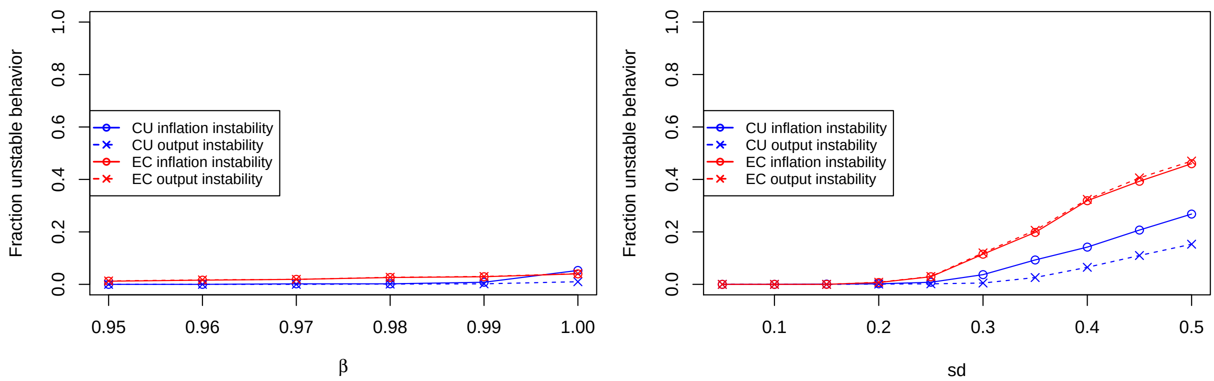


Figure 3: Instability Depending on β and the Standard Deviation of the Shocks

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25 (for CU), sd=$0.25\sqrt{3}$ (for HE).

We summarize the discussion thus far in the following result.

Result 1: *Economic behavior in currency unions can be very different from economic behavior in homogeneous economies. Specifically, in currency unions deviations from the steady states can be much more persistent and economic behavior can be considerably less stable.*

3.2 Economic Integration

We have discussed variations of most of the parameters. While how these parameters affect economic behavior may be of theoretical interest, this information is of limited use from a policy perspective. Influencing these parameters is very difficult, if not impossible, and outcomes of attempts to do so are uncertain (it is, for example, completely unclear what kind of reforms could decrease σ). This is different for γ, the parameter that is not part of the model equations in a homogeneous economy. This parameter measures how strongly one country reacts to (expected) price changes in other countries, and thus represents the economic integration of a country in the currency union. Thus, γ can be influenced by reforms aimed at increasing or decreasing economic integration.[9] These reforms may not have a strong effect in the short run, but in the long run they determine γ. This parameter thus deserves special attention.

Figure 4 shows the stability of economic behavior for different values of γ (similar to Figures 2 and 3; as there is no γ in the model of the homogeneous economy, the corresponding values are now shown by horizontal red lines). What can be seen is that for low values of γ, i.e. for a low level of economic integration, economic behavior in the currency union is unstable. When γ increases, the economic system becomes stable.

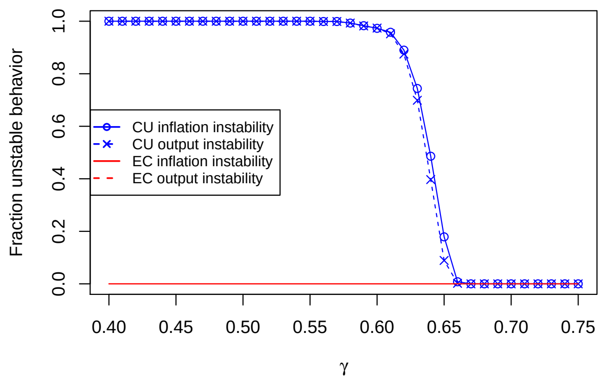


Figure 4: Instability Depending on Economic Integration

Notes: $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25 (for CU), sd=$0.25\sqrt{3}$ (for HE).

The stabilizing force of γ is that when some countries are in a boom and some others in a slump, expected increases in inflation in the boom countries lead to increased demand in the countries in a slump and vice versa, so that there exists a pull toward the steady state. This corresponds naturally to the intuition from the basic classes on international macroeconomics: In a currency union (or fixed exchange rater regime), goods in countries in an economic downturn become relatively cheaper and will be thus imported to a greater extent by other countries. This stabilizing effect will be stronger

[9]There are many such reforms. To name just a few possibilities, reforms could be aimed at removing obstacles to trade between the countries, or they could aim to ensure that firms from each country can compete in all countries. Reforms aimed at increasing labor mobility between countries also strengthen economic integration. One could even think of promoting websites in other languages, so that firms and consumers can easily compare prices. Reforms aimed at lightening economic integration could, for example, be the introduction of duties.

if the countries are more integrated economically.[10] We summarize this discussion as follows.

Result 2: *Economic integration plays a crucial rule in determining the stability of a currency union. More economic integration makes the currency union more stable.*

3.3 Monetary Policy

One might think that monetary policy can stabilize economic behavior in a currency union significantly. In particular, one might think that a stronger reaction of monetary policy to inflation and output gap could be stabilizing.[11] This is not the case, however. Increasing the reaction coefficients in the Taylor Rule, in general, does not stabilize the economy. Figure 5 shows output gap and inflation of simulations with very high Taylor Rule coefficients ($\Phi_\pi = 6$ and $\Phi_y = 2$). Except for the Taylor Rule coefficients, the simulations are identical to those in Figure 1.

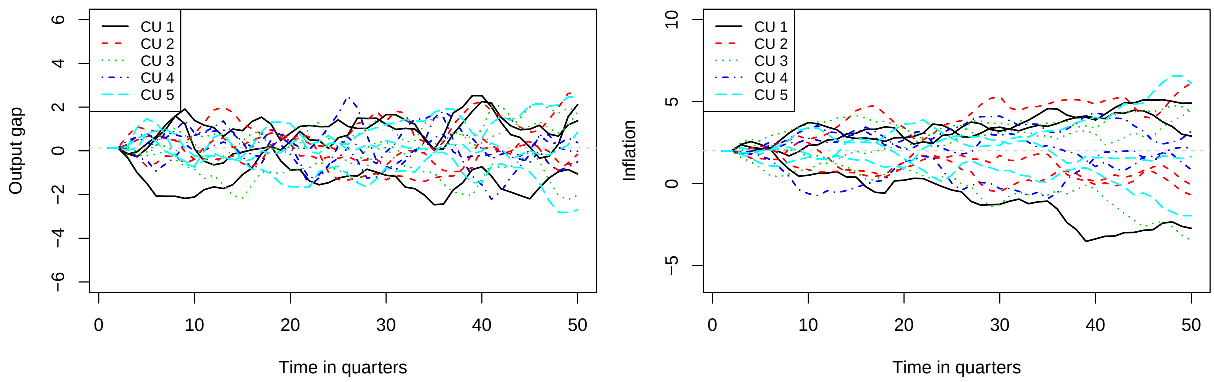


Figure 5: Output Gap and Inflation in Currency Unions with $\Phi_\pi = 6$ and $\Phi_y = 2$

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 6$, $\Phi_y = 2$, sd=0.25 (for CU), sd=$0.25\sqrt{3}$ (for HE).

As one can see, output gap and inflation are nearly identical to those found with regular monetary policy. More aggressive monetary policy does not help to make economic behavior more stable. The reason for this is that the central bank can only react to currency union aggregates. This means that monetary policy is "wrong" for most, usually even for all, countries. Imagine a currency union where two countries have output

[10] Note that for very high values of γ economic stability can decrease again. However, these values bear no economic relevance. We have relegated a more detailed discussion to Appendix C.2.

[11] As shown in model equation (6), we only consider a Taylor Rule as monetary policy. Other forms of conducting monetary policy, such as called quantitative easing, are not considered. However, as discussed below, we believe that the important part is not the exact specification of the monetary policy rule, but the property that the central bank can only react to currency union aggregates with monetary policy.

gaps and inflation below the steady state, and one country has them above it, in a way that the currency union aggregates are exactly at the steady states. The interest rate is then too high for the first countries, potentially leading their output gap and inflation to decrease even further. The interest rate is too low, however, for the country in a boom, which may then exhibit further increases of output gap and inflation. More activist monetary policy similarly cannot stabilize economic behavior when starting out from a relatively unstable calibration (see Appendix C.3). The findings discussed here lead us to the next result.

Result 3: *Monetary policy alone cannot ensure the stability of a currency union.*

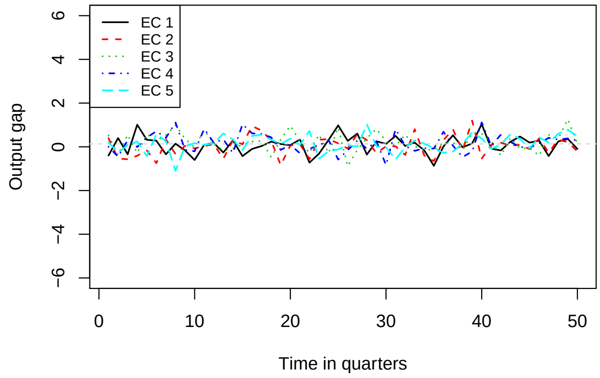
Note that this insufficiency is really due to the impossibility to react to country-level variables. If monetary policy were available at the country level, the currency union dynamic IS and New Keynesian Phillips curves, i.e. equations (4) and (5), would lead to very stable behavior, with high or low levels of economic integration alike. This can be seen in Figure 20 in Appendix C.4, where we show the results of an "artificial fixed exchange rate regime", that is, a situation identical to the currency union but with monetary policy available at the country level. The model equations for such a scenario can be found in Appendix B.3.

3.4 Rational Expectations

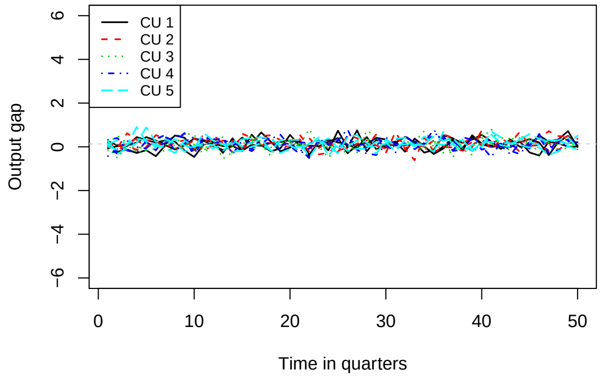
Despite the evidence that people do not form expectations rationally, modeling with a fully rational representative agent is still the standard in macroeconomics. One may wonder whether expectation formation is crucial for the results described so far, or if the same results are obtained under rational expectations. Figure 6 shows economic behavior under rational expectations in homogeneous economies and in currency unions for the extreme case that the countries in the currency union are not integrated economically in any way (i.e., $\gamma = 0$, which is extremely unstable under behavioral expectations).

It can be seen that, even in this extreme case, economic behavior is always very stable. Economists relying on the model with rational expectations may thus erroneously conclude that there are no problems when entering a currency union, at least from a viewpoint of stability or persistence of deviations from the steady state. This extreme stability holds for basically all possible parameters. Figures showing the effects of parameter changes can be found in Appendix C.5 (Figures 22, 23, and 24).

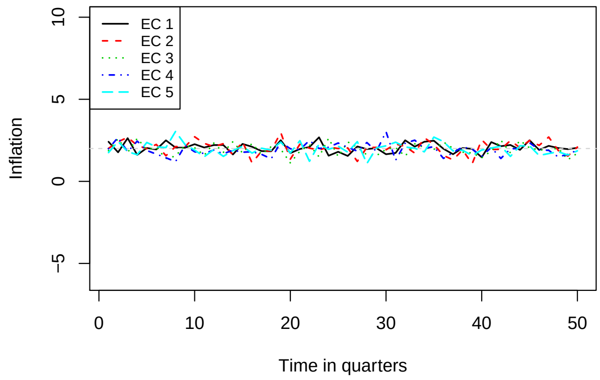
Result 4: *If currency unions are modeled with rational expectations, the problems of currency unions (Results 1-3) are overlooked.*



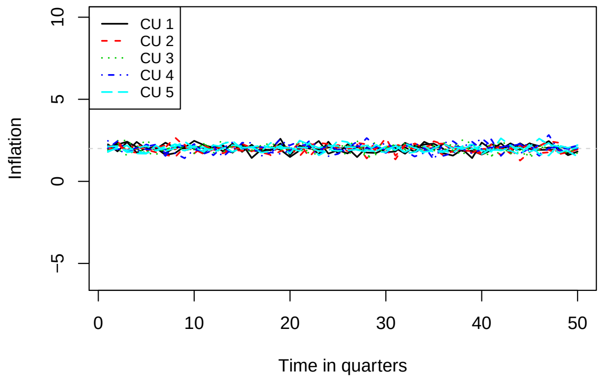
(a) Output gap, homogeneous economies



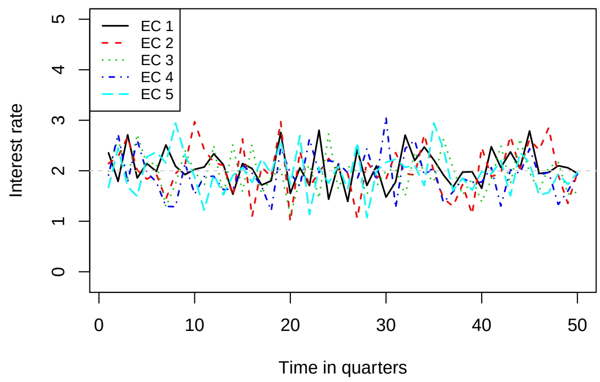
(b) Output gap, currency unions



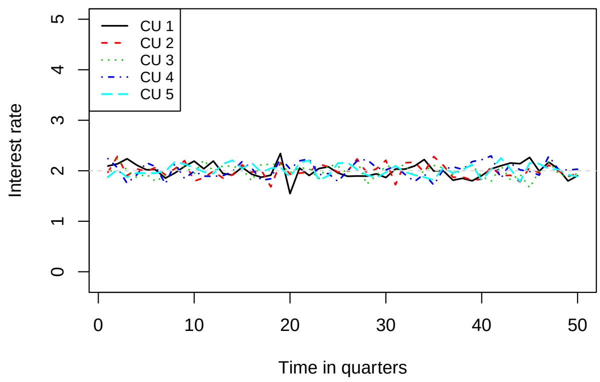
(c) Inflation, homogeneous economies



(d) Inflation, currency unions



(e) Interest rate, homogeneous economies



(f) Interest rate, currency unions

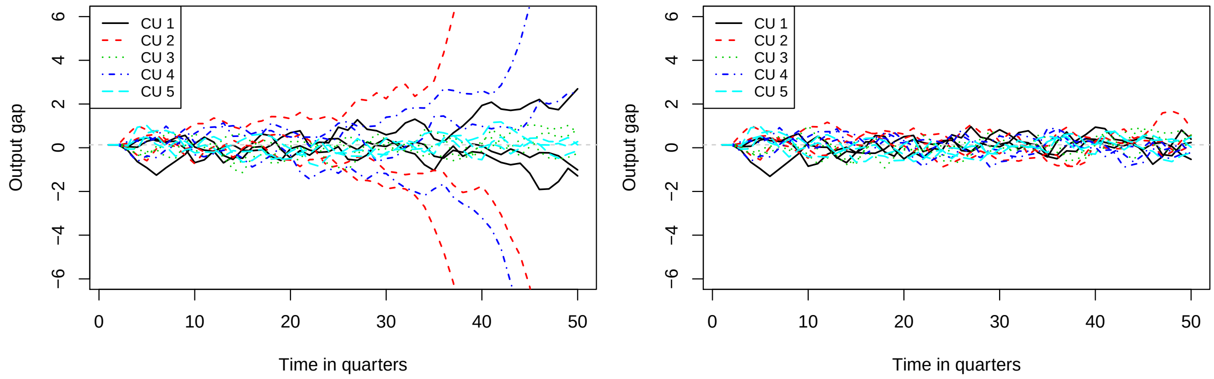
Figure 6: Economic Behavior in Homogeneous Economies and Currency Unions **under Rational Expectations**

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25 (for CU), sd=$0.25\sqrt{3}$ (for HE).

3.5 Alternative Behavioral Models of Expectation Formation

We have seen that the results we obtain with the behavioral heuristic switching model are not obtained under rational expectations. What about other models of expectation formation? As it turns out, the results from our main behavioral model are very robust. In addition to being robust to changes in the parameter values of this behavioral model (which we do not show here), the results are robust to assuming a variety of other models of expectation formation. Here, we show two such examples. The first of these is a simple heuristic switching model, similar to the one described in Section 2.3 but with the difference that agents can only use two very simple heuristics. These heuristics are naive expectations (naive expectations are expectations that always equal the last observations) and a simple trend-following rule with coefficient one. The second example involves no switching between heuristics but assumes that expectations are formed homogeneously according to the adaptive rule described in Section 2.3.

Figure 7 shows output gaps under the very simple heuristic switching model for $\gamma = 0.61$ and $\gamma = 0.66$. Also here, very unstable behavior arises for a low level of economic integration. Increasing the level of economic integration leads to more stable economic behavior.

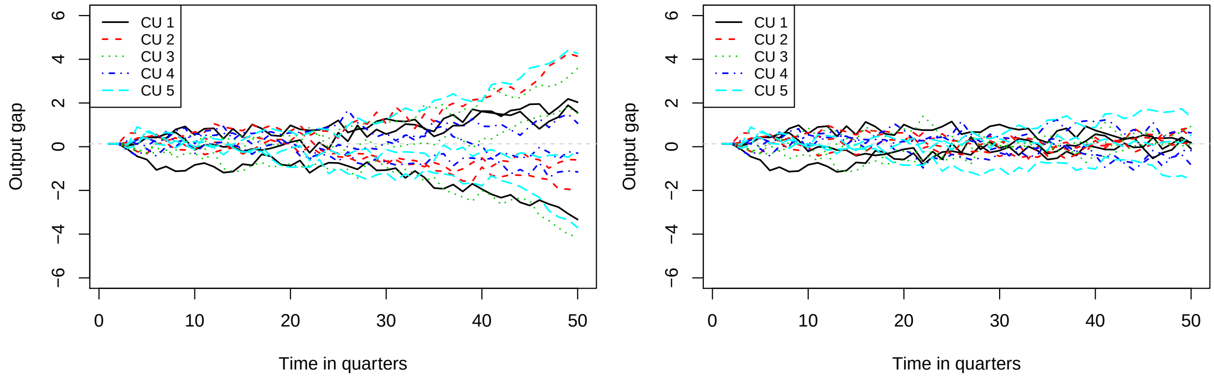


(a) $\gamma = 0.61$ (b) $\gamma = 0.66$

Figure 7: Output Gaps in Currency Unions with Expectations Formed According to a Simplistic Heuristic Switching Model for Different Values of γ

Notes: $\gamma = 0.61$ (left), $\gamma = 0.66$ (right), $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25.

Figure 8 shows output gaps assuming that expectations are formed according to the adaptive rule (again for $\gamma = 0.64$ and $\gamma = 0.7$). Again, we see unstable behavior for $\gamma = 0.64$ and that economic behavior becomes more stable when γ increases.



(a) $\gamma = 0.64$

(b) $\gamma = 0.7$

Figure 8: Output Gaps in Currency Unions under Adaptive Expectations for Different Values of γ

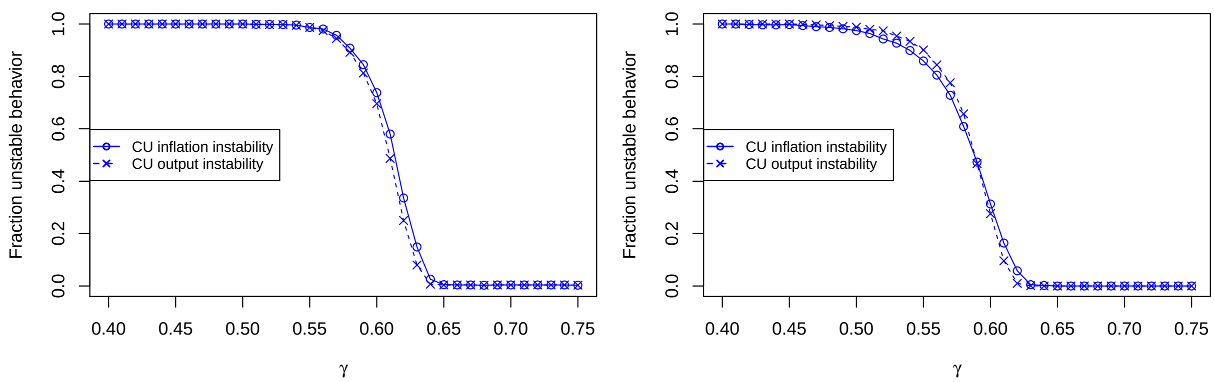
Notes: $\gamma = 0.64$ (left), $\gamma = 0.7$ (right), $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25.

Next we examine how the stability changes when γ changes as in Figure 4. This results in Figure 9a for the simple heuristic switching model and in Figure 9b for adaptive expectations. Similar to the results from the main behavioral model, increasing economic integration leads to more stable economic behavior. Interestingly, although these behavioral mechanisms of expectation formation are quite different, economic behavior becomes stable at similar values of γ (with other parameters at their standard calibration).[12] Appendix C.6 shows more graphs of simulations with the alternative mechanisms of expectation formation considered here.

3.6 Mathematical Stability of the Steady States under Naive Expectations

The stability of a steady state in a dynamical system is determined by the greatest absolute value of the eigenvalues of the Jacobian matrix. If this value is less than one, the steady state is stable. The macroeconomic model, with our main model of expectation formation, constitutes too large a dynamical system to be used here. Therefore, we show eigenvalues assuming naive expectations, which serve as a first indication of the mathematical stability of the steady states. As mentioned in the beginning of the results section, this mathematical stability of the dynamical system is not necessarily economically relevant in our case, but it nevertheless shows our results from an additional,

[12]Again, one could observe an increase in instability again when increasing γ to extremely high values. As discussed in Appendix C.2, such values bear no economic relevance.



(a) Simple HSM (b) Adaptive expectations

Figure 9: Instability Depending on Economic Integration for Different Behavioral Expectations

Notes: $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25.

interesting slant.

Figure 10 shows the absolute values of the eigenvalues for a homogeneous economy on the vertical axis (with the default calibration). The specification of monetary policy is shown on the horizontal axis; moving from left to right, both coefficients in the Taylor Rule increase simultaneously, while their ratio is fixed at $\Phi_\pi = 3 * \Phi_y$. The value on the horizontal axis corresponds to the value of Φ_π. In the homogeneous economy, a stronger reaction of monetary policy to deviations of output gap and inflation from the steady state increases stability. This can be seen in the graph by the decrease of the greatest absolute value of the eigenvalues when Φ_π increases (and Φ_y with it).

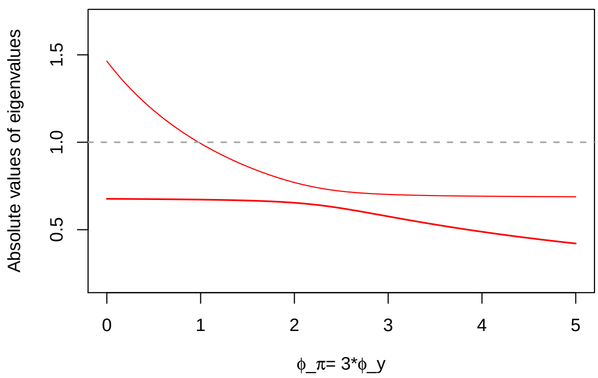
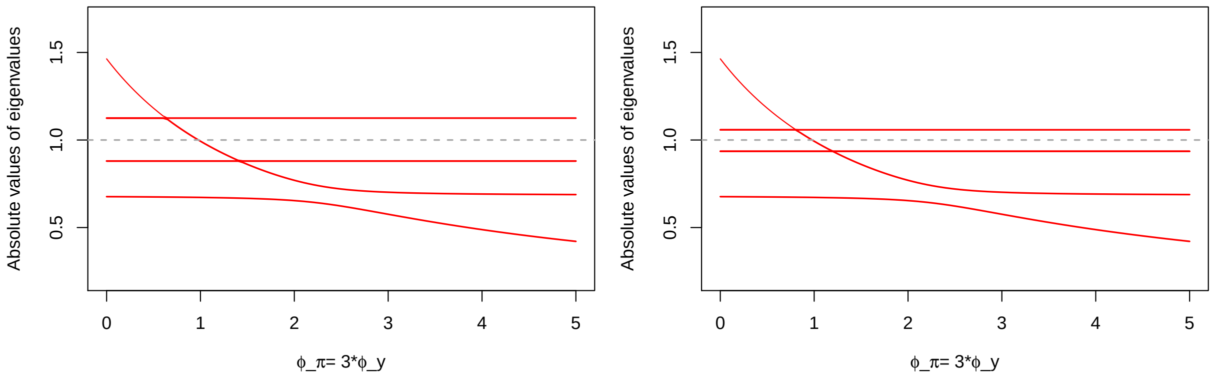


Figure 10: Absolute Values of Eigenvalues in Homogeneous Economies (Naive Expectations)

Notes: $\Phi_\pi = 3 * \Phi_y$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$.

Figure 11 shows that this is different in a currency union. A stronger reaction of the interest rate to deviations of output gap and inflation from their steady states does not increase the stability of the steady state (except for a short period with extremely low Taylor Rule coefficients). More aggresive monetary policy does not decrease the greatest eigenvalue after a short initial phase (note that the two horizontal lines represent two eigenvalues each).



(a) $\gamma = 0.6$ (b) $\gamma = 0.65$

Figure 11: Absolute Values of Eigenvalues in a Currency Union (Naive Expectations)

Notes: $\Phi_\pi = 3 * \Phi_y$, $\gamma = 0.6$ (left),$\gamma = 0.65$ (right), $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$.

However, economic integration does matter for the stability of the steady states. Figures 11a and 11b show the same graphs for two different values of γ. As we can see in the graphs, increasing γ leads to a decrease of the greatest absolute eigenvalues and makes the steady state more stable (further increasing γ here leads to the two horizontal lines collapsing a bit below one; this can be seen in Figure 28b, Appendix C.7). Appendix C.7 also shows the same graphs for currency unions with two countries, which look extremely similar (there are still two horizontal lines that then collapse, but these lines represent only one eigenvalue each before collapsing).

4 A Model with more Emphasis on Realized Variables

One might wonder whether our results persist when considering a different macroeconomic model. In this section, we briefly consider an aggregate model based more on the realizations of economic variables. The analysis generally confirms the results described in Section 3.

4.1 Model Equations

The model equations for a currency union are as follows:

$$y_t^i = a_1^i \tilde{E}_t y_{t+1}^i + (1-a_1^i) y_{t-1}^i + a_2^i (r_t - \tilde{E}_t \pi_{t+1}^i) + a_3^i s_t^i + m^i (y_t^{*i} - y_t^i) + g_t^i \tag{11}$$
$$\pi_t^i = b_1^i \tilde{E}_t \pi_{t+1}^i + (1-b_1^i) \pi_{t-1}^i + b_2^i y_t^i + u_t^i \tag{12}$$
$$r_t = max\{\bar{\pi} + c_1(\pi_t^{cu} - \bar{\pi}) + c_2(y_t^{cu} - \bar{y}^{cu}),\ 0\}. \tag{13}$$

This model is a modification of a two-country model without the real exchange rate, which is developed in De Grauwe and Ji (2017).[13] We use Latin rather than Greek characters for the parameters to avoid confusion of the two models we discuss in this paper (staying close to the notation in De Grauwe and Ji, 2017).

Inflation, output gap, interest rate, and their steady states have the same notation and meaning as before. s_t^i denotes the (log) real effective terms of trade of country i, $s_t^i = p_t^{*i} - p_t^i$. $0 < a_1^i < 1$, $a_2^i < 0$, $0 \le a_3^i$, $0 < b_1^i < 1$, $0 < b_2^i$, and $0 < m^i$ are parameters. $0 < c_1$ and $0 \le c_2$ are the reaction coefficients in the Taylor Rule and g_t^i and u_t^i are independent, identically distributed demand and supply shocks. Equations (11) and (12) describe again dynamic IS curves and New Keynesian Phillips curves. Equation (13) describes the behavior of the central bank following a Taylor Rule, as before.

The differences between this set of aggregate equations and Equations (4) to (6) are the following. (1) There is the term $m^i(y_t^{*i} - y_t^i)$ representing the effects of increased demand from abroad when the output gap is high abroad compared to the output gap at home (and vice versa). (2) It is now assumed that the (log) real effective terms of trade $s_t^i = p_t^{*i} - p_t^i$ matters instead of the expected change in the terms of trade $\tilde{E}_t \Delta s_{t+1}^i$. (3) Inflation and output gap now also depend on their own past realized values.

In the steady state different from the zero lower bound, inflation in all countries is again equal to the central bank's inflation target $\bar{\pi}$. The steady-state level of the output gap in all countries is equal to zero, $\bar{y}^i = 0$ (the term $\bar{y}^{cu}$ could thus be removed; we refrain from doing so in order to keep the similarities to the model discussed in Section 2).

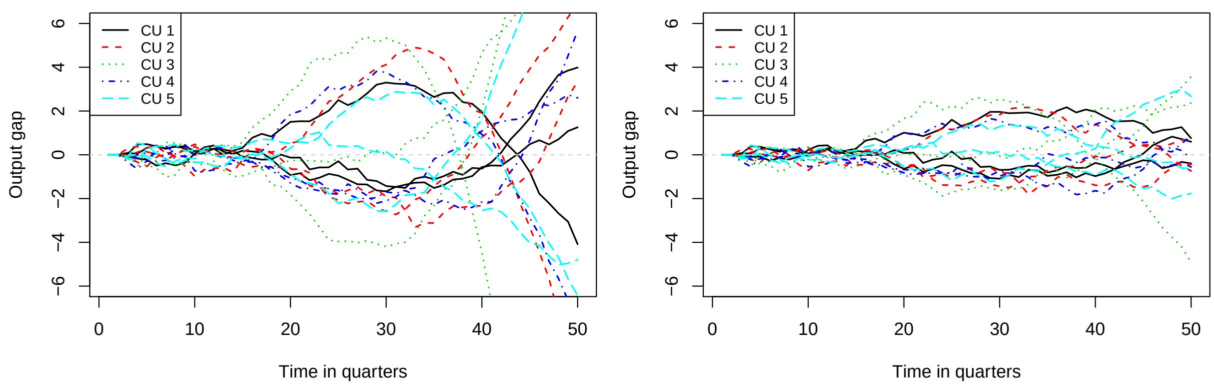
A model of a homogeneous economy in this style consists of the same equations (with $N = 1$) without the parts $a_3^i s_t^i$ and $m^i(y_t^{*i} - y_t^i)$. The equations are shown in Appendix D. This appendix furthermore shows the matrix forms of the currency union model and the calculations of the rational expectation solution.

[13] De Grauwe and Ji (2017) show that business cycles can be correlated assuming uncorrelated shocks.

4.2 Results

For the currency union we show again only the symmetric three country case. The calibration we use, unless stated otherwise, is the following: $a_1 = b_1 = 0.5$, $a_2 = -0.5$, $b_2 = 0.1$, and $a_3 = 0.05$. In this model, m turns out to play a crucial role; therefore, we show results with different values. Again we use policy parameters $c_1 = 1.5$ and $c_2 = 0.5$ and the same distributions of the shocks as before. We also use the same expectation formation mechanism, as described in Section 2.3. The results by and large confirm the results obtained in Section 3.

Figure 12 shows output gaps of currency unions for two values of m. Figure 12a shows the output gap for $m = 0.25$. There are sizable deviations from the steady state, with explosive behavior toward later periods. Figure 12b shows simulations with $m = 0.5$. Economic behavior in this case is much more stable.



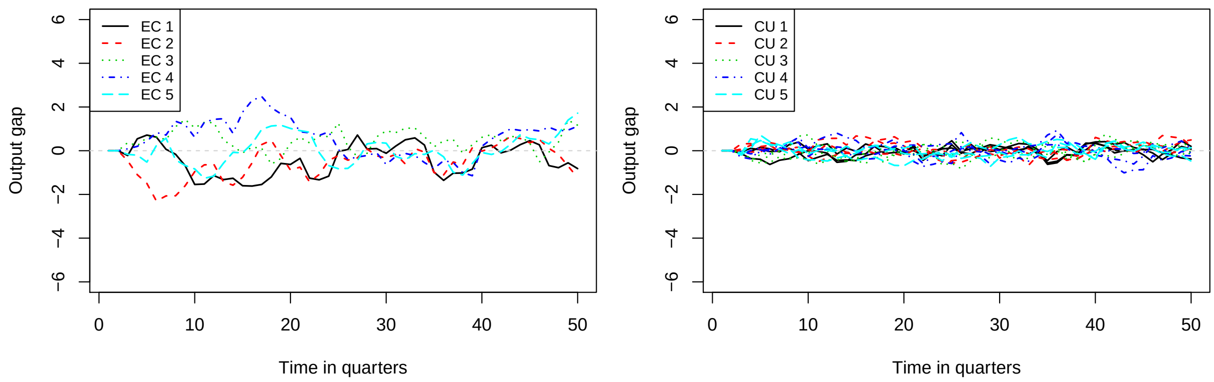
(a) $m = 0.25$

(b) $m = 0.5$

Figure 12: Output Gaps in Currency Unions for Different Values of m

Notes: $a_3 = 0.05$, $m = 0.25$ (left), $m = 0.5$ (right), $a_1 = 0.5$, $b_1 = 0.5$, $a_2 = -0.5$, $b_2 = 0.1$, $c1 = 1.5$, $c2 = 0.5$, $sd = 0.25$.

Figure 13a shows the output gap in a homogeneous economy. Economic behavior is stable. We can already see the confirmation of some of the results from the main model. Economic behavior in a currency union can be very different from what one would expect when modeling it with a homogeneous economy. Furthermore, in the additional macroeconomic model as well, economic integration plays a crucial role when determining the stability of a currency union. This is illustrated in Figure 12 and it can also be seen when examining the stability of the currency union for different values of m, which is shown in Figure 14a (using the same measure of stability as in Section 3). This graph shows that economic behavior is more stable for economically more integrated countries.



(a) Homogeneous economy, behavioral expectations (b) Currency union, rational expectations

Figure 13: Output Gaps in Homogeneous Economies under Behavioral Expectations (Left) and in a Currency Union under Rational Expectations (Right)

Notes: $m = 0$, $a_3 = 0$, $a_1 = 0.5$, $b_1 = 0.5$, $a_2 = -0.5$, $b_2 = 0.1$, $c1 = 1.5$, $c2 = 0.5$, $sd = 0.25$ (for CU), $sd = 0.25\sqrt{3}$ (for HE).

Note that there are two parameters representing economic integration in this model, m and a_3 and that the effect here stems from m. However, a_3 can also play a role for the stability of the currency union. Figure 14b shows stability in dependence of a_3. While the overall level of instability that we observe is rather low, we can note a decrease of instability when a_3 increases for low values of a_3.

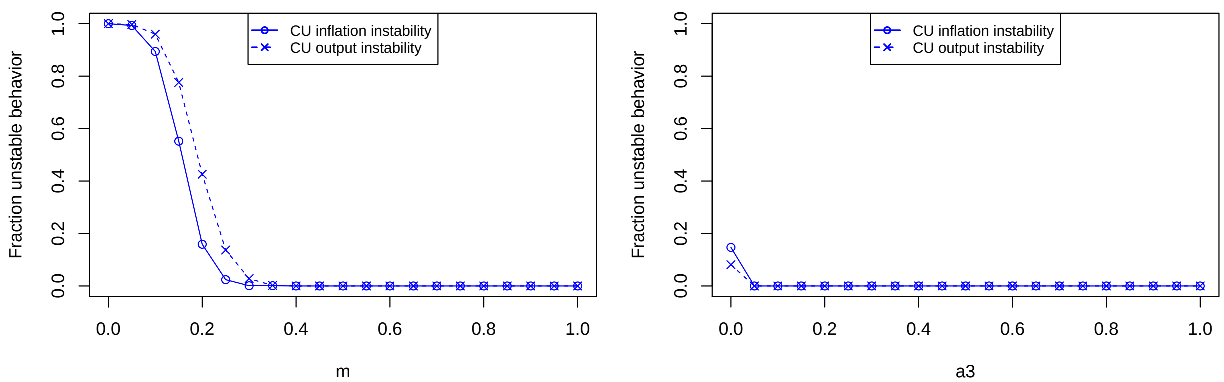


Figure 14: Instability of Currency Unions Depending on Model Parameters

Notes: $a_3 = 0.05$ (left), $m = 0.5$ (right), $a_1 = 0.5$, $b_1 = 0.5$, $a_2 = -0.5$, $b_2 = 0.1$, $c1 = 1.5$, $c2 = 0.5$, $sd = 0.25$.

Moreover, the results concerning monetary policy persist. When economic behavior is unstable, monetary policy cannot stabilize it. The corresponding graphs illustrating this are shown in Appendix E.2 (Figure 32). As in the main model, monetary policy would be enough to stabilize the economies of the currency union if the central bank could set different interest rates in the different countries, all else being equal. For illustrations of such an artificial exchange rate regime, see Figure 33 in Appendix E.3.

We also consider this model with the assumption of rational expectations. Again, the model under rational expectations does not reveal much; behavior in currency unions is very stable. Graphs of simulations with rational expectations can be found in Figure 34 in Appendix E.4. A brief discussion of the mathematical stability of the steady states with results similar to the ones in Section 3.6 can be found in Appendix E.5.

5 Conclusion

Our results are important for both policy makers and research economists. For the former, there are two crucial points. The first of these is that economic integration is of great importance in determining the functioning of a currency union. While this may be the healthy intuition of some economists working on currency unions, in this paper we provide a dynamic modeling framework to explain it. A policy recommendation that arises directly from this finding is that reforms that deepen economic integration ought to be undertaken. It should be kept in mind, though, that in general, these reforms will only bear fruit in the long run.

This leads us to the second finding of relevance to policy makers. Monetary policy alone is not enough to guarantee the functioning of a currency union. This does not mean that monetary policy is irrelevant. The central bank still needs to appropriately react to currency union aggregates; poor interest rate decisions, for example, can cause problems for the currency union. However, monetary policy alone is not enough to stabilize a troubled currency union. Thus, another policy tool to stabilize the economies in the short and medium run is called for. While there is no activist fiscal policy in our modeling framework, fiscal policy seems to be the best (if not the only) candidate. Taking our findings together with other economic research on stabilization policy and currency unions suggests that fiscal policy is indispensable as a stabilization tool in a currency union.

For research economists, there are two additional important findings. The first of these is that it is problematic to model currency unions as homogeneous economies. Economic behavior in a currency union can be very different from that in a homogeneous economy and policy recommendations built upon models of homogeneous economies may be misguided.

The second finding is that the assumption of rational expectations is not an innocent simplifying assumption (this may not come as news to many). In a setting such as ours, this assumption can decisively drive the results. Great care ought to be exercised, then, when deciding whether or not to model agents as having rational expectations. This is especially the case given the availability of behavioral models on expectation formation built on microeconomic evidence, such as the one that we use in this paper.

References

Anufriev, M. and Hommes, C. H. (2012). Evolutionary selection of individual expectations and aggregate outcomes. *American Economic Journal: Microeconomics*, 4:35–64.

Assenza, T., Heemeijer, P., Hommes, C., and Massaro, D. (2014). Managing self-organization of expectations through monetary policy: a macro experiment. CeNDEF Working Paper 14-07, University of Amsterdam.

Blanchflower, D. G. and MacCoille, C. (2009). The formation of inflation expectations: an empirical analysis for the UK. *NBER Working Paper 15388*.

Branch, W. (2004). The Theory of Rationally Heterogeneous Expectations: Evidence from Survey Data on Inflation Expectations. *The Economic Journal*, 114(497):592–621.

Branch, W. and McGough, B. (2009). A New Keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 33:1036–1051.

Branch, W. and McGough, B. (2010). Dynamic predictors selection in a New Keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 34(8):1492–1508.

Branch, W. A. and Evans, G. W. (2011). Monetary policy and heterogeneous expectations. *Economic Theory*, 47(2-3):365–393.

Brock, W. A. and Hommes, C. H. (1997). A rational route to randomness. *Econometrica*, 65(5):1059–1095.

Brock, W. A. and Hommes, C. H. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control*, 22:1235–1274.

Busetti, F., Delle Monache, D., Gerali, A., and Locarno, A. (2015). Trust, but verify. De-anchoring of inflation expectations under learning and heterogeneity. *Working Paper*.

Carroll, C. (2003). Macroeconomic expectations of households and professional forecasters. *Quarterly Journal of Economics*, 118:269–298.

Cornea, A., Hommes, C., and Massaro, D. (2017). Behavioral heterogeneity in U.S. inflation dynamics. *Journal of Business and Economic Statistics*. Forthcoming.

De Grauwe, P. (2010). The scientific foundation of dynamic stochastic general equilibrium (DSGE) models. *Public Choice*, 144(3-4):413–443.

De Grauwe, P. (2011). Animal spirits and monetary policy. *Economic theory*, 47(2-3):423–457.

De Grauwe, P. (2012a). Booms and busts in economic activity: A behavioral explanation. *Journal of Economic Behavior & Organization*, 83(3):484–501.

De Grauwe, P. (2012b). *Lectures on behavioral macroeconomics*. Princeton University Press, New Jersey.

De Grauwe, P. and Ji, Y. (2017). The international synchronisation of business cycles: the role of animal spirits. *Open Economies Review*, 28(3):383–412.

Diks, C. and van der Weide, R. (2005). Herding, a-synchronous updating and heterogeneity in memory in a CBS. *Journal of Economic Dynamics and Control*, 29:741–763.

Evans, G. and Honkapohja, S. (2006). Monetary policy, expectations and commitment. *The Scandinavian Journal of Economics*, 108:15–38.

Galí, J. and Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies*, 72(3):707–734.

Galì, J. and Monacelli, T. (2008). Optimal monetary and fiscal policy in a currency union. *Journal of International Economics*, 76(1):116 – 132.

Gigerenzer, G. and Selten, R. (2002). *Bounded Rationality: The Adaptive Toolbox*. MIT Press.

Gigerenzer, G. and Todd, P. M. (1999). *Simple Heuristics That Make Us Smart*. Oxford University Press, USA.

Grether, D. and Plott, C. (1979). Economic theory of choice and the preference reversal phenomenon. *American Economic Review*, 69(4):623–638.

Hommes, C. H. (2011). The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control*, 35:1–24.

Hommes, C. H., Huang, H., and Wang, D. (2005a). A robust rational route to randomness in a simple asset pricing model. *Journal of Economic Dynamics and Control*, 29:1043–1072.

Hommes, C. H., Massaro, D., and Weber, M. (2017). Monetary policy under behavioral expectations: Theory and experiment. *Bank of Lithuania Working Paper No 42/2017*.

Hommes, C. H., Sonnemans, J., Tuinstra, J., and van de Velden, H. (2005b). Coordination of expectations in asset pricing experiments. *Review of Financial Studies*, 18(3):955–980.

Hommes, C. H., Sonnemans, J., Tuinstra, J., and van de Velden, H. (2008). Expectations and bubbles in asset pricing experiments. *Journal of Economic Behavior and Organization*, 67(1):116–133.

Kurz, M., Piccillo, G., and Wu, H. (2013). Modeling diverse expectations in an aggregated New Keynesian model. *Journal of Economic Dynamics and Control*, 37:1403–1433.

Malmendier, U. and Nagel, S. (2016). Learning from inflation experiences. *The Quarterly Journal of Economics*, 131(1):53–87.

Massaro, D. (2013). Heterogeneous expectations in monetary dsge models. *Journal of Economic Dynamics and Control*, 37(3):680–692.

Pfajfar, D. and Santoro, E. (2010). Heterogeneity, learning and inflation stickiness in inflation expectations. *Journal of Economic Behavior and Organization*, 75(3):426–444.

Pfajfar, D. and Zakelj, B. (2014). Experimental evidence on inflation expectations formation. *Journal of Economic Dynamics and Control*, 44:147–168.

Richman, B. D. (2005). Behavioral economics and health policy: Understanding medicaid's failure. *Cornell Law Review*, 90(3).

Thaler, R. H. and Benartzi, S. (2004). Save more tomorrow™: Using behavioral economics to increase employee saving. *Journal of Political Economy*, 112(S1):S164–S187.

Tversky, A. and Kahneman, D. (1974). Judgement under uncertainty: Heuristics and biases. *Science*, 185:1124–1130.

Weber, M. (2017). Choosing the rules: Preferences over voting systems for assemblies of representatives. *Journal of Economic Behavior and Organization*. Forthcoming.

Weber, M. and Schram, A. (2017). The non-equivalence of labour market taxes: A real-effort experiment. *The Economic Journal*, 127(604):2187–2215.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

A Appendix: Microfoundations of the Behavioral Macroeconomic Currency Union Model

The following derivation is based on the rational expectations open economy model in Galí and Monacelli (2005) and on the behavioral closed economy model in Kurz et al. (2013, see also Galì and Monacelli, 2008, and Hommes et al., 2017).

A.1 Households

Consider a generic country i part of the currency union. Household j in country i chooses consumption $C^i_{j,t}$, labor $N^i_{j,t}$ and bond holdings $B^i_{j,t}$ to maximise

$$E^i_{j,t}\sum_{\tau=0}^{\infty}\beta^{\tau}\left(\frac{(C^i_{j,t+\tau})^{1-\sigma}}{1-\sigma}-\frac{(N^i_{j,t+\tau})^{1+\eta}}{1+\eta}\right) . \tag{14}$$

The variable $C^i_{j,t}$ is a composite consumption index of household j in country i defined by

$$C^i_{j,t}=\frac{(C^i_{j,i,t})^{1-\alpha}(C^i_{j,F,t})^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}} . \tag{15}$$

Variable $C^i_{j,i,t}$ in Equation (15) denotes consumption of household j, living in country i (superscript), of goods produced in country i (subscript), given by the CES aggregator

$$C^i_{j,i,t}=\left(\int_0^1 C^i_{j,i,t}(z)^{\frac{\varepsilon-1}{\varepsilon}}dz\right)^{\frac{\varepsilon}{\varepsilon-1}} ,$$

where $z\in[0,1]$ indexes the type of good, within the set produced in country i. Following Galí and Monacelli (2005), we assume that each country produces a continuum of differentiated goods, that each good is produced by a distinct firm, and that no good is produced in more than one country.

Variable $C^i_{j,F,t}$ in Equation (15) is an index of consumer j's consumption of imported goods given by

$$C^i_{j,F,t}=\exp\int_0^1 c^i_{j,f,t}df ,$$

where $c^i_{j,f,t}=\log C^i_{j,f,t}$ is the log of an index of the quantity of goods consumed by household j in country i, and produced in country f. In what follows, lower case letters will denote logs of the respective variables.

The index $C^i_{j,f,t}$ is defined symmetrically to $C^i_{j,i,t}$, namely

$$C^i_{j,f,t} = \left(\int_0^1 C^i_{j,f,t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

Denoting the index of prices of *domestically produced* goods as $P^i_t = \left(\int_0^1 P^i_t(z)^{1-\varepsilon} dz \right)^{1/(1-\varepsilon)}$, and the index of prices of goods *imported from country f* as $P^f_t = \left(\int_0^1 P^f_t(z)^{1-\varepsilon} dz \right)^{1/(1-\varepsilon)}$, the optimal allocation of expenditures on the goods produced in a certain country gives the following demand functions

$$C^i_{j,i,t}(z) = \left(\frac{P^i_t(z)}{P^i_t} \right)^{-\varepsilon} C^i_{j,i,t}$$
$$C^i_{j,f,t}(z) = \left(\frac{P^f_t(z)}{P^f_t} \right)^{-\varepsilon} C^i_{j,f,t} .$$

Therefore we have that $\int_0^1 P^i_t(z) C^i_{j,i,t}(z) dz = P^i_t C^i_{j,i,t}$ and $\int_0^1 P^f_t(z) C^i_{j,f,t}(z) dz = P^f_t C^i_{j,f,t}$. Moreover, optimal allocation of expenditures on imported goods by country of origins yields

$$P^f_t C^i_{j,f,t} = P^*_t C^i_{j,F,t} ,$$

where $P^*_t = \exp \int_0^1 p^f_t df$ is the union-wide price index.[14] Finally, defining

$$P^i_{c,t} = (P^i_t)^{1-\alpha} (P^*_t)^{\alpha} \tag{16}$$

as the *consumer price index* (CPI) for country i, we can write the optimal allocation of expenditures between imported and domestically produced goods as

$$\begin{aligned} P^i_t C^i_{j,i,t} &= (1-\alpha) P^i_{c,t} C^i_{j,t} \\ P^*_t C^i_{j,F,t} &= \alpha P^i_{c,t} C^i_{j,t} . \end{aligned} \tag{17}$$

We can then write the households' budget constraint as

$$P^i_{c,t} C^i_{j,t} + B^i_{j,t} = B^i_{j,t-1} R_{t-1} + W^i_t N^i_{j,t} - T^i_{j,t} , \tag{18}$$

where R_t is the union-wide gross interest, W^i_t is the gross wage, and $T^i_{j,t}$ are lump sum transfers including profits from firms. The first order conditions for household's j

[14] As there is a continuum of countries, $P^*_t = P^{*i}_t$. This means that the union-wide price index is identical to the price index "abroad", that is, excluding country i.

optimisation problem are given by[15]

$$(C^i_{j,t})^{-\sigma} = \beta R_t E^i_{j,t}\left((C^i_{j,t+1})^{-\sigma}\frac{P^i_{c,t}}{P^i_{c,t+1}}\right) \tag{19}$$

$$(C^i_{j,t})^{-\sigma}\frac{W^i_t}{P^i_{c,t}} = (N^i_{j,t})^{\eta}\ . \tag{20}$$

A.2 Domestic and CPI Inflation

Define the *bilateral terms of trade* between countries i and f as $S^i_{f,t} \equiv P^f_t/P^i_t$. The *effective terms of trade* can then be written as

$$S^i_t = P^*_t/P^i_t = \exp\int_0^1 (p^f_t - p^i_t)df = \exp\int_0^1 s^i_{f,t}df\ .$$

Therefore taking logs we have $s^i_t = \int_0^1 s^i_{f,t}df$.
Using Equation (16) we can derive the following relationship between CPI and domestic price levels

$$P^i_{c,t} = P^i_t(S^i_t)^{\alpha}\ , \tag{21}$$

which can be rewritten in logs as

$$p^i_{c,t} = p^i_t + \alpha s^i_t\ .$$

From the previous equation it follows that the relationship between *domestic inflation*, defined as $\pi^i_t = p^i_t - p^i_{t-1}$, and *CPI inflation*, defined as $\pi^i_{c,t} = p^i_{c,t} - p^i_{c,t-1}$, is given by

$$\pi^i_{c,t} = \pi^i_t + \alpha\Delta s^i_t\ . \tag{22}$$

A.3 Firms

Each firm in country i produces a differentiated good, indexed by z, with a linear technology

$$Y^i_t(z) = A^i_t N^i_t(z)\ ,$$

[15]As standard, we require that agents' subjective transversality condition is satisfied, $\lim_{\tau\to\infty} E^i_{j,t}\beta^{t+\tau}(C^i_{j,t+\tau})^{-\sigma}\frac{B^i_{j,t+\tau}}{P^i_{c,t+\tau}} \leq 0$ (see Branch and McGough, 2009).

where A_t^i is a country-specific productivity process. Real marginal costs are therefore common across firms in country i and given by

$$mc_t^i = w_t^i - p_t^i - a_t^i \,. \tag{23}$$

Moreover, denoting the aggregate output index for country i as $Y_t^i = \left(\int_0^1 Y_t^i(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right)^{\frac{\varepsilon}{\varepsilon-1}}$, we can derive the following log-linearized relationship between aggregate output and aggregate employment in country i:[16]

$$y_t^i = a_t^i + n_t^i \,. \tag{24}$$

We assume a staggered price setting *à la* Calvo, where only a fraction $1-\omega$ of prices are readjusted in every period. Moreover, we consider a scenario in which households have equal ownership shares in all firms (so that income effects of random price adjustments are removed), though each household manages only one firm (i.e., makes price decisions for only one firm).

Let $Q_t^{i,opt}(z) = P_t^{i,opt}(z)/P_t^i$ denote the optimal price set by firm producing good z relative to the aggregate price level. The optimal price-setting strategy for a firm resetting its price in period t log-linearized around steady state is given by

$$\hat{q}_t^{i,opt}(z) = (1-\omega\beta)E_{z,t}^i \sum_{\tau=0}^{\infty} (\omega\beta)^\tau (\widehat{mc}_{t+\tau}^i + \pi_{t+\tau}^i) \,, \tag{25}$$

where $\hat{x}_t = x_t - x$ denotes log deviations from steady state for a generic variable X_t.

A.4 Aggregate Demand and New Keynesian Phillips Curve

Aggregating individual demands across households, the market clearing for good z produced in country i implies that

$$\begin{aligned} Y_t^i(z) &= \int_0^1 C_{j,i,t}^i(z)dj + \int_0^1 \int_0^1 C_{j,i,t}^f(z)djdf \\ &= \left(\frac{P_t^i(z)}{P_t^i}\right)^{-\varepsilon} \left(\int_0^1 C_{j,i,t}^i dj + \int_0^1 \int_0^1 C_{j,i,t}^f djdf\right) \\ &= \left(\frac{P_t^i(z)}{P_t^i}\right)^{-\varepsilon} \left(C_{i,t}^i + \int_0^1 C_{i,t}^f df\right) \,. \end{aligned} \tag{26}$$

[16]See Galí and Monacelli (2005) for a detailed derivation.

Aggregating the equations in (17) across households yields

$$\begin{aligned} P^i_t C^i_{i,t} &= (1-\alpha) P^i_{c,t} C^i_t \\ P^*_t C^i_{F,t} &= \alpha P^i_{c,t} C^i_t \,. \end{aligned} \tag{27}$$

Substituting (27) in (26) gives

$$\begin{aligned} Y^i_t(z) &= \left(\frac{P^i_t(z)}{P^i_t}\right)^{-\varepsilon} \left((1-\alpha)\frac{P^i_{c,t}}{P^i_t}C^i_t + \alpha \int_0^1 \frac{P^f_{c,t}}{P^i_t} C^f_t \, df\right) \\ &= \left(\frac{P^i_t(z)}{P^i_t}\right)^{-\varepsilon} \left((1-\alpha)(S^i_t)^\alpha C^i_t + \alpha (S^i_t)^\alpha \int_0^1 \frac{P^f_{c,t}}{P^i_{c,t}} C^f_t \, df\right). \end{aligned} \tag{28}$$

Combining individual first order conditions for households in country i and f yields

$$\beta E^i_{j,t}\left(\left(\frac{C^i_{j,t+1}}{C^i_{j,t}}\right)^{-\sigma}\left(\frac{P^i_{c,t}}{P^i_{c,t+1}}\right)\right) = \beta E^f_{j,t}\left(\left(\frac{C^f_{j,t+1}}{C^f_{j,t}}\right)^{-\sigma}\left(\frac{P^f_{c,t}}{P^f_{c,t+1}}\right)\right). \tag{29}$$

Integrating across households in both countries,[17] we get

$$C^i_t = \left(\frac{P^f_{c,t}}{P^i_{c,t}}\right)^{\frac{1}{\sigma}} C^f_t \,. \tag{30}$$

Substituting (30) in (28) we obtain

$$\begin{aligned} Y^i_t(z) &= \left(\frac{P^i_t(z)}{P^i_t}\right)^{-\varepsilon} \left((1-\alpha)(S^i_t)^\alpha C^i_t + \alpha (S^i_t)^\alpha C^i_t \int_0^1 \left(\frac{P^f_{c,t}}{P^i_{c,t}}\right)^{\frac{\sigma-1}{\sigma}} df\right) \\ &= \left(\frac{P^i_t(z)}{P^i_t}\right)^{-\varepsilon} \left((1-\alpha)(S^i_t)^\alpha C^i_t + \alpha (S^i_t)^\alpha C^i_t \int_0^1 \left(S^i_{f,t}\right)^{\frac{(1-\alpha)(\sigma-1)}{\sigma}} df\right). \end{aligned} \tag{31}$$

Plugging (31) into the definition of of country i's aggregate output $Y^i_t = \left(\int_0^1 (Y^i_t(z))^{\frac{\varepsilon-1}{\varepsilon}} dz\right)^{\frac{\varepsilon}{\varepsilon-1}}$ we obtain the aggregate goods market clearing condition for country i

$$Y^i_t = C^i_t\left((1-\alpha)(S^i_t)^\alpha + \alpha (S^i_t)^\alpha \int_0^1 \left(S^i_{f,t}\right)^{\frac{(1-\alpha)(\sigma-1)}{\sigma}} df\right), \tag{32}$$

[17] We assume $\vartheta \equiv \frac{\int_0^1 E^i_{j,t}\left(C^i_{j,t+1}(P^i_{c,t+1})^{1/\sigma}\right)dj}{\int_0^1 E^f_{j,t}\left(C^f_{j,t+1}(P^f_{c,t+1})^{1/\sigma}\right)dj} = 1$, so that our derivation stays close to that in Galí and Monacelli (2005).

which can be log-linearized around a symmetric steady state as[18]

$$\hat{y}_t^i = \hat{c}_t^i + \frac{\alpha\delta}{\sigma} s_t^i \,, \tag{33}$$

where $\delta = \sigma + (1-\alpha)(\sigma - 1)$. Log-linearizing the Euler equation (19) yields

$$\hat{c}_{j,t}^i = E_{j,t}^i \hat{c}_{j,t+1}^i - \sigma^{-1}(i_t - E_{j,t}^i \pi_{c,t+1}^i - \rho) \,, \tag{34}$$

where $\rho = -\log\beta$ is the time discount rate, which can be rewritten as

$$\hat{c}_{j,t}^i = E_{j,t}^i \hat{c}_{t+1}^i + (E_{j,t}^i \hat{c}_{j,t+1}^i - E_{j,t}^i \hat{c}_{t+1}^i) - \sigma^{-1}(i_t - E_{j,t}^i \pi_{c,t+1}^i - \rho) \,.$$

Aggregating the previous equation across households we get

$$\hat{c}_t^i = \bar{E}_t^i \hat{c}_{t+1}^i - \sigma^{-1}(i_t - \bar{E}_t \pi_{c,t+1}^i - \rho) + \Phi_t^i(\hat{c}) \,, \tag{35}$$

where $\bar{E}_t^i$ is the aggregate expectation operator defined as $\bar{E}_t^i(x_{t+1}) = \int_0^1 E_{j,t}^i x_{t+1} dj$ for a generic variable x and the term $\Phi_t^i(\hat{c}) = \int_0^1 (E_{j,t}^i \hat{c}_{j,t+1}^i - E_{j,t}^i \hat{c}_{t+1}^i) dj$ denotes the difference between the average expectation of individual consumption and average consumption. Substituting (33) in (35), we obtain

$$\hat{y}_t^i = \bar{E}_t^i \hat{y}_{t+1}^i - \sigma^{-1}(i_t - \bar{E}_t \pi_{c,t+1}^i - \rho) - \frac{\alpha\delta}{\sigma} \bar{E}_t^i \Delta s_{t+1}^i + \Phi_t^i(\hat{c}) \,,$$

which can be rewritten using (22) as

$$\hat{y}_t^i = \bar{E}_t^i \hat{y}_{t+1}^i - \sigma^{-1}(i_t - \bar{E}_t \pi_{t+1}^i - \rho) + \frac{\alpha(1-\delta)}{\sigma} \bar{E}_t^i \Delta s_{t+1}^i + \Phi_t^i(\hat{c}) \,. \tag{36}$$

We now turn to the supply side of the economy. Since each firm produces a single good z and is managed by a single household j (with subjective expectations j), we can, without loss of generality, use a single index, say j, to denote the produced good and the subjective expectations of the firm. We can therefore write the individual pricing rule in (25) as

$$\hat{q}_t^{i,opt}(j) = (1-\omega\beta)\widehat{mc}_t^i + \omega\beta E_{j,t}^i (\hat{q}_t^{i,opt}(j) + \pi_{t+1}^i) \,. \tag{37}$$

Given the Calvo pricing scheme, in each period only a set of firms $F_t \in [0,1]$ of measure $1-\omega$ adjust prices, while a set $F_t^c \in [0,1]$ of measure ω do not adjust. Assuming that the sample of firms allowed to adjust prices in each period is selected independently

[18] We used the fact that in a symmetric steady state $S^i = 1$ and therefore $s^i = 0$.

across agents so that the distribution of firms in terms of beliefs is the same whether we consider firms that adjust prices or firms that do not adjust prices, we can write using the aggregate price definition

$$(P_t^i)^{1-\varepsilon} = \int_{F_t} (P_t^{i,opt}(j))^{1-\varepsilon} dj + \int_{F_t^c} (P_{t-1}^i(j))^{1-\varepsilon} dj \,,$$

which can be rewritten as

$$1 = (1-\omega) \int_0^1 (Q_t^{i,opt}(j))^{1-\varepsilon} dj + \omega (P_t^i / P_{t-1}^i)^{\varepsilon - 1} \,.$$

Log-linearizing the relation above we get

$$\pi_t^i = \frac{1-\omega}{\omega} \int_0^1 \hat{q}_t^{i,opt}(j) dj \,. \tag{38}$$

Denoting $\hat{q}_t^i = \int \hat{q}_t^{i,opt}(j)dj$ and integrating Equation (37) on both sides we get

$$\hat{q}_t^i = (1-\omega\beta)\widehat{mc}_t^i + \omega\beta \int_0^1 E_{j,t}^i (\hat{q}_{t+1}^{i,opt}(j) + \pi_{t+1}^i) dj \,,$$

which can be rewritten as

$$\hat{q}_t^i = (1-\omega\beta)\widehat{mc}_t^i + \omega\beta \int_0^1 E_{j,t}^i (\hat{q}_{t+1}^{i,opt}(j) + \hat{q}_{t+1}^i - \hat{q}_{t+1}^i + \pi_{t+1}^i) dj \,.$$

Recalling from Equation (38) that $\hat{q}_t^i = \omega/(1-\omega)\pi_t^i$ and substituting in the equation above we get

$$\pi_t^i = \frac{(1-\omega)(1-\omega\beta)}{\omega} \widehat{mc}_t^i + \beta \bar{E}_t^i \pi_{t+1}^i + \Phi_t^i(\hat{q}) \,, \tag{39}$$

where again $\bar{E}_t^i$ is the aggregate expectation operator and $\Phi_t^i(\hat{q}) = \beta(1-\omega)\int_0^1 (E_{j,t}^i \hat{q}_{t+1}^{i,opt}(j) - E_{j,t}^i \hat{q}_{t+1}^i) dj$ denotes the difference between the average expectation of individual price and average price.

Log-linearizing Equation (20), and using (23) and (24) combined with market clearing result (33), we can express real marginal costs as a function of output, productivity and the effective terms of trade

$$\widehat{mc}_t^i = (\sigma+\eta)\hat{y}_t^i - (1+\eta)a_t^i + \alpha(1-\delta)s_t^i \,. \tag{40}$$

Recall from (30) that

$$C_t^i = (S_{f,t}^i)^{\frac{1-\alpha}{\sigma}} C_t^f \,.$$

Log-linearizing the above expression gives

$$\hat{c}_t^i = \hat{c}_t^f + \frac{1-\alpha}{\sigma} s_{f,t}^i \ ,$$

and integrating over f yields

$$\hat{c}_t^i = \hat{c}_t^{cu} + \frac{1-\alpha}{\sigma} s_t^i \ , \tag{41}$$

where $\hat{c}_t^{cu}$ is consumption at the currency union level. Integrating Equation (33) over i we get $\hat{y}_t^{cu} = \hat{c}_t^{cu}$ since $\int_0^1 s_t^i di = 0$. Therefore, substituting (41) in (33) we get

$$\hat{y}_t^i = \hat{y}_t^{cu} + \frac{1}{\sigma_\alpha} s_t^i \ , \tag{42}$$

where $\sigma_\alpha = \frac{\sigma}{1+\alpha(\delta-1)}$.

Using (42) in (40) to substitute for s_t^i we get

$$\widehat{mc}_t^i = (\sigma+\eta)\hat{y}_t^i - (1+\eta)a_t^i + \alpha(1-\delta)(\hat{y}_t^i - \hat{y}_t^{cu})\sigma_\alpha \ . \tag{43}$$

Equation (43) implies a natural level of output under flexible prices given by

$$\hat{y}_t^{i,n} = \frac{(1+\eta)}{\sigma+\eta+\alpha(1-\delta)\sigma_\alpha} a_t^i + \frac{\alpha(1-\delta)\sigma_\alpha}{\sigma+\eta+\alpha(1-\delta)\sigma_\alpha} y_t^{cu} \ . \tag{44}$$

Defining the output gap as $\tilde{y}_t^i = \hat{y}_t^i - \hat{y}_t^{i,n}$ we can write Equation (43) as

$$\widehat{mc}_t^i = (\sigma_\alpha + \eta)\tilde{y}_t^i \ . \tag{45}$$

Therefore, substituting (45) in (39) we get

$$\pi_t^i = k\tilde{y}_t^i + \beta \bar{E}_t^i \pi_{t+1}^i + \Phi_t^i(\hat{q}) \ , \tag{46}$$

where $k = \frac{(1-\omega)(1-\omega\beta)(\sigma_\alpha+\eta)}{\omega}$.

Rewriting Equation (36) in terms of the output gap yields

$$\tilde{y}_t^i = \bar{E}_t^i \tilde{y}_{t+1}^i - \sigma^{-1}(r_t - \bar{E}_t \pi_{t+1}^i) + \gamma \bar{E}_t^i \Delta s_{t+1}^i + \Phi_t^i(\hat{c}) + v_t^i \ , \tag{47}$$

where $\gamma = \frac{\alpha(1-\delta)}{\sigma}$, r_t is defined as $r_t = i_t - \rho$, and $v_t^i = \frac{(1+\eta)}{\sigma+\eta+\alpha(1-\delta)\sigma_\alpha} \bar{E}_t^i (a_{t+1}^i - a_t^i) + \frac{\alpha(1-\delta)\sigma_\alpha}{\sigma+\eta+\alpha(1-\delta)\sigma_\alpha} \bar{E}_t^i (y_{t+1}^{cu} - y_t^{cu})$. We assume that expectations of changes in productivity and currency union level output gap are white noise. Assuming that agents' expectations of future individual consumption and price will coincide with their expectations about future average consumption and price (see Hommes et al., 2017), the terms $\Phi_t(\hat{c})$ and $\Phi_t(\hat{q})$ are equal to zero. We can therefore rewrite the aggregate demand and supply equations as

$$\begin{aligned} \tilde{y}_t^i &= \bar{E}_t^i \tilde{y}_{t+1}^i - \sigma^{-1}(r_t - \bar{E}_t \pi_{t+1}^i) + \gamma \bar{E}_t^i \Delta s_{t+1}^i + v_t^i \qquad (48) \\ \pi_t^i &= k\tilde{y}_t^i + \beta \bar{E}_t^i \pi_{t+1}^i + \xi_t^i \,, \qquad (49) \end{aligned}$$

where ξ_t^i is a cost-push shock at the country level.

B Appendix (for Online Publication): Matrix Forms of the Macroeconomic Equations and Additional Information

B.1 Matrix Form of the Homogeneous Economy

When the zero lower bound is not binding, the model of the homogeneous closed economy consisting of equations (1)–(3) can be rewritten in matrix form as

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \Omega^{-1} \begin{bmatrix} (\bar{\pi}(\Phi_\pi - 1) + \Phi_y \bar{y})/\sigma \\ (\kappa\bar{\pi}(\Phi_\pi - 1) + \kappa\Phi_y \bar{y})/\sigma \end{bmatrix} + \Omega^{-1} \begin{bmatrix} 1 & (1-\Phi_\pi\beta)/\sigma \\ \kappa & \kappa/\sigma + \beta + \beta\Phi_y/\sigma \end{bmatrix} \begin{bmatrix} \bar{y}^e_{t+1} \\ \bar{\pi}^e_{t+1} \end{bmatrix} + \Omega^{-1} \begin{bmatrix} 1 & -\Phi_\pi/\sigma \\ \kappa & 1+\phi_y/\sigma \end{bmatrix} \begin{bmatrix} \nu_t \\ \xi_t \end{bmatrix},$$

where $\Omega = 1 + \frac{\kappa\Phi_\pi + \Phi_y}{\sigma}$.

B.2 Matrix Form of the Currency Union

The model for a currency union of N countries consisting of Equations (4)–(5) for each country i, $i = 1, \ldots, N$ and Equation (6) can be rewritten in matrix form as

$$x_t = M x^e_{t+1} + B + R\varepsilon_t \tag{50}$$

with the following notation:

$$x_t := \begin{bmatrix} y^1_t \\ \vdots \\ y^N_t \\ \pi^1_t \\ \vdots \\ \pi^N_t \end{bmatrix}, \quad x^e_{t+1} := \begin{bmatrix} \tilde{E}_t y^1_{t+1} \\ \vdots \\ \tilde{E}_t y^N_{t+1} \\ \tilde{E}_t \pi^1_{t+1} \\ \vdots \\ \tilde{E}_t \pi^N_{t+1} \end{bmatrix}, \quad \varepsilon_t := \begin{bmatrix} \nu^1_t \\ \vdots \\ \nu^N_t \\ \xi^1_t \\ \vdots \\ \xi^N_t \end{bmatrix},$$

$$\Omega := \begin{bmatrix} 1 + \frac{\phi_y w(1)}{\sigma^1 \sum_{k=1}^N w(k)} & \cdots & \frac{\phi_y w(N)}{\sigma^1 \sum_{k=1}^N w(k)} & \frac{\phi_\pi w(1)}{\sigma^1 \sum_{k=1}^N w(k)} & \cdots & \frac{\phi_\pi w(N)}{\sigma^1 \sum_{k=1}^N w(k)} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ \frac{\phi_y w(1)}{\sigma^N \sum_{k=1}^N w(k)} & \cdots & 1 + \frac{\phi_y w(N)}{\sigma^N \sum_{k=1}^N w(k)} & \frac{\phi_\pi w(1)}{\sigma^N \sum_{k=1}^N w(k)} & \cdots & \frac{\phi_\pi w(N)}{\sigma^N \sum_{k=1}^N w(k)} \\ -\kappa^1 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\kappa^N & 0 & \cdots & 1 \end{bmatrix},$$

$$
M := \Omega^{-1}\begin{bmatrix}
1 & \cdots & 0 & \frac{1}{\sigma^1} - \gamma^1 & \cdots & \frac{\gamma^1 w(N)}{\sum_{k=1}^{N,k\neq 1} w(k)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 1 & \frac{\gamma^N w(1)}{\sum_{k=1}^{N,k\neq N} w(k)} & \cdots & \frac{1}{\sigma^N} - \gamma^N \\
0 & \cdots & 0 & \beta^1 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & \beta^N
\end{bmatrix},
$$

$$
B := \Omega^{-1}\begin{bmatrix}
\frac{\Phi_y w(1)}{\sigma^1 \sum_{k=1}^{N} w(k)} & \cdots & \frac{\Phi_y w(N)}{\sigma^1 \sum_{k=1}^{N} w(k)} & -\frac{1-\Phi_\pi}{\sigma^1} & \cdots & 0 \\
\vdots & & \vdots & \vdots & \ddots & \vdots \\
\frac{\Phi_y w(1)}{\sigma^N \sum_{k=1}^{N} w(k)} & \cdots & \frac{\Phi_y w(N)}{\sigma^N \sum_{k=1}^{N} w(k)} & 0 & \cdots & -\frac{1-\Phi_\pi}{\sigma^N} \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\bar{y}^1 \\ \vdots \\ \bar{y}^N \\ \bar{\pi} \\ \vdots \\ \bar{\pi}
\end{bmatrix},
$$

$$
R := \Omega^{-1}\begin{bmatrix}
1 & \cdots & 0 & 0 & 0 & \\
\vdots & \ddots & \vdots & \vdots & & \vdots \\
0 & \cdots & 1 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 1 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 1
\end{bmatrix}.
$$

This matrix notation describes the model economy correctly when the zero lower bound is not binding. When it is binding a similar form can be derived easily.

It can easily be verified that expectations are rational if inflation expectations equal the central bank's inflation target in all countries and each country's output gap expectation equals its own steady state value of the output gap.

B.3 An Artificial Fixed Exchange Rate Regime

Economic behavior under an artificial fixed exchange rate regime can be described by the following equations:

$$y_t^i = \tilde{E}_t y_{t+1}^i - \frac{1}{\sigma^i}(r_t^i - \tilde{E}_t \pi_{t+1}^i) + \gamma^i \tilde{E}_t^i \Delta s_{t+1}^i + \nu_t^i \tag{51}$$

$$\pi_t^i = \beta^i \tilde{E}_t \pi_{t+1}^i + \kappa_\alpha^i y_t^i + \xi_t^i \tag{52}$$

$$r_t^i = max\{\bar{\pi}^i + \Phi_\pi^i(\pi_t^i - \bar{\pi}^i) + \Phi_y^i(y_t^i - \bar{y}^i),\ 0\}. \tag{53}$$

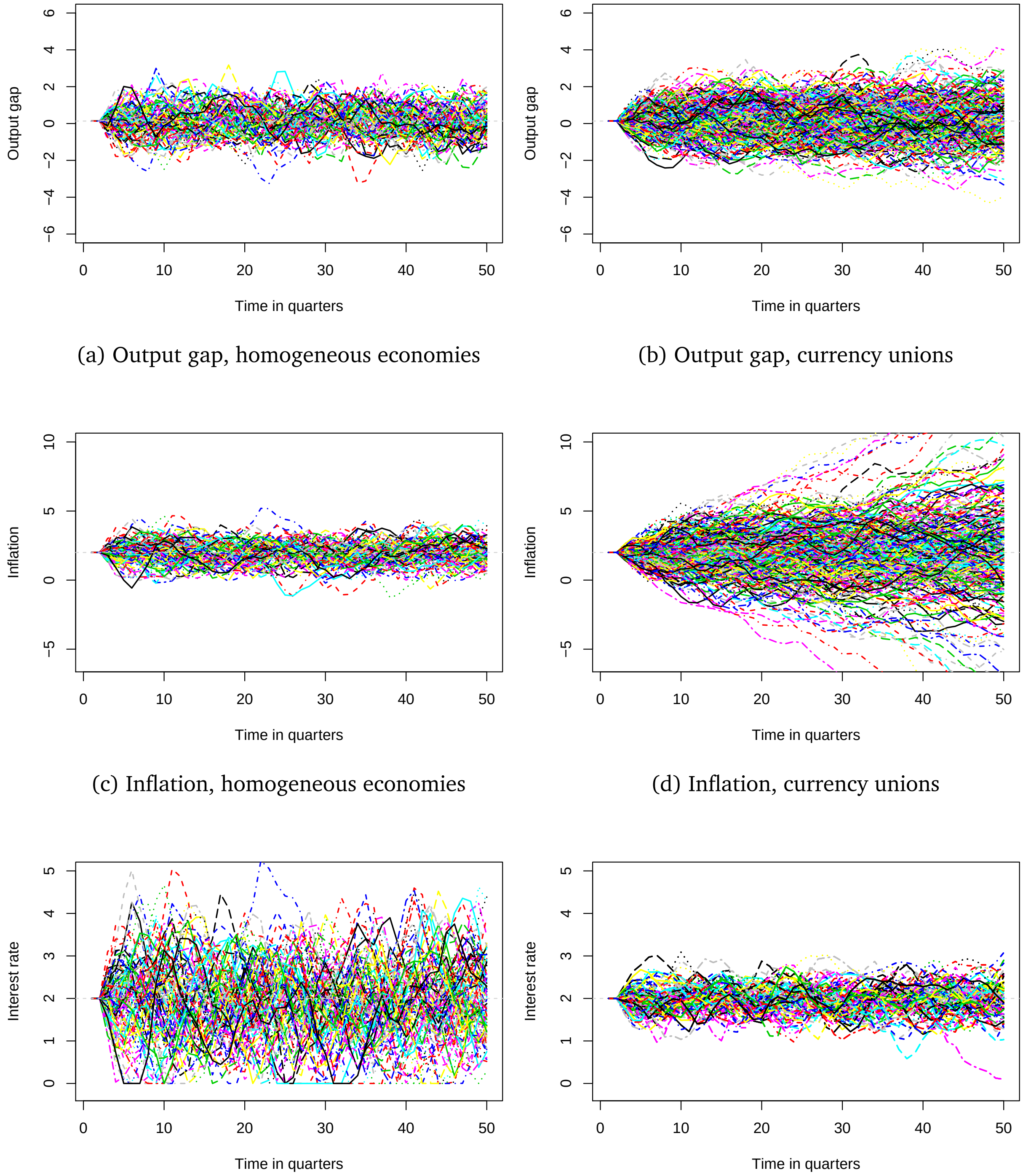
The equations are identical to the currency union equations except for the description of monetary policy. In model (51)-(53), there are different interest rates in different countries. We say that this model represents an "artificial" fixed exchange rate regime, because it does not require the central banks to react with monetary policy decisions to pressures on the exchange rate.

Matrix forms of this model can be derived in a way similar to the one in Appendix B.2. A small additional difficulty arises in this model when some countries hit the zero lower bound while others do not. However, this problem can be solved by creating a short algorithm (essentially in each step replacing the rows in the matrix corresponding to the countries at the zero lower bound).

C Appendix (for Online Publication): Additional Graphs and Analysis

C.1 Economic Behavior in Homogeneous Economies and Currency Unions

Figure 15 illustrates economic behavior in 100 homogeneous closed economies (on the left hand side) and 100 currency unions (on the right hand side). All parameter values are as for Figure 1 in Section 3.1.



(a) Output gap, homogeneous economies
(b) Output gap, currency unions
(c) Inflation, homogeneous economies
(d) Inflation, currency unions
(e) Interest rate, homogeneous economies
(f) Interest rate, currency unions

Figure 15: Economic Behavior in Homogeneous Economies and Currency Unions)

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd =0.25 (for CU), sd =$0.25\sqrt{3}$ (for HE).

C.2 Economic Integration

Figure 16 shows instability depending on γ for more values of γ (the calibration is the same as for Figure 4). What can be seen is that for low values of γ, i.e. for a low level of economic integration, economic behavior in the currency union is unstable. When γ increases, the economic system becomes stable. However, if γ becomes too high, the economic system becomes unstable again. This U-shape can be explained in the following way, which also illustrates the qualitatively different behavior that can be observed in the model.

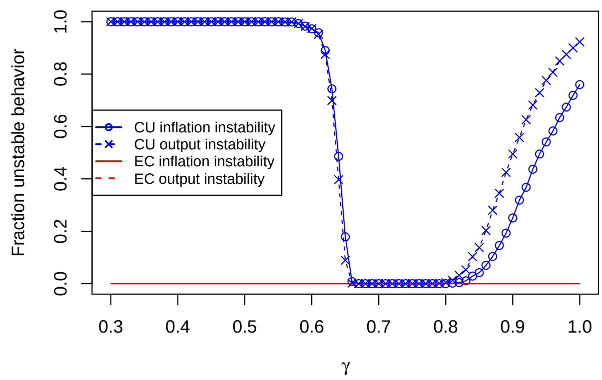
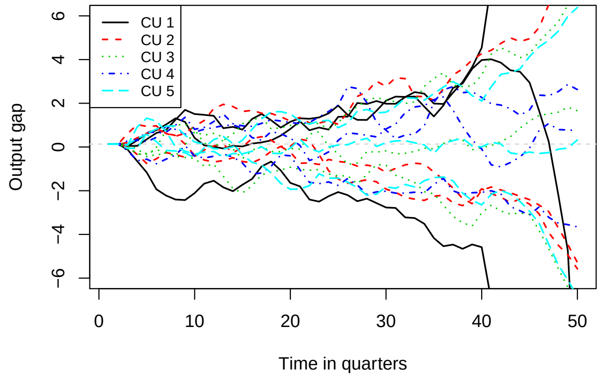


Figure 16: Instability Depending on Economic Integration

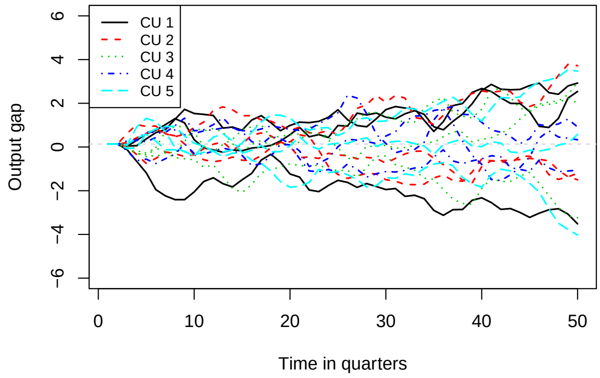
Notes: $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd=0.25 (for CU), sd=$0.25\sqrt{3}$ (for HE).

First, in the lowest range of γ-values, economic variables diverge (see Figure 17a). Ties between the countries are not so strong that a boom in some of the countries influences the countries in a slump enough to move back toward the steady states, and vice versa. For values of γ that are around the border of the stable region, smaller but still persistent deviations from the steady state can be observed (Figure 17b). For some levels of γ, very stable behavior can be observed (Figure 17c). The stabilizing force of γ is that when some countries are in a boom and some others in a slump, expected increases in inflation in the boom countries lead to increased demand in the countries in a slump and vice versa, so that there exists a pull toward the steady state. If γ is very high, it can (in theory) not only pull countries toward the steady state but also to the other side of the steady state. In this case, oscillations arise with countries' output gap and inflation switching frequently between below and above the steady states. These oscillations can have increasing amplitudes, so that countries spiral away from the steady states (see Figure 17d with a value of γ at the beginning of the unstable region and Figure 17e with even greater γ and very unstable oscillations).

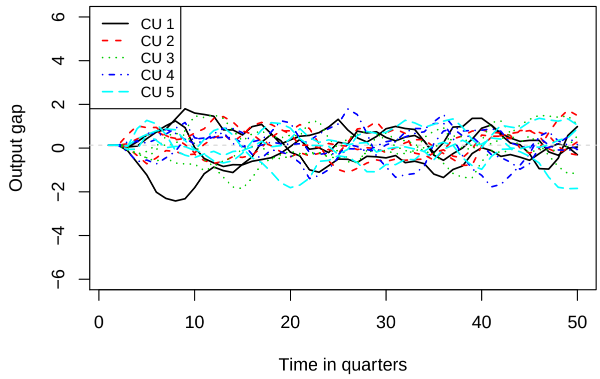
We do not observe fully explosive behavior or high-frequency oscillations in real-world currency unions, and they are hard to imagine. Thus, the values of γ that are economically relevant are those around the left side of the "U" in Figure 4. This leads to the policy implication that in order to make economic behavior in a currency union more stable, economic integration should be deepened.



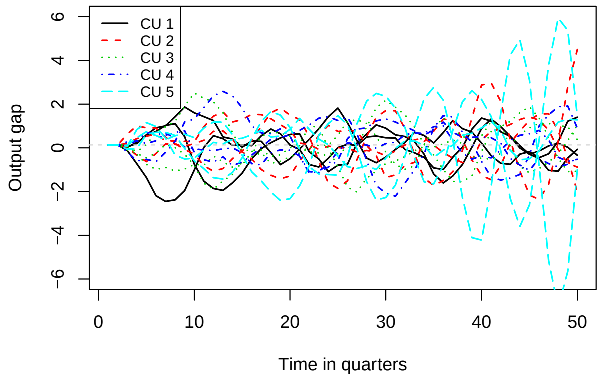
(a) γ=0.64



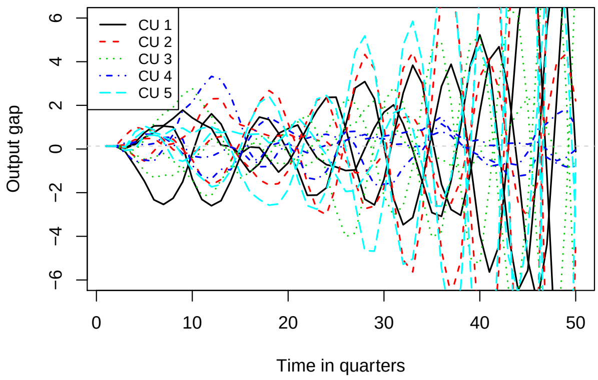
(b) γ=0.65



(c) γ=0.69



(d) $\gamma = 0.8$



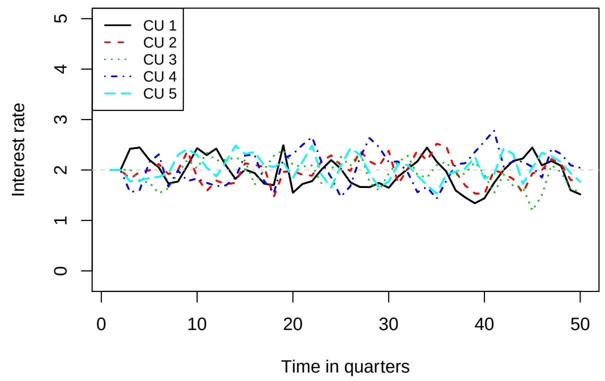
(e) $\gamma = 0.9$

Figure 17: Different Output Gaps in a CU

Notes: $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, $sd = 0.25$.

C.3 Extreme Monetary Policy

Figure 18 shows the behavior of the interest rate in 5 currency unions with high reaction coefficients in the Taylor rule ($\Phi_\pi = 6$ and $\Phi_y = 2$), corresponding to output gaps and inflation in Figure 5.

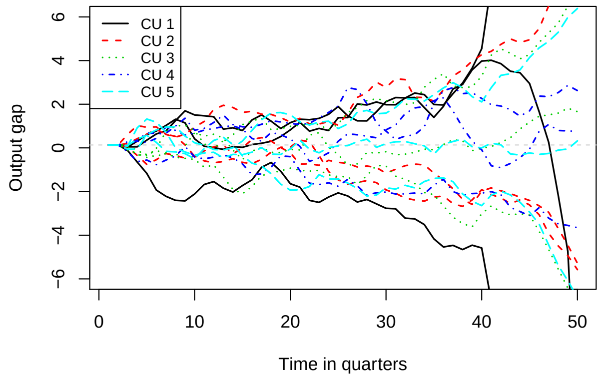


(a) Interest rate

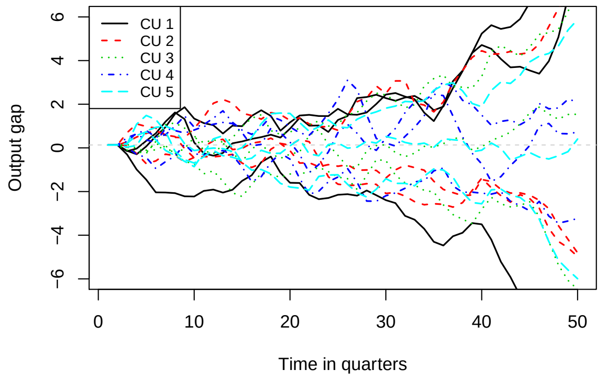
Figure 18: Interest Rate in Currency Unions with Taylor Rule coefficients $\Phi_\pi = 6$ and $\Phi_y = 2$

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 6$, $\Phi_y = 2$, sd =0.25.

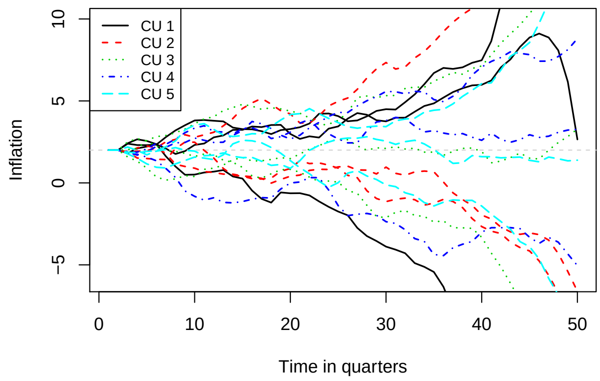
Figure 19 shows that a more activist monetary policy does not stabilize economic behavior with a less stable calibration. The figure illustrates economic behavior in 5 currency unions with standard ($\Phi_\pi = 1.5$ and $\Phi_y = 0.5$) and high reaction coefficients ($\Phi_\pi = 6$ and $\Phi_y = 2$) in the Taylor rule. In this figure, $\gamma = 0.64$.



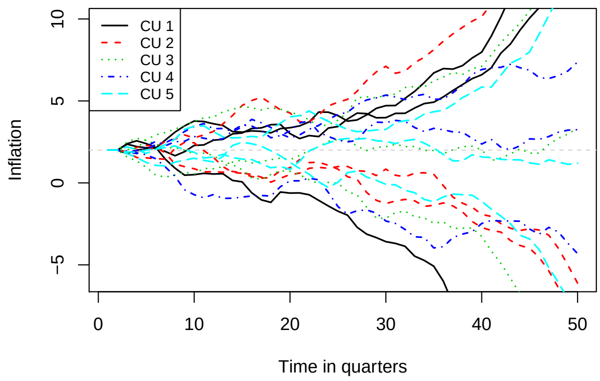
(a) Output gap, $\Phi_\pi = 1.5$ and $\Phi_y = 0.5$



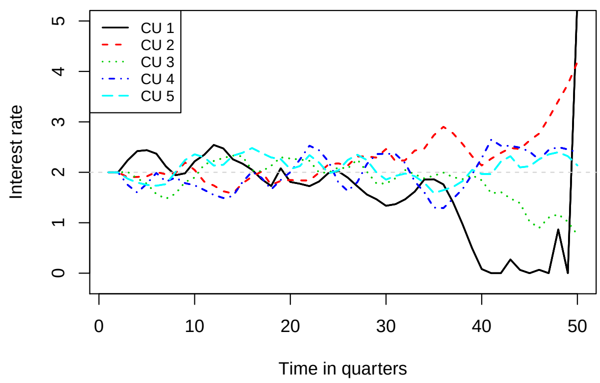
(b) Output gap, $\Phi_\pi = 6$ and $\Phi_y = 2$



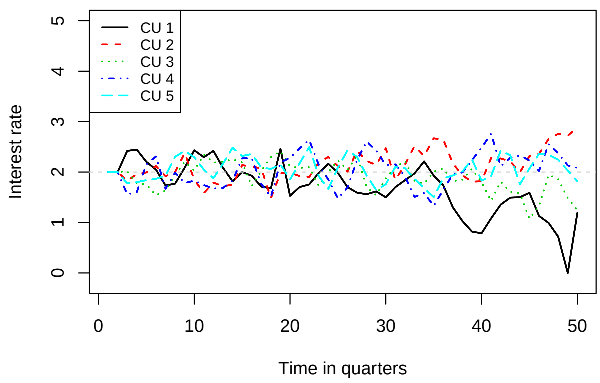
(c) Inflation, $\Phi_\pi = 1.5$ and $\Phi_y = 0.5$



(d) Inflation, $\Phi_\pi = 6$ and $\Phi_y = 2$



(e) Interest rate, $\Phi_\pi = 1.5$ and $\Phi_y = 0.5$



(f) Interest rate, $\Phi_\pi = 6$ and $\Phi_y = 2$

Figure 19: Economic Behavior in Currency Unions with $\gamma = 0.64$ and Taylor Rule coefficients $\Phi_\pi = 1.5$ and $\Phi_y = 0.5$ (left) and $\Phi_\pi = 6$ and $\Phi_y = 2$ (right).

Notes: $\gamma = 0.64$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$ (left), $\Phi_y = 0.5$ (left), $\Phi_\pi = 6$ (right), $\Phi_y = 2$ (right), sd =0.25.

C.4 Artificial Fixed Exchange Rate Regime

Figure 20 illustrates economic behavior under an artificial fixed exchange rate regime.

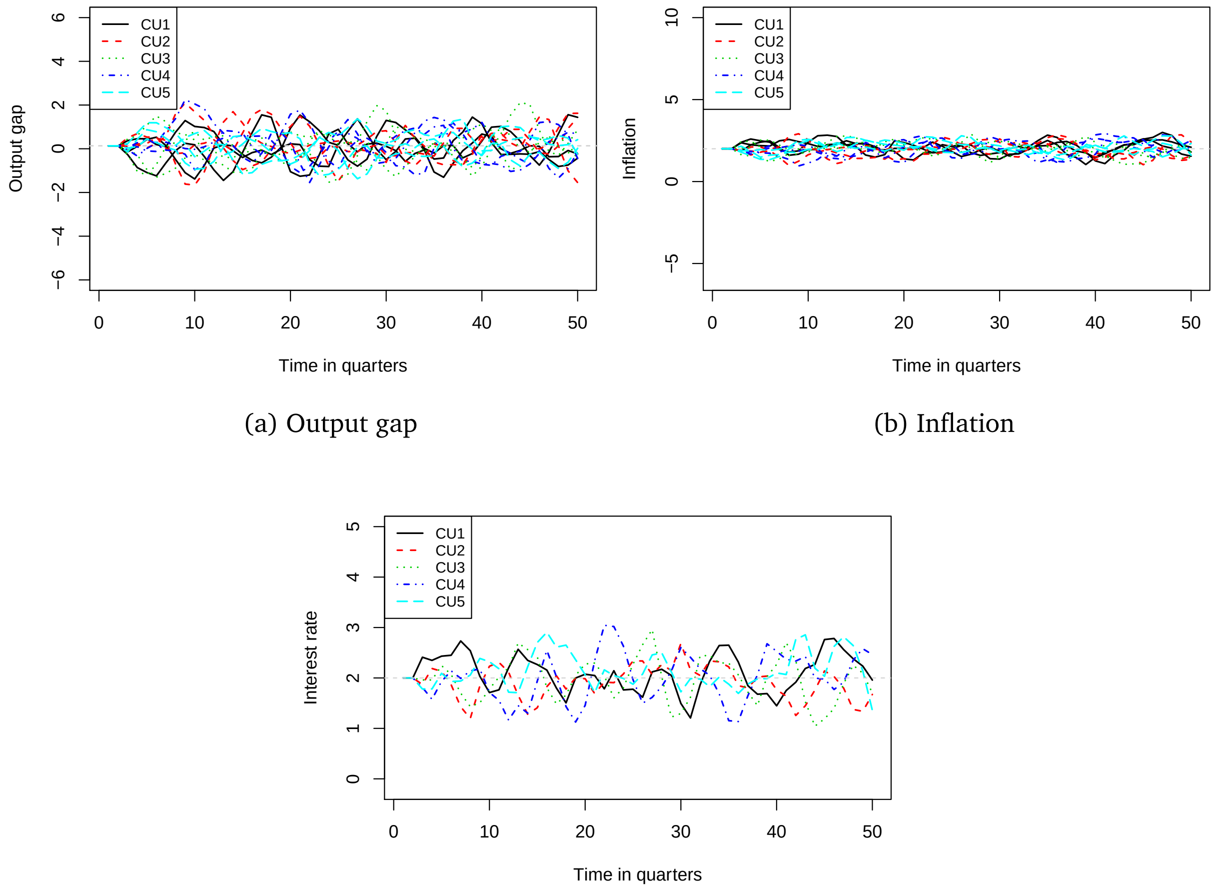


Figure 20: Economic Behavior under an Artificial Fixed Exchange Rate Regime

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, $sd = 0.25$.

Figure 21 depicts the stability of economic behavior in currency unions modeled as an artificial fixed exchange rate regime for different values of γ.

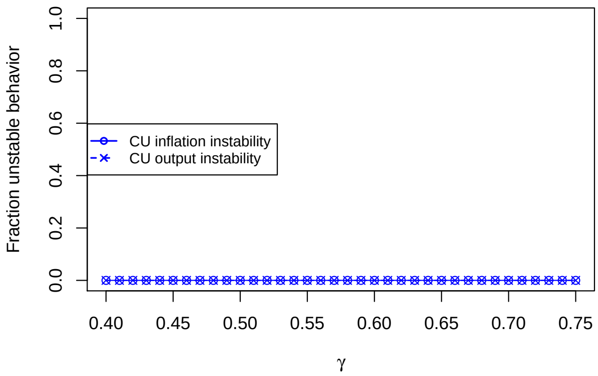


Figure 21: Instability of Artificial Fixed Exchange Rate Regimes Depending on Economic Integration

Notes: $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, $sd = 0.25$.

C.5 Rational Expectations

Figure 22 and Figure 23 investigate the stability of economic behavior for different values of κ, σ, the standard deviation and β in currency unions (blue) and homogeneous economies (red) under rational expectations.

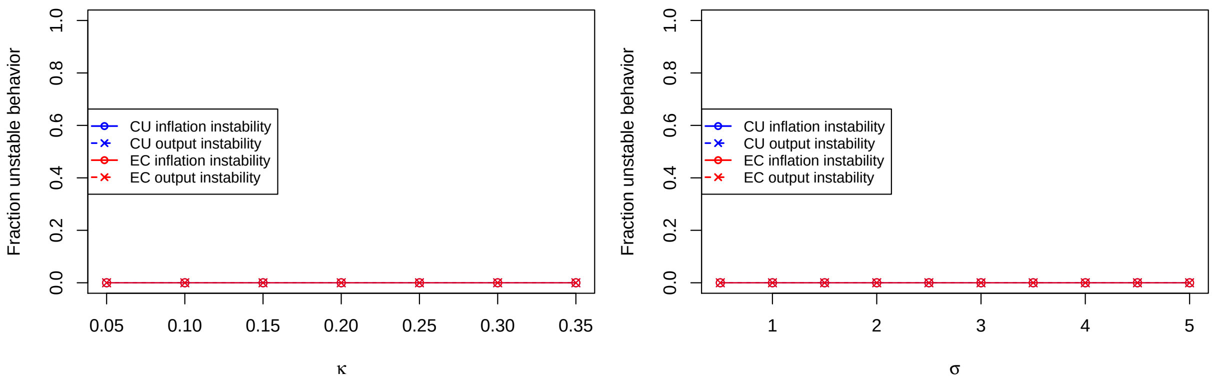


Figure 22: Instability Depending on κ and σ under Rational Expectations

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd =0.25 (for CU), sd =$0.25\sqrt{3}$ (for HE).

Figure 24 investigates the stability of economic behavior in currency unions modeled under rational expectations for different values of γ.

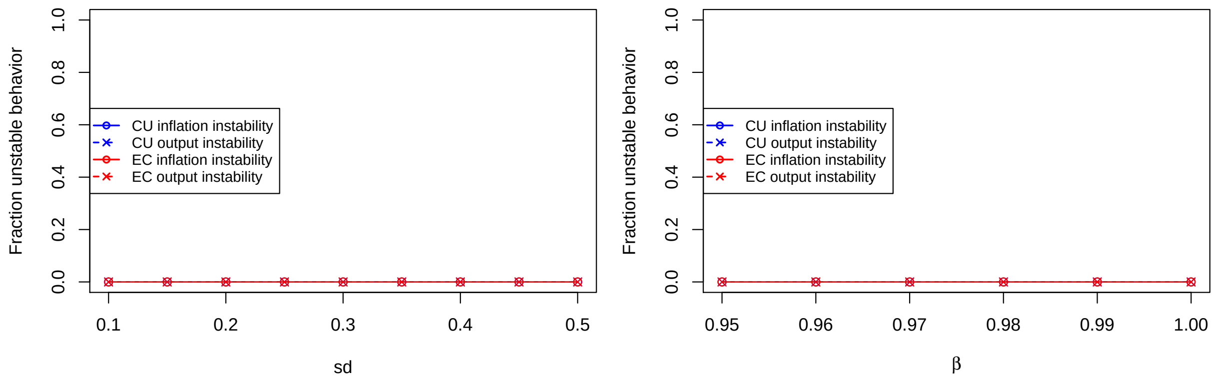


Figure 23: Instability Depending on the Standard Deviations of the Shocks and β under Rational Expectations

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd =0.25 (for CU), sd =0.25$\sqrt{3}$ (for HE).

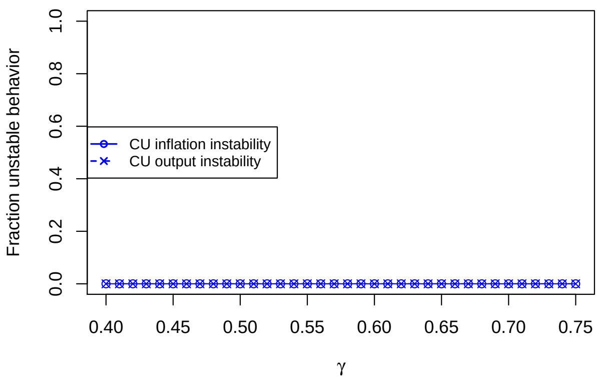


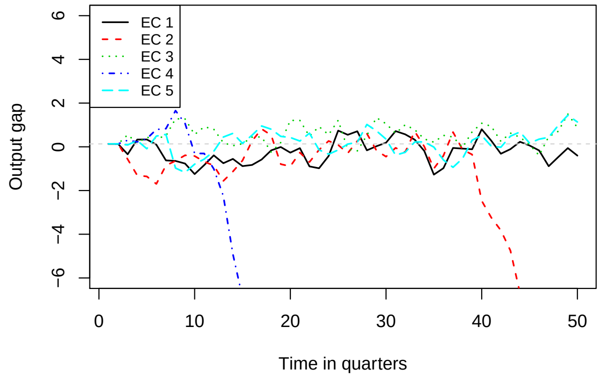
Figure 24: Instability of Currency Unions Depending on γ under Rational Expectations

Notes: $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd =0.25.

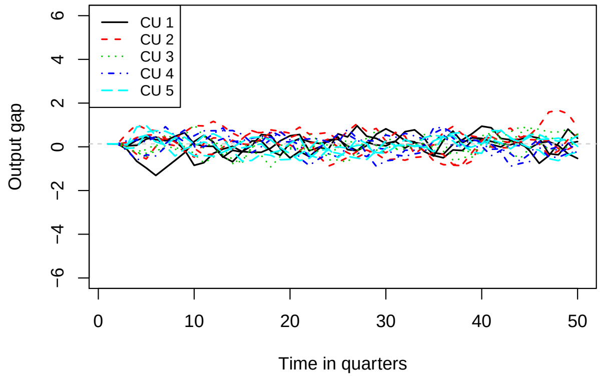
C.6 Alternative Behavioral Models of Expectation Formation

Figure 25 illustrates economic behavior in homogeneous economies (left) and currency unions (right) with a simple heuristic switching model (switching occurs only between naive expectations and a trend-following rule with coefficient one).

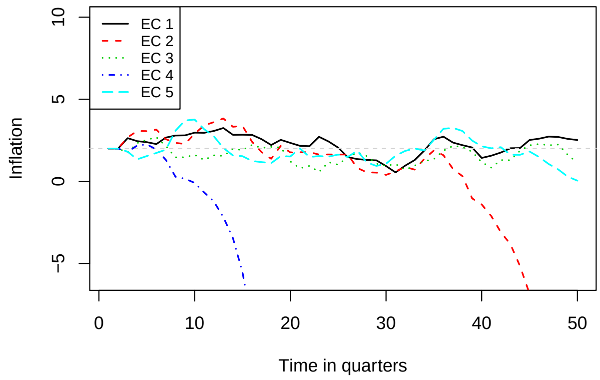
Figure 26 shows economic behavior in homogeneous economies (left) and currency unions (right) with homogeneous expectations formed according to the adaptive rule.



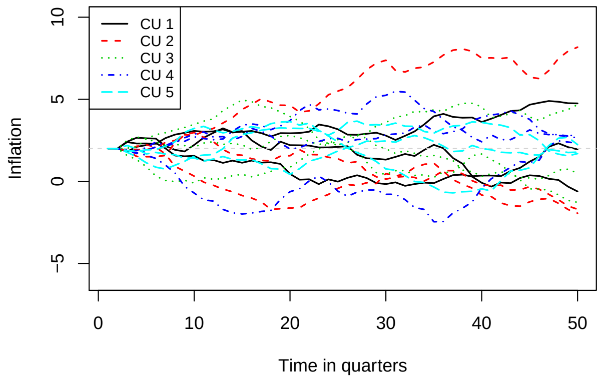
(a) Output gap, homogeneous economies



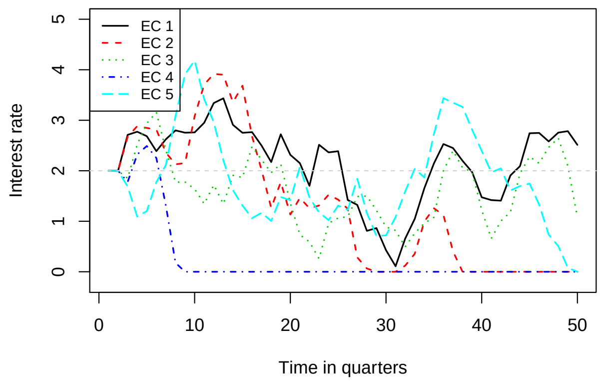
(b) Output gap, currency unions



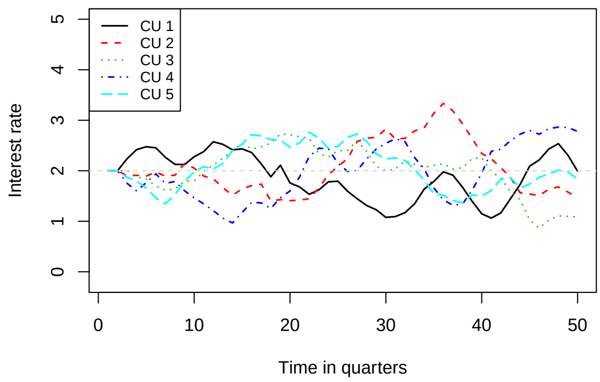
(c) Inflation, homogeneous economies



(d) Inflation, currency unions



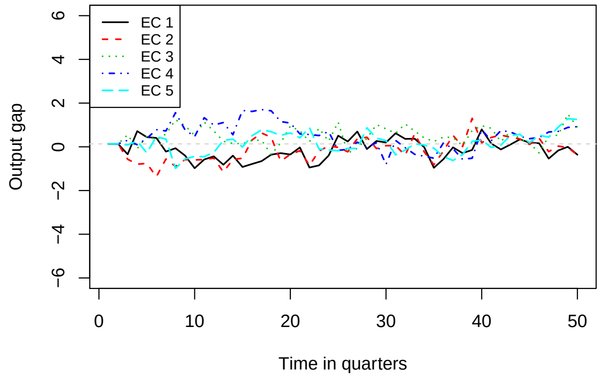
(e) Interest rate, homogeneous economies



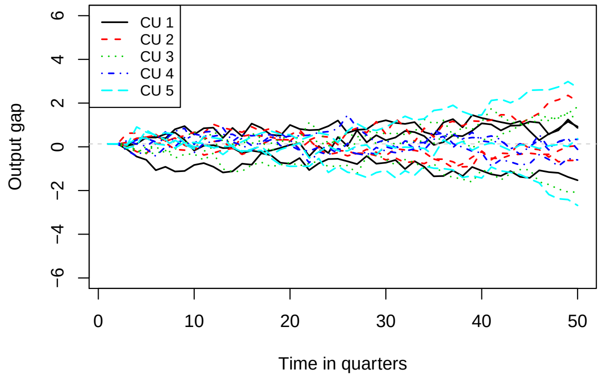
(f) Interest rate, currency unions

Figure 25: Economic Behavior in Homogeneous Economies and Currency Unions with Switching only between Naive Expectations and Trend-following

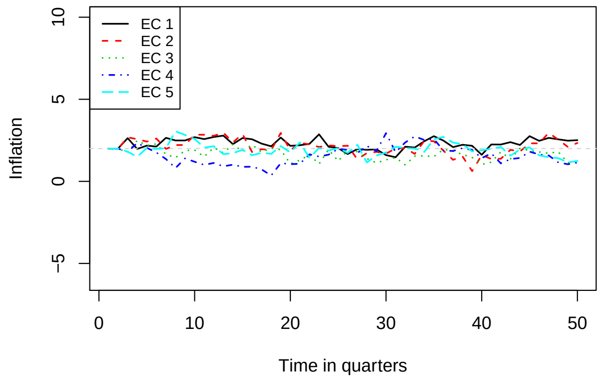
Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd =0.25 (for CU), sd =$0.25\sqrt{3}$ (for HE).



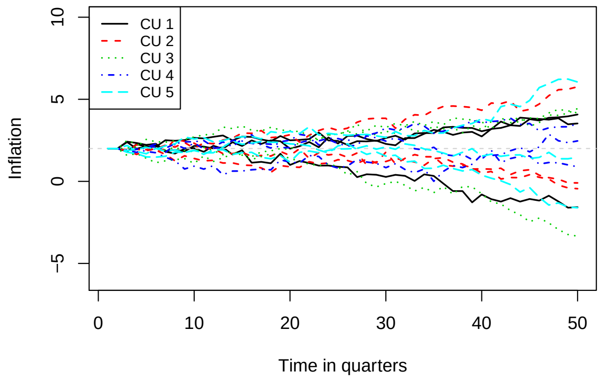
(a) Output gap, homogeneous economies



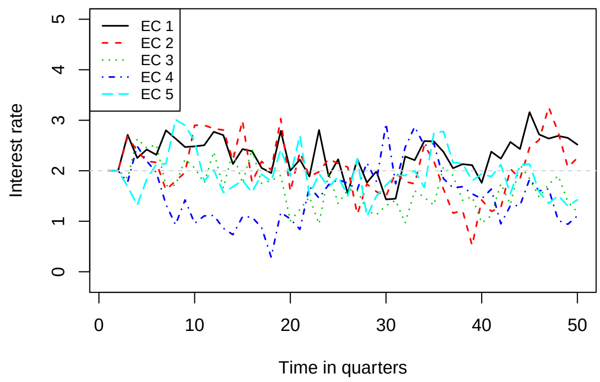
(b) Output gap, currency unions



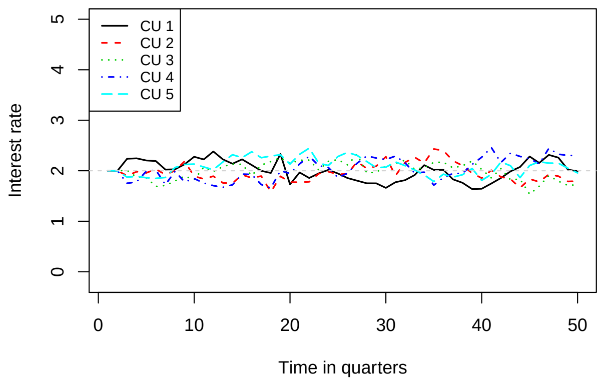
(c) Inflation, homogeneous economies



(d) Inflation, currency unions



(e) Interest rate, homogeneous economies



(f) Interest rate, currency unions

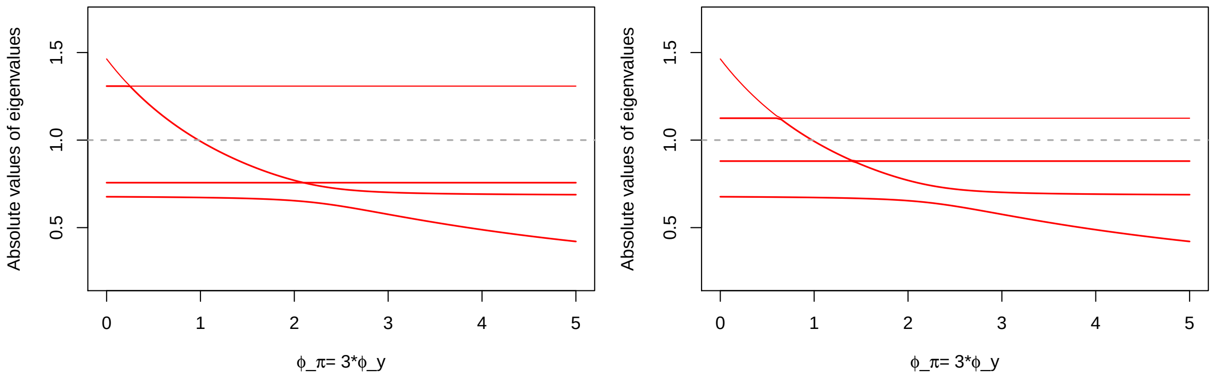
Figure 26: Economic Behavior in Homogeneous Economies and Currency Unions under Adaptive Expectations

Notes: $\gamma = 0.66$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$, $\Phi_\pi = 1.5$, $\Phi_y = 0.5$, sd =0.25 (for CU), sd =$0.25\sqrt{3}$ (for HE).

C.7 Mathematical Stability of the Steady States under Naive Expectations

Figure 27 shows the absolute values of the eigenvalues of the Jacobian matrix in a currency union consisting of only 2 countries (under naive expectations) with $\gamma = 0.25$ and $\gamma = 0.45$, respectively.

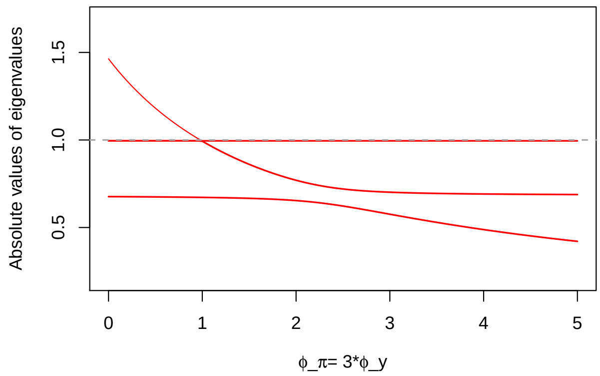
Figure 28 shows the absolute values of the eigenvalues in a currency union of 2 countries (on the left hand side) and in a currency union of 3 countries (on the right hand side) with $\gamma = 0.7$.



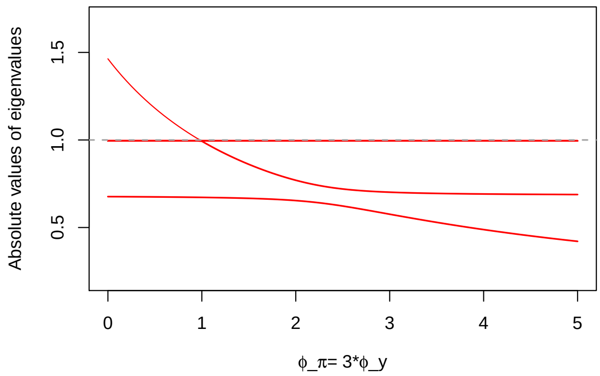
(a) $\gamma = 0.25$ (b) $\gamma = 0.45$

Figure 27: Absolute Values of Eigenvalues in a Currency Union of 2 Countries (Naive Expectations)

Notes: $\gamma = 0.25$ (left), $\gamma = 0.45$ (right), $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$.



(a) 2 countries



(b) 3 countries

Figure 28: Absolute Values of Eigenvalues in Currency Unions of 2 and 3 Countries with $\gamma = 0.7$ (Naive Expectations)

Notes: $\gamma = 0.7$, $\sigma = 1$, $\kappa = 0.15$, $\beta = 0.99$.

D Appendix (for Online Publication): Macroeconomic Equations, Matrix Forms, and Additional Information for the Additional Macroeconomic Model

D.1 Homogeneous Economy

A model for a homogeneous closed economy corresponding to the currency union model presented in Section 4, Equations (11) to (13), consists of the following equations:

$$y_t = a_1 \tilde{E}_t y_{t+1} + (1-a_1) y_{t-1} + a_2 (r_t - \tilde{E}_t \pi_{t+1}) + g_t \tag{54}$$

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1-b_1) \pi_{t-1} + b_2 y_t + u_t \tag{55}$$

$$r_t = max\{\bar{\pi} + c_1 (\pi_t - \bar{\pi}) + c_2 (y_t - \bar{y}),\ 0\}. \tag{56}$$

Again, $\bar{y} = 0$, so that one could leave this term out (we abstain from removing the term to make the comparison to the main model discussed in this paper easier). The matrix form of the homogeneous closed economy model can easily be derived.

D.2 Matrix Form of the Currency Union Model

The macroeconomic model for a currency union consisting of Equations (11)–(13) can be rewritten in matrix form in the following way. This matrix form is again valid as long as the zero lower bound is not binding. When the zero lower bound is binding, a similar form can be derived easily.

$$\begin{bmatrix} 1 - \frac{a_2^1 c_2 w(1)}{\Sigma_{k=1}^N w(k)} + m^1 & \cdots & \frac{-m^1 w(N)}{\Sigma_{k=1}^{N,k\neq 1} w(k)} + \frac{-a_2^1 c_2 w(N)}{\Sigma_{k=1}^N w(k)} & a_3^1 - \frac{a_2^1 c_1 w(1)}{\Sigma_{k=1}^N w(k)} & \cdots & \frac{-a_3^1 w(N)}{\Sigma_{k=1}^{N,k\neq 1} w(k)} + \frac{-a_2^1 c_1 w(N)}{\Sigma_{k=1}^N w(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{-m^N w(1)}{\Sigma_{k=1}^{N,k\neq N} w(k)} + \frac{-a_2^N c_2 w(1)}{\Sigma_{k=1}^N w(k)} & \cdots & 1 - \frac{a_2^N c_2 w(N)}{\Sigma_{k=1}^N w(k)} + m^N & \frac{-a_3^N w(1)}{\Sigma_{k=1}^{N,k\neq N} w(k)} + \frac{-a_2^N c_1 w(1)}{\Sigma_{k=1}^N w(k)} & \cdots & a_3^N - \frac{a_2^N c_1 w(N)}{\Sigma_{k=1}^N w(k)} \\ -b_2^1 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -b_2^N & 0 & \cdots & 1 \end{bmatrix} * \begin{bmatrix} y_t^1 \\ \vdots \\ y_t^N \\ \pi_t^1 \\ \vdots \\ \pi_t^N \end{bmatrix}$$

$$
= \begin{bmatrix} a_1^1 & \cdots & 0 & -a_2^1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & a_1^N & 0 & \cdots & -a_2^N \\ 0 & \cdots & 0 & b_1^1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & b_1^N \end{bmatrix} \begin{bmatrix} \tilde{E}_t y_{t+1}^1 \\ \vdots \\ \tilde{E}_t y_{t+1}^N \\ \tilde{E}_t \pi_{t+1}^1 \\ \vdots \\ \tilde{E}_t \pi_{t+1}^N \end{bmatrix}
$$

$$
+ \begin{bmatrix} 1-a_1^1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1-a_1^N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1-b_1^1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1-b_1^N \end{bmatrix} \begin{bmatrix} y_{t-1}^1 \\ \vdots \\ y_{t-1}^N \\ \pi_{t-1}^1 \\ \vdots \\ \pi_{t-1}^N \end{bmatrix}
$$

$$
+ \begin{bmatrix} -\frac{a_2^1 c_2 w(1)}{\sum_{k=1}^N w(k)} & \cdots & -\frac{a_2^1 c_2 w(N)}{\sum_{k=1}^N w(k)} & a_2^1(1-c_1) & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -\frac{a_2^N c_2 w(1)}{\sum_{k=1}^N w(k)} & \cdots & -\frac{a_2^N c_2 w(N)}{\sum_{k=1}^N w(k)} & 0 & \cdots & a_2^N(1-c_1) \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \bar{y}^1 \\ \vdots \\ \bar{y}^N \\ \bar{\pi}^1 \\ \vdots \\ \bar{\pi}^N \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0 & \cdots & 0 & -a_3^1 & \cdots & \frac{a_3^1 w(N)}{\sum_{k=1}^{N,k\neq 1} w(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{a_3^N w(1)}{\sum_{k=1}^{N,k\neq N} w(k)} & \cdots & -a_3^N \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ p_{t-1}^1 \\ \vdots \\ p_{t-1}^N \end{bmatrix} + \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} g_t^1 \\ \vdots \\ g_t^N \\ u_t^1 \\ \vdots \\ u_t^N \end{bmatrix}
$$

In addition to the currency union model, an artificial fixed exchange rate model, similar to the model shown in Appendix B.3, can be constructed analogously.

D.3 Rational Expectation Solutions

In order to find the rational expectation solutions, it is useful to rewrite the model of Equations (11)–(13) slightly. The following matrix notation is not particularly elegant but convenient as the price level is in the model determined simultaneously with inflation but does not only influence economic behavior via inflation but also through the real effective exchange rate.

The model of Equations (11)–(13) can be written as

$$x_t = L + Bx_{t-1} + Mx^e_{t+1} + Rz_t \tag{57}$$

with the following notation (please note that the letters denoting matrices do now represent different matrices than in Appendix B).

$$x_t := \begin{bmatrix} y_t^1 \\ \vdots \\ y_t^N \\ \pi_t^1 \\ \vdots \\ \pi_t^N \\ p_t^1 \\ \vdots \\ p_t^N \end{bmatrix}, x^e_{t+1} := \begin{bmatrix} \tilde{E}_t y^1_{t+1} \\ \vdots \\ \tilde{E}_t y^N_{t+1} \\ \tilde{E}_t \pi^1_{t+1} \\ \vdots \\ \tilde{E}_t \pi^N_{t+1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, z_t := \begin{bmatrix} g_t^1 \\ \vdots \\ g_t^N \\ u_t^1 \\ \vdots \\ u_t^N \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Omega := \begin{bmatrix}
1 - \frac{a_2^1 c_2 w(1)}{\sum_{k=1}^N w(k)} + m^1 & \cdots & \frac{-m^1 w(N)}{\sum_{k=1}^{N,k\neq 1} w(k)} + \frac{-a_2^1 c_2 w(N)}{\sum_{k=1}^N w(k)} & \frac{-a_2^1 c_1 w(1)}{\sum_{k=1}^N w(k)} & \cdots & \frac{-a_2^1 c_1 w(N)}{\sum_{k=1}^N w(k)} & a_3^1 & \cdots & \frac{-a_3^1 w(N)}{\sum_{k=1}^{N,k\neq 1} w(k)} \\
\vdots & \ddots & \vdots & \vdots & & \vdots & \vdots & \ddots & \vdots \\
\frac{-m^N w(1)}{\sum_{k=1}^{N,k\neq N} w(k)} + \frac{-a_2^N c_2 w(1)}{\sum_{k=1}^N w(k)} & \cdots & 1 - \frac{a_2^N c_2 w(N)}{\sum_{k=1}^N w(k)} + m^N & \frac{-a_2^N c_1 w(1)}{\sum_{k=1}^N w(k)} & \cdots & \frac{-a_2^N c_1 w(N)}{\sum_{k=1}^N w(k)} & \frac{-a_3^N w(1)}{\sum_{k=1}^{N,k\neq N} w(k)} & \cdots & a_3^N \\
-b_2^1 & \cdots & 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\
0 & \cdots & -b_2^N & 0 & \cdots & 1 & 0 & \cdots & 0 \\
0 & \cdots & 0 & -1 & \cdots & 0 & 1 & \cdots & 0 \\
\vdots & & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & -1 & 0 & \cdots & 1
\end{bmatrix},$$

$$M := \Omega^{-1} \begin{bmatrix} a_1^1 & \cdots & 0 & -a_2^1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & a_1^N & 0 & \cdots & -a_2^N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & b_1^1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & b_1^N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$B := \Omega^{-1} \begin{bmatrix} 1-a_1^1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 1-a_1^N & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1-b_1^1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1-b_1^N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 1 \end{bmatrix},$$

$$L := \begin{bmatrix} -\frac{a_2^1 c_2 w(1)}{\sum_{k=1}^N w(k)} & \cdots & -\frac{a_2^1 c_2 w(N)}{\sum_{k=1}^N w(k)} & a_2^1(1-c_1^1) & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ -\frac{a_2^N c_2 w(1)}{\sum_{k=1}^N w(k)} & \cdots & -\frac{a_2^N c_2 w(N)}{\sum_{k=1}^N w(k)} & 0 & \cdots & a_2^N(1-c_1^N) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \bar{y}^1 \\ \vdots \\ \bar{y}^N \\ \bar{\pi}^1 \\ \vdots \\ \bar{\pi}^N \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$R := \Omega^{-1} \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Assume that $x^e_{t+1} = S + Ux_{t-1}$, where S and U are $3Nx3N$ matrices. Leading Equation (57) one period ahead and inserting the expectations term yields:

$$x_{t+1} = L + Bx_t + M(S + Ux_t) + Rz_{t+1}. \tag{58}$$

Taking mathematical expectations of this equation leads to

$$Ex_{t+1} = L + BE'x_t + M(S + UE'x_t), \tag{59}$$

where $E'x_t$ denotes the mathematical expectation of x_t right before time t (recall that the expectations of future variables are formed right before the current variables are known, so that the expectations $\tilde{E}x_{t+1}$ or Ex_{t+1} are formed before x_t is known; the prime is added to make clear that the expectations have a different time horizon while being formed at the same point in time). With $E'z_t = 0$ it follows that

$$E'x_t = L + Bx_{t-1} + M(S + Ux_{t-1}) + RE'z_t = L + Bx_{t-1} + M(S + Ux_{t-1}). \tag{60}$$

Inserting this expression (and simplifying slightly) leads to

$$Ex_{t+1} = (I + B + MU)(L + MS) + (B + MU)(B + MU)x_{t-1}. \tag{61}$$

These expectations are equal to the expectations of economic agents if

$$(I + B + MU)(L + MS) + (B + MU)(B + MU)x_{t-1} = S + Ux_{t-1}. \tag{62}$$

Thus, expectations are rational if

$$S = (I + B + MU)(L + MS), \tag{63}$$

$$U = (B + MU)(B + MU). \tag{64}$$

With the notation $Y := B + MU$, Equation (64) becomes $U = Y^2$. The problem can then be reformulated. As $B + MY^2 = B + MU = Y$, the problem becomes one of finding Y with $MY^2 - Y + B = 0$. We solve this equation using the eigenvalue approach (see, e.g., Higham and Kim, 2000; for a particularly simple description, see McCandless, 2016). After finding Y, U and S can be calculated as $U = Y^2$ and $S = (I - (I + B + MU)M)^{-1}(I + B + MU)L$.

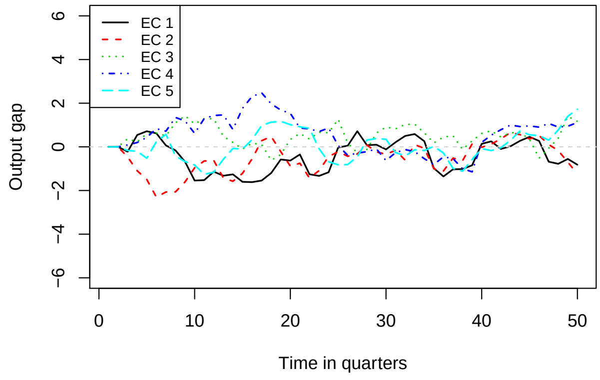
E Appendix (for Online Publication): Graphs of the Additional Macroeconomic Model

E.1 Economic Behavior in Homogeneous Economies and Currency Unions (Additional Model)

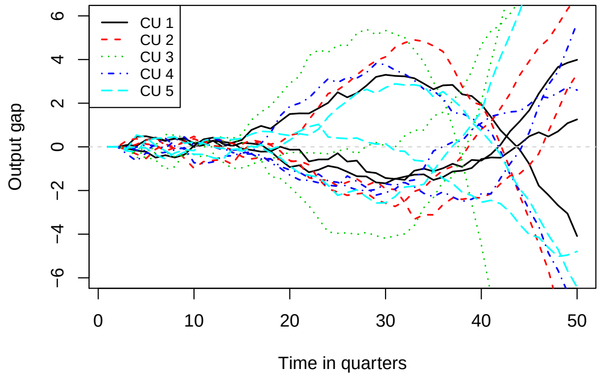
Figure 29 illustrates economic bahavior in 5 homogeneous economies (left) and currency unions (right).

Figure 30 shows economic behavior in 5 currency unions with parameter $m = 0.5$ (instead of $m = 0.25$).

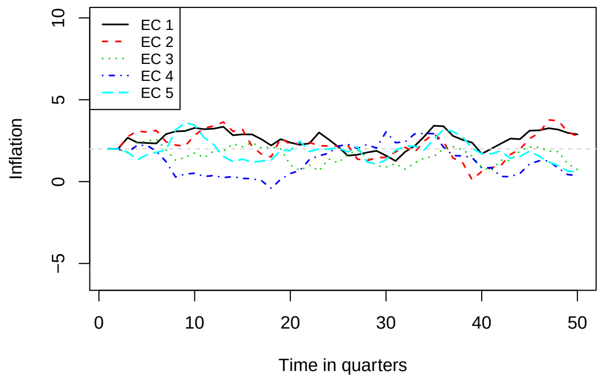
Figure 31 shows the stability of economic behavior for different values of a_2 and b_2 in currency unions.



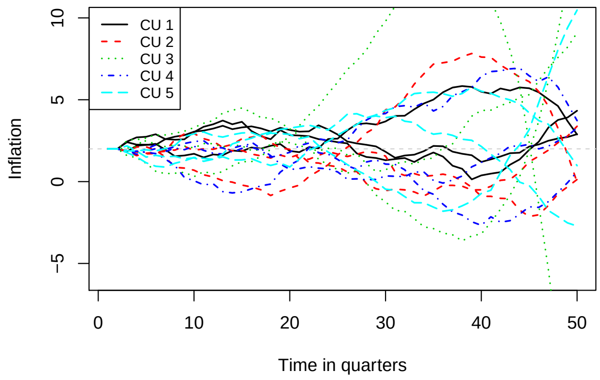
(a) Output gap, homogeneous economies



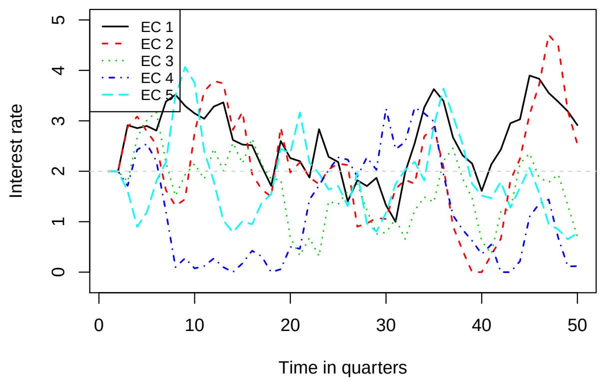
(b) Output gap, currency unions



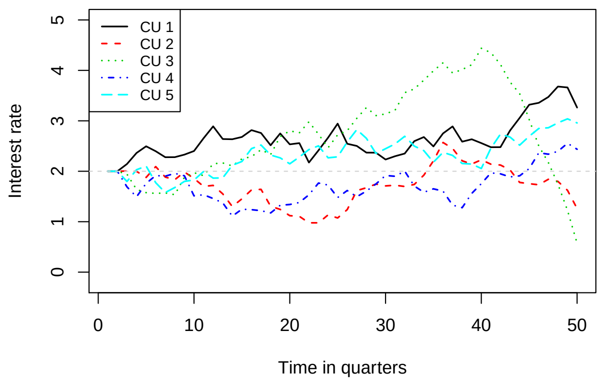
(c) Inflation, homogeneous economies



(d) Inflation, currency unions



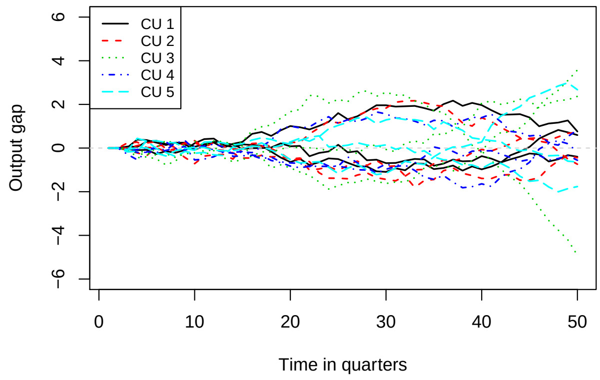
(e) Interest rate, homogeneous economies



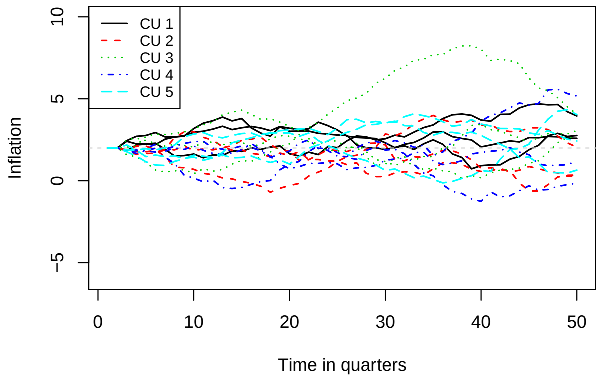
(f) Interest rate, currency unions

Figure 29: Economic Behavior in Homogeneous Economies and Currency Unions (Additional Model)

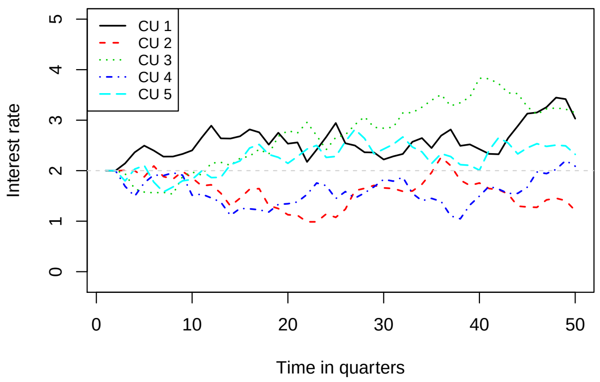
Notes: $m = 0.25$ (for CU), $a_3 = 0.05$ (for CU), $a_1 = 0.5$, b_1=0.5, a_2=-0.5, b_2=0.1, c_1=1.5, c_2=0.5, sd =0.25 (for CU), sd =$0.25\sqrt{3}$ (for HE).



(a) Output gap, currency unions



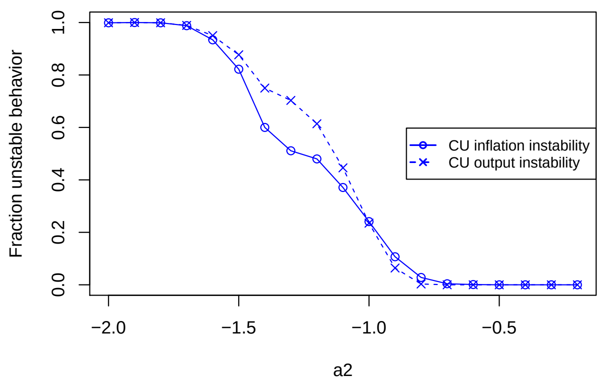
(b) Inflation, currency unions



(c) Interest rate, currency unions

Figure 30: Economic Behavior in Currency Unions with $m = 0.5$ (Additional Model)

Notes: $m = 0.5$, $a_3 = 0.05$, $a_1 = 0.5$, b_1=0.5, a_2=-0.5, b_2=0.1, c_1=1.5, c_2=0.5, sd =0.25.



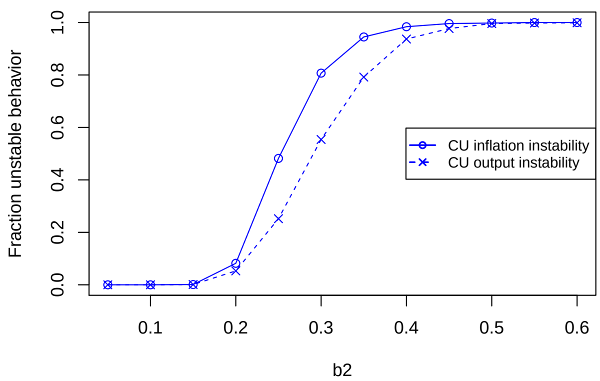
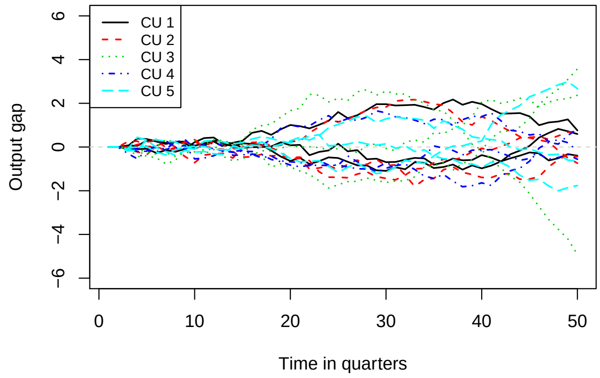


Figure 31: Instability of Currency Unions Depending on a_2 and b_2 (Additional Model)

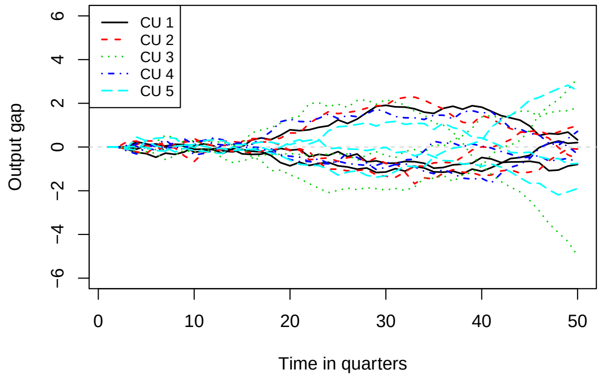
Notes: $m = 0.5$, $a_3 = 0.05$, $a_1 = 0.5$, b_1=0.5, a_2=-0.5, b_2=0.25, c_1=1.5, c_2=0.5, $sd = 0.25$.

E.2 Extreme Monetary Policy (Additional Model)

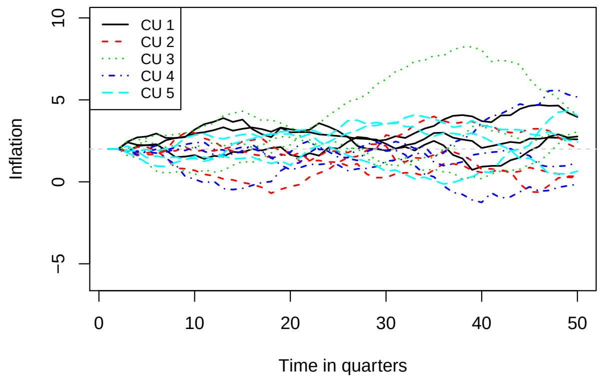
Figure 32 illustrates economic bahavior in 5 currency unions with standard ($c_1 = 1.5$ and $c_2 = 0.5$) and increased reaction coefficients ($c_1 = 6$ and $c_2 = 2$) in the Taylor rule.



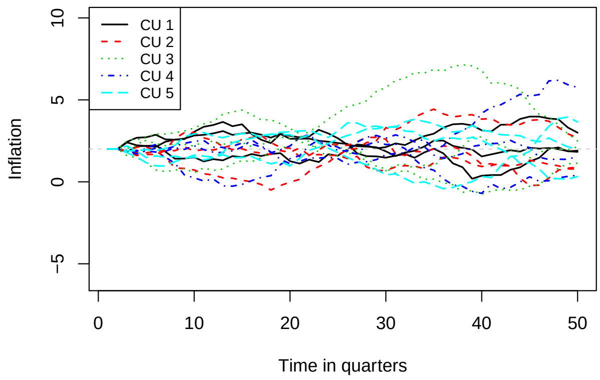
(a) Output gap, $c_1 = 1.5$ and $c_2 = 0.5$



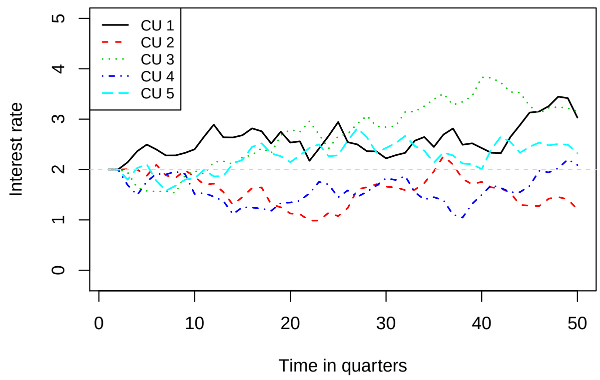
(b) Output gap, $c_1 = 6$ and $c_2 = 2$



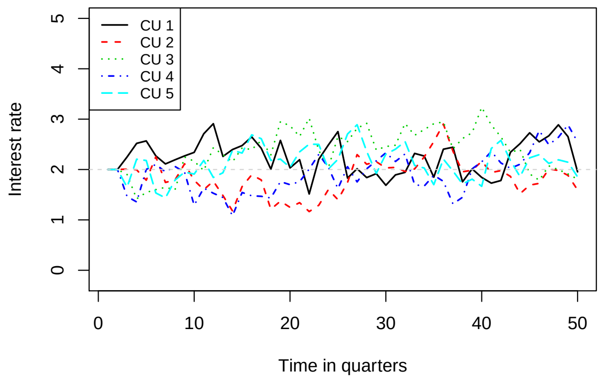
(c) Inflation, $c_1 = 1.5$ and $c_2 = 0.5$



(d) Inflation, $c_1 = 6$ and $c_2 = 2$



(e) Interest rate, $c_1 = 1.5$ and $c_2 = 0.5$



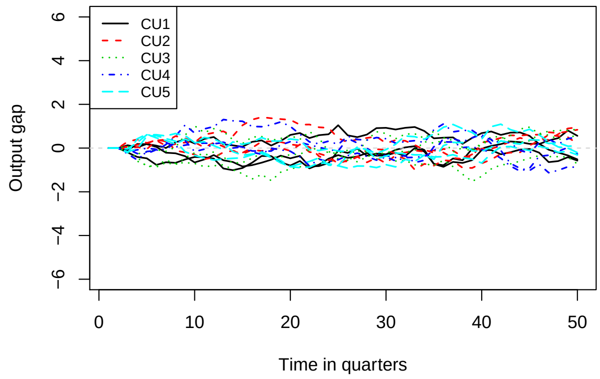
(f) Interest rate, $c_1 = 6$ and $c_2 = 2$

Figure 32: Economic Behavior in Currency Unions with Taylor Rule coefficients $c_1 = 1.5$ and $c_2 = 0.5$ on the left hand side and $c_1 = 6$ and $c_2 = 2$ on the right hand side (Additional Model)

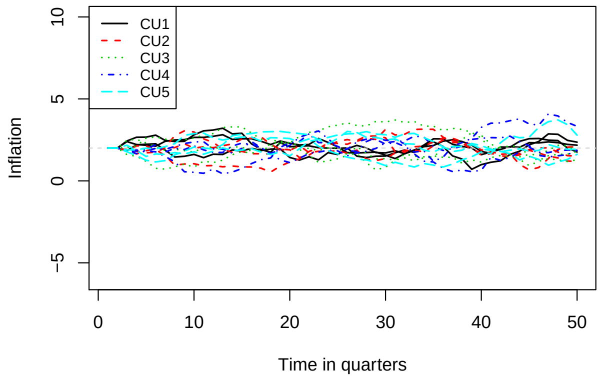
Notes: $m = 0.5$, $a_3 = 0.05$, $a_1 = 0.5$, b_1=0.5, a_2=-0.5, b_2=0.1, sd =0.25, $c_1 = 1.5$ (left), $c_2 = 0.5$ (left), $c_1 = 6$ (right), $c_2 = 2$ (right).

E.3 Artificial Exchange Rate Regime (Additional Model)

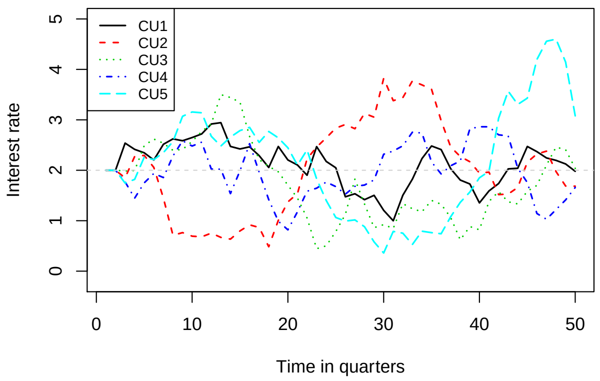
Figure 33 shows economic behavior in 5 "currency unions" modeled as an artificial fixed exchange rate regime.



(a) Output gap, $c_1 = 1.5$ and $c_2 = 0.5$



(b) Inflation, $c_1 = 1.5$ and $c_2 = 0.5$



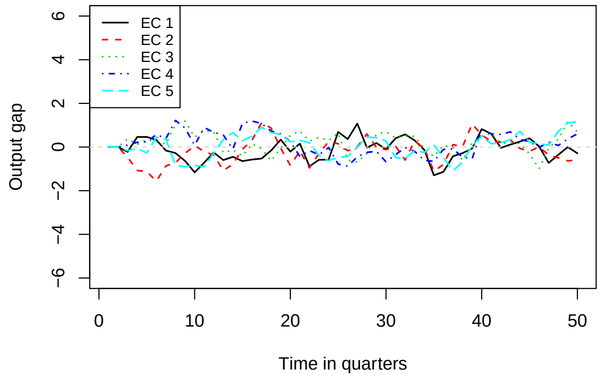
(c) Interest rate, $c_1 = 1.5$ and $c_2 = 0.5$

Figure 33: Economic Behavior under an Artificial Fixed Exchange Rate Regime (Additional Model)

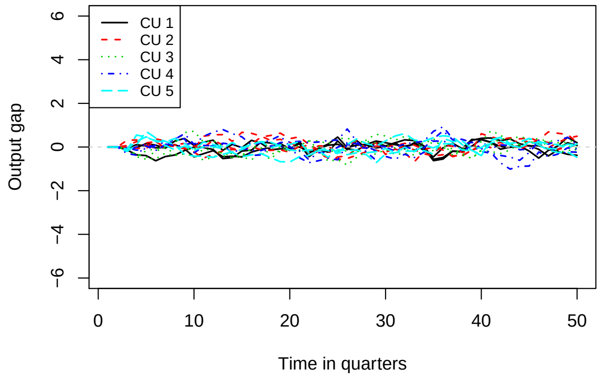
Notes: $m = 0.5$, $a_3 = 0.05$, $a_1 = 0.5$, b_1=0.5, a_2=-0.5, b_2=0.1, sd =0.25, $c_1 = 1.5$, $c_2 = 0.5$.

E.4 Rational Expectations (Additional Model)

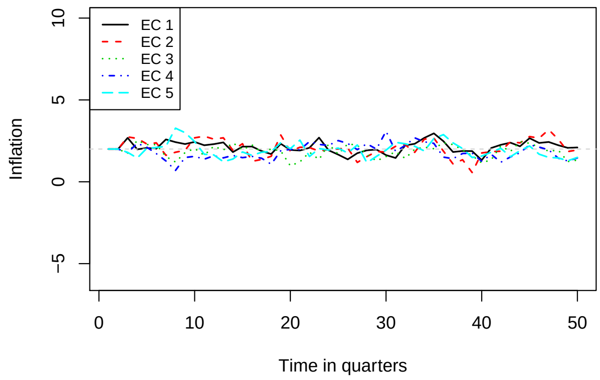
Figure 34 illustrates economic bahavior in 5 homogeneous economies (left) and currency unions (right) under rational expectations.



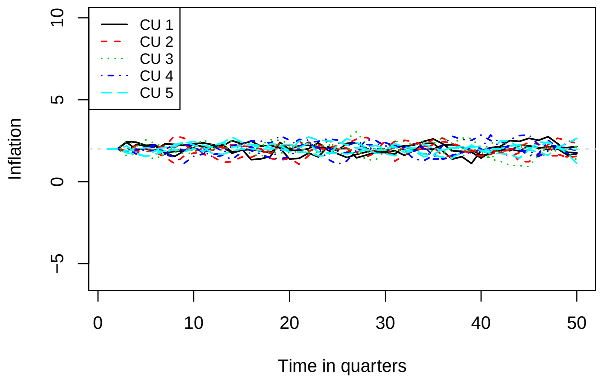
(a) Output gap, homogeneous economies



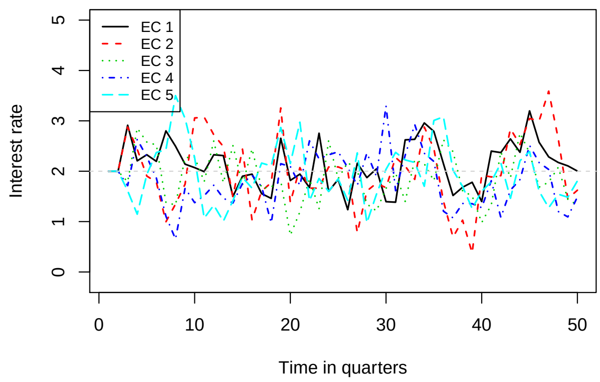
(b) Output gap, currency unions



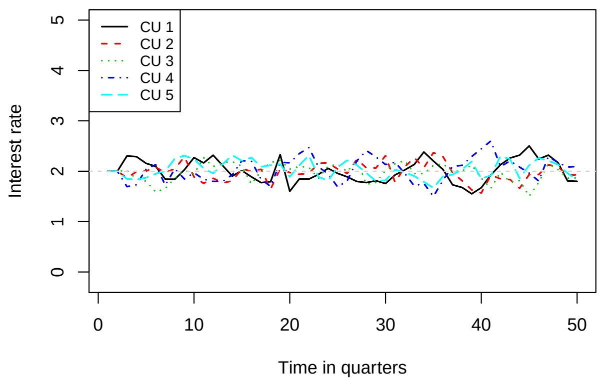
(c) Inflation, homogeneous economies



(d) Inflation, currency unions



(e) Interest rate, homogeneous economies



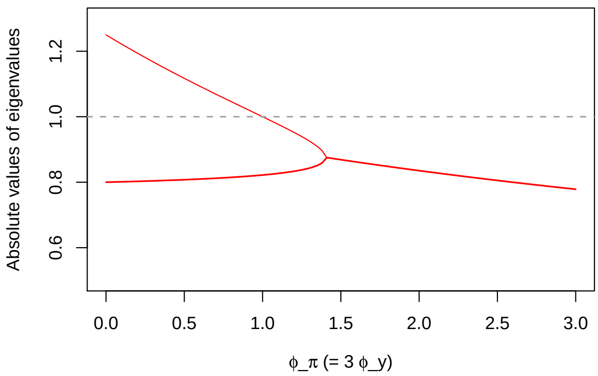
(f) Interest rate, currency unions

Figure 34: Economic Behavior in Homogeneous Economies and Currency Unions under Rational Expectations (Additional Model)

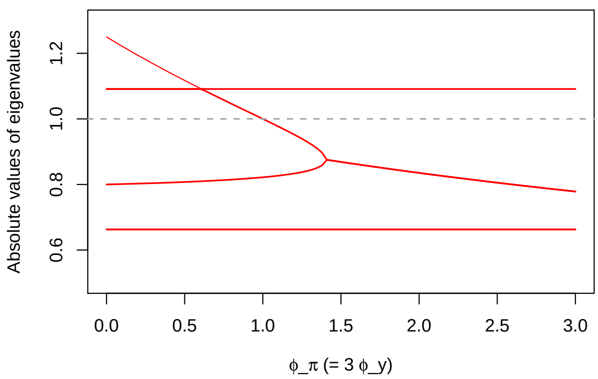
Notes: $m = 0.5$, $a_3 = 0.05$, $a_1 = 0.5$, b_1=0.5, a_2=-0.5, b_2=0.1, c_1=1.5, c_2=0.5, sd =0.25 (for CU), sd =$0.25\sqrt{3}$ (for HE).

E.5 Mathematical Stability of the Steady States under Naive Expectations (Additional Model)

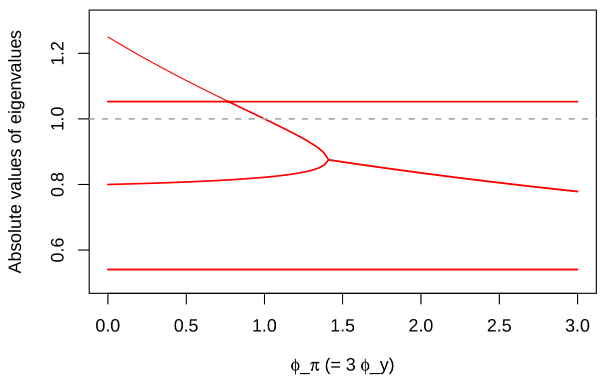
Figure 35 show the absolute values of the eigenvalues in a currency union and in a homogeneous closed economy under naive expectations. One can see that also in the additional model, more active monetary policy leads to more stable economic behavior in the homogeneous economy (Figure 35a). Again, monetary policy does not have a great influence on the stability of the steady states in a currency union. The greatest absolute value of the eigenvalues doe not decrease with monetary policy (after a short initial phase; e.g., Figure 35b). Increasing economic integration via m, however, leads to a decrease of the greatest absolute eigenvalue (Figures 35b and 35c).



(a) Homogeneous economy



(b) $m = 0.25$



(c) $m = 0.5$

Figure 35: Absolute Values of Eigenvalues in a Homogeneous Economy and in a Currency Unions of three Countries (Naive Expectations, Additional Model)

Notes: $m = 0.25$ (bottom left), $m = 0.5$ (bottom right), $a_3 = 0.05$, $a_1 = 0.5$, b_1=0.5, a_2=-0.5, b_2=0.1, $c1 = 1.5$, $c2 = 0.5$.

F Appendix (for Online Publication): References in the Online Appendix

Higham, N. J. and Kim, H. M. (2000). Numerical analysis of a quadratic matrix equation. *IMA Journal of Numerical Analysis*, 20(4):499–519.

McCandless, G. T. (2016). Solving matrix quadratic equations. *Lecture Notes*. Available at: http://www.ucema.edu.ar/u/gtm/Course_Material/macroeconomia_2_de_MEA/8a_solving_matrix_quadratic_equations.pdf (accessed on October 17, 2016).